A Structural Model for Capitalization Rate[1]

A Research Report to Real Estate Research Institute (RERI)

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Executive Summary

Capitalization rate (cap rate) is a critical variable in commercial real estate valuations. To form beliefs of cap rate under a certain market situation, investors look into the required return of property investment and expectations of rental income. For example, when required return is high and expected rental growth is weak, investors apply a high cap rate to value a property because they know cash flows of the property is not likely to grow and they need deep discount on future cash flows. Therefore, cap rate is a compact indicator that expresses investors' expectations about investment return and/or rental growth. In this project, we intend to tease out those implied expectations through a cap rate anatomy. In addition, we model dynamics of those expectations so that investors can use our model to predict cap rates based on past information of return and rental growth.

We build a dynamic cap rate model that links cap rate to multi-period expected returns and rental growths. In our model, cap rate is the weighted average of all future "growth-adjusted discount rates" and those "growth-adjusted discount rates" can have substantial variations over time. We estimate our structural model with Kalman filter. Results show that cap rate is significantly related to both future expected return and expected rental growth. Therefore, overlooking either component will lead to a biased assessment regarding cap rate movements. Further, investors weigh expectations about return and rental growth in different future periods differently. They place more weights on the nearer future. Third, expected return is significantly mean-reversion. Because of this mean-reverting property in expected return and because of the positive relationship between cap rate and expected return, investors should look for upward sloping cap rates if the current and past returns are low. Finally, a comparison between our structural estimates and an VAR estimates shows that our structural model captures the relationship between cap rate and NOI growth that is not captured by reduced-form models.

1. Introduction

Capitalization rate (cap rate), measured as the ratio of net rental income to property value is a critical variable in valuations of commercial real estate and Real Estate Investment Trusts (REITs). Investors use it as a denominator to find out the value of a property when they know the net operating income (NOI) of that property. To form beliefs of cap rate under a certain market situation, investors look into the required return of property investment and expectations of rental income. For example, when required return is high and expected rental growth is weak, investors apply a high cap rate to value a property because they know cash flows of the property is not likely to grow and they need deep discount on future cash flows. From another perspective, cap rate is a compact indicator of the property market. It expresses investors' expectations about investment return and/or rental growth. For example, a low cap rate implies that investors are optimistic about the market, either applying low discount on future income or possessing strong expectations about rental growth. In fact, the textbook relationship between cap rate c , required return r and expected rental growth g is $c = r - g$. However, as shown in figure 1, $r - g$ does not coincide with c even in terms of time trends, not to mention the levels. Therefore, an investigation of cap rate and its determinants is in order.

The purpose of this study is to conduct an anatomy of cap rate. We intend to tease out the implied expectations in cap rate about property return and rental growth. These expectations include magnitude and speed of change in those variables. Further, we model dynamics of those expectations and structures of interactions between cap rate and those expectations so that reversely investors can use our model to predict future cap rates based on past information of return and rental growth.

A number of studies have been undertaken on cap rate. For example, Froland (1987), Evans (1990), Ambrose and Nourse (1993), and Jud and Winkler (1995) regress cap rate on bond market and stock market returns to seek relationship between cap rate and capital market returns. More recent studies such as Sivitanidou and Sivitanides (1999), Sivitanides, Southard, Torto and Wheaton (2001), and Hendershott and MacGregor (2005) incorporate rental growth into analysis to examine not only the relationship between cap rate and return, but also the relationship between cap rate and rental growth. However, those studies are based on the relationship $c = r - g$ derived from the static Gordon (1962) model, which assumes that looking forward at each point in time both expected return and future rental growth are constant. An important feature that distinguishes our study from those existing studies is that we turn to a rational expectations model. By stating "rational expectations", we mean: 1) investors expect return and rental growth will change from time to time rather than stay constant. Particularly, we believe those two variables are both mean-reverting in the long run; 2) cap rate should incorporate investors' expectations about returns and rental growths in multiple periods and given that expected return and rental growth will fluctuate, the relationship between cap rate, return and rental growth will not be as simple as $c = r - g$. In that regard, we build a dynamic cap rate model following Campbell and Shiller (1989). This yields a structural model that links cap rate to multi-period expected returns and rental growths. In our model, cap rate is the weighted average of all future "growth-adjusted discount rates" and those "growth-adjusted discount rates" can have substantial variations over time.

We further put our structural model into state space form and estimate the structural model using Kalman filter. Reduced-form estimations of the relationships between cap rate, return and growth can yield very different results according to various model specifications as evidenced in the literature. Using sophisticated econometric modeling, we seek to adduce a "best fit" based on our structural model which more accurately describes the interactions among the aforementioned variables. More importantly, our structural estimates should provide us a better understanding of the economic relationships rather than the statistical relationships between cap rate, return and rental growth as provided by linear regressions.

Our estimation of the dynamic cap rate model is mainly based on a rich data set of quarterly observations on *ex-post* cap rate, property return, NOI growth and vacancy rate from the National Council of Real Estate Investment Fiduciaries (NCREIF). We also utilize information such as *ex-ante* cap rate from the Real Estate Research Corporation (RERC) cap rate survey and risk-free interest rate from the Federal Reserve.

Several interesting results arise from our empirical work with the NCREIF *ex-post* data. First, estimation results show that cap rate is significantly related to both future expected return and expected rental growth. This finding suggests that overlooking either component will lead to a biased assessment regarding cap rate movements. Expected return is positively related to cap rate while rental growth is negatively related to cap rate, consistent with the common wisdom that low cap rates imply expectations of higher rent and/or diminishing returns. Second, investors weigh expectations about return and rental growth in different future periods differently. They place more weights on the nearer future. Third, expected return is significantly mean-reverting according to our estimation. This reflects the fact that commercial real estate market is cyclical but supply/demand adjustments always force the market towards equilibrium in the long run. An implication of this result is: Because of this mean-reverting property in expected return and because of the positive relationship between cap rate and expected return, investors should look for upward sloping cap rates if the current and past returns are low. Fourth, the volatilities of expected return and rental growth are substantially smaller than those of the observed return and rental growth. This could be because investors are sluggish in adjusting their expectations or they have rational expectations about the cyclicality of the real estate market and thus smooth their expectations about return and rental growth. Finally, prediction errors of NOI growth and cap rate within our model decrease over time from 1980s to the most present, suggesting that NOI growth and cap rates are becoming more predictable.

We also compare our structural estimates with those from a reduced-form VAR model. A VAR estimation by Shilling and Sing (2007) shows that cap rate is positively related to *past* property excessive return, which contradicts the theoretical prediction. We thus estimate a VAR model with the information set we use in our structural model estimation. Our estimates show a negative relationship between cap rate and property return. Given that property return is again shown to be mean-reverting, we infer a positive relationship between cap rate and *future* property return, consistent with the theory. However, the VAR estimates show that there is no significant relationship between cap rate and NOI growth, either statistically or economically. This result is actually similar to the reduced-form estimate by Plazzi, Torous and Valkanov (2008), which finds that cap rate is not useful in predicting NOI growth. The bottom line here is our structural model captures the relationship between cap rate and NOI growth that is not captured by reduced-form models.

Our estimation with the RERC *ex-ante* cap rates shows similar results with those of the NCREIF *ex-post* data. Estimations by property type find variations in the mean-reverting speed of expected returns. Apartments have the highest speed in terms of return mean-reversion. This may be because adjustments in supply/demand of apartments are easier than those of office, retail and industrial properties. Industrial spaces have the lowest mean-reverting speed in expected return possibly because investors know many industrial spaces are customizes and thus it is hard to adjust inventory, which finally affects return. However, the relationships between cap rate and "growth-adjusted discount rates" are very similar across different property types.

The rest of the report is organized as follows: the next section reviews related literature and explains how our research fits into the literature and constitutes an extension of previous research. Sections 3 and 4 present our theoretical model and empirical approach, respectively. Section 5 reports data and estimation results. Conclusions and discussions are in a final section.

2. Related Literature

Cap rate has long been a subject of research. Froland (1987) compares cap rate movements with yields of other assets trading in the capital market. He finds strong correlation of cap rate with mortgage rates, ten-year bond rates and stock market earnings/price ratio. Evans (1990) empirically investigates the time series properties of commercial real estate cap rate using an ARIMA model and particularly studies its linkage with stock market earnings/price ratio with a transfer function analysis. He finds that the earnings/price ratio affects cap rate with a one quarter lag, and thus concludes that real estate investors respond slowly to general business cycle and monetary conditions. Ambrose and Nourse (1993) study the relationship between cap rate and capital market variables that are both from the debt side and from the equity side based on the WACC argument, which states that the overall cost of capital is the weighted average of debt cost and equity cost. However, using a seemingly unrelated regression, they find that cap rate is not closely tied to either the S&P 500 earnings/price ratio or bond market risk premium. Jud and Winkler (1995) follow the same approach and regress the excess cap rate (cap rate minus the risk free rate) on debt market excess return (corporate debt minus treasury rate) and excess equity return (S&P 500 return minus the treasury rate). They report that both the debt market excess return and equity market excess return are significantly correlated with excess cap rate. They also find significant lags for excess cap rate to respond to changes in capital market spreads.

Based on a simple static Gordon (1962) model, cap rate equals to the difference between property return and rental growth. Obviously, the aforementioned studies only put emphasis on the first part – the property return, and its relations with broader capital market returns. Sivitanidou and Sivitanides (1999) study both of those two components. Using a reduced form model, they regress office property cap rate on both the capital market variables such as inflation and yield slope, and location variables such as MSA dummies and MSA office vacancy rate that serve as real estate space market indicators. They find that location variables play a pivotal role in determining cap rate, while national capital market features have a lesser role. Sivitanides, Southard, Torto and Wheaton (2001) study the determinants of appraisal based cap rates, trying to reveal how cap rates move with the opportunity cost of capital and whether they reflect realistic expectations about future income growth and risk. Interestingly, they find that cap rates do move exactly as price/earnings ratios do, but only if appraisers form expectations about future income growth by looking myopically backward. Because of this, they conclude that appraisal based cap rate is forecast-able.

A more recent study by Hendershott and MacGregor (2005) also follows the simple Gordon model to look at the importance of capital market variables and rental growth on cap rate in the United Kingdom. They assume there is a long term equilibrium relationship between current cap rate, current capital market returns (stock market return and risk free rate), and current rental growth rate. However, in the short term, the relationship can deviate from the long term equilibrium. They use an error correction model to study the adjustments of cap rates in response to rental growth change and capital market return change. Proxies for rental growth rate are used. They find that cap rate is linked to stock market dividend/price ratio and expected rental growth rate in the expected manner. However, their model focuses on the current states of the variables and thus is a static model. It does not take into consideration of investors' expectations about future return and rental growth.

While carrying out this study, we notice that there are two ongoing studies that go beyond the static Gordon (1962) model framework. Plazzi, Torous and Valkanov (2008) and Shilling and Sing (2007) both apply the Campbell and Shiller (1989) log linearization to cap rate, expressing cap rate as a function of multi-period expected return and rental growth. However, Plazzi, Torous and Valkanov (2008) focus on whether cap rate can forecast expected return and rental growth. In their empirical work, they leave the expected return and expected rental growth dynamics unspecified and run two separate regressions of cap rate on expected return and of cap rate on rental growth. Therefore, the interactions between cap rate, return and rental growth are not fully explored in their study. Shilling and Sing (2007) use a VAR model to study the interdependence of cap rate, property return and rental growth. Their VAR model is a reduced-form simplification of the relationship among aforementioned variables they obtain from the Campbell and Shiller (1989) log linearization.

Our study differs from existing research in significant ways. First, different from most studies that are based on the static Gordon (1962) model, we develop a dynamic cap rate model. We fully incorporate investors' expectations about multi-period future returns and rental growths in forming cap rate, and incorporate possible time variations in return and rental growth. Second, with advanced time series techniques we are able to estimate our model based on the structural form of our theoretical model. This differentiates our study from Plazzi, Torous and Valkanov (2008) and Shilling and Sing (2007). In so doing, we seek to adduce a “best fit” based on our structural model which theoretically describes the interactions among cap rate, return and rental growth more accurately. In fact, we compare our structural estimates with those of a reduced-form VAR model in this study. Last but not least, we estimate the dynamics of expected return and rental growth based on a mean-reverting structure together with the relationship between cap rate, return and rental growth. This completes our dynamic cap rate model and we can form predictions of cap rate with the system we estimate.

3. The Dynamic Cap Rate Model

To better see the difference between a dynamic cap rate model and a static cap rate model, let’s first review the static Gordon (1962) model.

Suppose P_t is the value of the commercial property at time t , and D_t is the cash flow (e.g. NOI) of the property during the period $\left[t, t+1\right)$, and g_{t+1} is the NOI growth during the period $\left[t+1, t+2\right)$. Denote r_{t+1} as the return of the commercial property during the period $\left[t, t+1\right)$. Using a discounted cash flow (DCF) approach, the value of the property is:

$$P_t = \frac{D_t}{1+r_{t+1}} + \frac{D_t\left(1+g_{t+1}\right)}{\left(1+r_{t+2}\right)^2} + \frac{D_t\left(1+g_{t+2}\right)^2}{\left(1+r_{t+3}\right)^3} + \cdots \qquad (1)$$

Assuming that g_{t+1} and r_{t+1} stay constant, we can easily simplify equation (1) and get:

$$P_t = \frac{D_t}{r-g} \qquad (2)$$

By definition, $c_t = \frac{D_t}{P_t}$ is capitalization rate. Immediately we obtain the textbook relationship between cap rate, return and rental growth:

$$c = r - g \qquad (3)$$

Notice that equation (3) implies that cap rate is a compact measure for real estate valuation and that there is nothing made explicit in DCF that is not implicit in the cap rate.

However, equation (3) is based on assumptions that both future required returns and expected rental growths are constant, which apparently do not hold in reality.

A very important purpose of a dynamic cap rate model is to relax these assumptions. We start with the Euler equation:

$$E_t\left[m_{t+1}\cdot R_{t+1}\right]=1 \tag{4}$$

Here $R_{t+1}=\frac{P_{t+1}+D_{t+1}}{P_t}$ is the gross return of the commercial property during the period $\left[t,t+1\right)$. m_{t+1} is the stochastic discount factor $m_{t+1}=\beta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}$.

Ex-post, i.e. we observe P_{t+1}, P_t and D_{t+1}, we study $R_{t+1}=\frac{P_{t+1}+D_{t+1}}{P_t}$. Assuming $R_{t+1}>0$ for sure, we can rewrite the equation as:

$$\log R_{t+1}=\log\left(P_{t+1}+D_{t+1}\right)-\log P_t \tag{5}$$

Following Campbell and Shiller (1989), we linearize the above equation and get:

$$\begin{aligned}\log R_{t+1}&\simeq k+\rho\log P_{t+1}+(1-\rho)\log D_{t+1}-\log P_t\\&\simeq k-\rho\log\frac{D_{t+1}}{P_{t+1}}+\log\frac{D_t}{P_t}+\log\frac{D_{t+1}}{D_t}\end{aligned} \tag{6}$$

Where k is a constant, ρ is the long-run average ratio of price and NOI to price, i.e. $\rho=\overline{P_t/\left(P_t+D_t\right)}$, which is close to but a little smaller than 1.

Notice that $C_t=\frac{D_t}{P_t}$ is the capitalization rate, $G_{t+1}=\frac{D_{t+1}}{D_t}$ is the rental growth rate, and R_{t+1} is property gross return. We use small letters to denote the log of all above variables, then we have:

$$r_{t+1}\simeq k-\rho c_{t+1}+c_t+g_{t+1} \tag{7}$$

Here r_{t+1} is log return of the commercial property during the period $[t, t+1)$, c_{t+1} and c_t are log of cap rates in periods t and $t+1$, and g_{t+1} is the log growth rate of NOI during $[t, t+1)$.

Ex-ante, we impose expectations on all terms, which yields:

$$Er_{t+1} \simeq k - \rho Ec_{t+1} + c_t + Eg_{t+1} \tag{8}$$

This equation links cap rate and expected property return and expected rental growth rate. To better see the economic meaning of this equation. We solve for c_t by iterating forward, and get:

$$c_t \simeq E_t \sum_{j=0}^{\infty} \rho^j \left(r_{t+j} - g_{t+j} \right) - \frac{k}{1-\rho} \qquad (9)^4$$

We see that the cap rate is the weighted average of all future "growth-adjusted discount rates" and that the future "growth-adjusted discount rates" can have substantial variations over time. It is different from the static Gordon (1962) model in that in this model investors form their cap rate in valuating properties by taking into consideration of all future expected return and rental growth rate. However, the model includes the static Gordon (1962) model as a special case. If r_t and g_t are constant over time, i.e. we are in the steady state, as shown in Campbell and Shiller (1989), we have $\rho = \frac{P}{P+D}$, $k = -\log \rho - (1-\rho)c$ and we can easily verify that $c = r - g$, which is exactly the result of the static Gordon (1962) model.

To see how the dynamic cap rate model can be estimated, let's work on equation (5). To avoid confusion, let's denote $\tilde{r}_{t+1}$ and $\tilde{g}_{t+1}$ as the expected return and expected rental growth rate (both in log forms) of the commercial property during the

[4] Here we impose the terminal condition that $\lim_{j \to \infty} \rho^j c_{t+j} = 0$, which says that the cap rate c_{t+j} does not explode. This is apparently the case in reality.

period $\left[t, t+1\right)$ when we are standing at time t. $\tilde{c}_{t+1}$ is the expected cap rate for the period $t+1$. From equation (5), we have:

$$\rho\tilde{c}_{t+1} - \tilde{c}_t + \tilde{r}_{t+1} - \tilde{g}_{t+1} - k \simeq 0 \qquad (10)$$

Assuming the approximation error is random with respect to time, we have the following model that is estimate-able:

$$\rho\tilde{c}_{t+1} - \tilde{c}_t + \tilde{r}_{t+1} - \tilde{g}_{t+1} - k + \varepsilon_t = 0, \quad \varepsilon_t \sim N\left(0, \sigma_c^2\right) \qquad (11)$$

This is a nice simple autoregressive model for the expected cap rate $\tilde{c}_t$.

Now let's consider the dynamics of the expected return and expected rental growth rate of a commercial property. In the literature, it is usually assumed that property value follows a Geometric Brownian Motion with a constant expected return (see, for example, Schwartz and Torous 1989):

$$dB_t = \left(r - b\right) B_t dt + \sigma_B B_t dW_t \qquad (12)$$

where r is the expected return of the property, and b is the payout ratio. We relax the assumption of constant expected return to model r_t to be time-varying but following a mean-reverting process:

$$d\tilde{r}_t = \kappa\left(\bar{r} - \tilde{r}_t\right) dt + \sigma_{\tilde{r}} dW_t \qquad (13)$$

where $\bar{r}$ is the long term mean of expected return, $\sigma_{\tilde{r}}$ is the volatility of expected return and κ measures the mean-reverting speed. This specification reflects the supply-demand equilibrium in the commercial space market: developers and investors adjust their supply and demand of spaces according to the abnormal return and thus the expected property return fluctuates around a long term mean.

In discrete time, we have:

$$\tilde{r}_{t+1} = \left(1 - \kappa\right)\tilde{r}_t + \kappa\bar{r} + \sigma_{\tilde{r}}\varepsilon_t^1, \varepsilon_t^1 \sim N\left(0,1\right) \qquad (14)$$

By the same token, we have:

$$\tilde{g}_{t+1} = \left(1 - \lambda\right)\tilde{g}_t + \lambda\bar{g} + \sigma_{\tilde{g}}\varepsilon_t^2, \varepsilon_t^2 \sim N\left(0,1\right) \qquad (15)$$

Taking equations (9), (12) and (13) together, we have the following system:

$$\tilde{r}_{t+1} = (1-\kappa)\tilde{r}_t + \kappa\bar{r} + \sigma_{\tilde{r}}\varepsilon_t^1, \varepsilon_t^1 \sim N(0,1)$$
$$\tilde{g}_{t+1} = (1-\lambda)\tilde{g}_t + \lambda\bar{g} + \sigma_{\tilde{g}}\varepsilon_t^2, \varepsilon_t^2 \sim N(0,1) \tag{16}$$
$$\tilde{c}_{t+1} + \gamma\tilde{r}_{t+1} - \gamma\tilde{g}_{t+1} = \gamma\tilde{c}_t + \gamma k + \sigma_{\tilde{c}}\varepsilon_t^3, \ \ \varepsilon_t^3 \sim N(0,1)$$

Here $\gamma = \frac{1}{\rho}$, and ε_t^1, ε_t^2 and ε_t^3 are assumed to be uncorrelated. This is the full dynamic cap rate model we present.

4. Empirical Methodology

To empirically estimate the system in equation (16), we need data on expected cap rate $\tilde{c}_t$, expected return $\tilde{r}_t$ and expected rental growth rate $\tilde{g}_t$. However, we generally do not observe these data *ex-ante*. Alternatively, we observe *ex-post* cap rates from real estate transactions/valuations. For example, NCREIF provides the data series on the realized cap rate. We can decompose the realized cap rate as the sum of the expected cap rate and a random shock, which leads to:

$$c_t = \tilde{c}_t + \eta_c\varepsilon_t^4, \varepsilon_t^4 \sim N(0,1) \tag{17}$$

in which c_t is the realized cap rate and η_c is the volatility of the shock.

For property return, we can again utilize the NCREIF observed return and assume:

$$R_t = \tilde{r}_t + \eta_R\varepsilon_t^5, \varepsilon_t^5 \sim N(0,1) \tag{18}$$

where R_t is the NCREIF total return in log form.

In order to help identify the model, we also introduce the relationship between property return and risk-free rate. Real estate as an investment instrument is competing funds with other investment instruments such as stocks and bonds in the capital market. Therefore, it's reasonable to expect that the expected property return is the expected risk free rate plus a risk premium, which we assume is normally distributed w.r.t. time:

$$\tilde{r}_t = \mu_t + \tilde{r}_t^f = \mu + \tilde{r}_t^f + \eta_r\varepsilon_t^6, \varepsilon_t^6 \sim N(0,1) \tag{19}$$

Further, every month we observe realized risk-free rate, which is expected risk free rate plus a shock:

$$r_t^f = \tilde{r}_t^f + \eta_f \varepsilon_t^7, \varepsilon_t^7 \sim N(0,1) \quad (20)$$

Now, we can simplify (19) and (20) and obtain a relationship between observed risk-free rate and expected property return:

$$r_t^f = \theta + \tilde{r}_t + \eta_{r^f} \varepsilon_t^8, \varepsilon_t^8 \sim N(0,1) \quad (21)$$

Theoretically, the expected rental growth rate can be extracted from the realized rental growth rate. However, the NOI growth rates reported by NCREIF are notoriously volatile due to the accounting noise, such as accounting lags, and ad hoc treatment on capital expenditure[5]. To mitigate the measurement problem, we take two measures. First, we use the four quarter moving average of observed NOI growths as a measurement variable for expected NOI growth and assume:

$$g_t = \tilde{g}_t + \eta_g \varepsilon_t^9, \varepsilon_t^9 \sim N(0,1) \quad (22)$$

where g_t is the four quarter moving average of observed NOI growths and $\tilde{g}_t$ is expected NOI growth.

Second, we use vacancy rates to extract information about expected rental growth. In practice, vacancy has the first order impact on rental growth and investors weigh vacancy and its trend heavily in forming their rental growth expectations. We model the vacancy-rental growth relationship as:

$$o_t = \alpha + \beta \tilde{g}_t + \eta_o \varepsilon_t^{10}, \varepsilon_t^{10} \sim N(0,1) \quad (23)$$

where o_t is the observed change in occupancy rate (=1-vacancy rate) at time t . α is a constant and β is the correlation coefficient between occupancy change and expected rental growth.

We now have a structural time series model set up as:

[5] In figure 6, we show that the NOI growth rates provided by NCREIF are unreasonably volatile.

$$
\begin{aligned}
c_t &= \tilde{c}_t + \eta_c \varepsilon_t^4 \\
R_t &= \tilde{r}_t + \eta_R \varepsilon_t^5 \\
r_t^f &= \theta + \tilde{r}_t + \eta_{rf} \varepsilon_t^8 \\
g_t &= \tilde{g}_t + \eta_g \varepsilon_t^9 \\
o_t &= \alpha + \beta \tilde{g}_t + \eta_o \varepsilon_t^{10} \\
\tilde{r}_{t+1} &= (1-\kappa)\tilde{r}_t + \kappa \bar{r} + \sigma_{\tilde{r}} \varepsilon_t^1 \\
\tilde{g}_{t+1} &= (1-\lambda)\tilde{g}_t + \lambda \bar{g} + \sigma_{\tilde{g}} \varepsilon_t^2 \\
\tilde{c}_{t+1} + \gamma \tilde{r}_{t+1} - \gamma \tilde{g}_{t+1} &= \gamma \tilde{c}_t + \gamma k + \sigma_{\tilde{c}} \varepsilon_t^3
\end{aligned}
\tag{24}
$$

During each time period, we observe the realized cap rate c_t, property return R_t, risk-free rate r_t^f, NOI growth rate g_t and occupancy rate change o_t. These observable variables are functions of the state variables, expected cap rate $\tilde{c}_t$, expected property return $\tilde{r}_t$ and expected rental growth rate $\tilde{g}_t$, which evolve following autoregressive processes. As econometrician, we can use time series data on c_t, R_t, r_t^f, g_t and o_t to infer the processes of $\tilde{r}_t$ and $\tilde{g}_t$, as well as the parameters in the model such as γ, k, θ, α and β. With the parameters estimated, we have a known system for cap rate, which describes the time series dynamics of cap rate.

To estimate the model, we further put the above structural model into state space form. Denote

$$
y_t = \begin{pmatrix} c_t \\ R_t \\ r_t^f \\ g_t \\ v_t \end{pmatrix},\ s_t = \begin{pmatrix} \tilde{c}_t \\ \tilde{r}_t \\ \tilde{g}_t \end{pmatrix},\ a_0 = \begin{pmatrix} 0 \\ 0 \\ \theta \\ 0 \\ \alpha \end{pmatrix},\ Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \beta \end{bmatrix},\ \eta = \begin{bmatrix} \eta_c & 0 & 0 & 0 & 0 \\ 0 & \eta_R & 0 & 0 & 0 \\ 0 & 0 & \eta_{rf} & 0 & 0 \\ 0 & 0 & 0 & \eta_g & 0 \\ 0 & 0 & 0 & 0 & \eta_o \end{bmatrix},\ \varepsilon_t = \begin{pmatrix} \varepsilon_t^4 \\ \varepsilon_t^5 \\ \varepsilon_t^8 \\ \varepsilon_t^9 \\ \varepsilon_t^{10} \end{pmatrix},
$$

$$
b_0 = \begin{pmatrix} \gamma k \\ \kappa \bar{r} \\ \lambda \bar{g} \end{pmatrix},\ T = \begin{bmatrix} 1 & \gamma & -\gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & 1-\kappa & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix},\ \sigma = \begin{bmatrix} \sigma_{\tilde{c}} & 0 & 0 \\ 0 & \sigma_{\tilde{r}} & 0 \\ 0 & 0 & \sigma_{\tilde{g}} \end{bmatrix},\ \text{and } \xi_t = \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \end{pmatrix}.
$$

Then we have:

$$y_t = a_0 + Zs_t + \eta\varepsilon_t,\ \ \varepsilon_t \sim N\left(\vec{0},\Sigma\right) \quad (25)$$

$$s_{t+1} = T^{-1}b_0 + T^{-1}Bs_t + T^{-1}\sigma\xi_t,\ \xi_t \sim N\left(\vec{0},\Omega\right) \quad (26)$$

where Σ and Ω are 5×5 and 3×3 identity matrices, respectively.

Here, equation (26) is the state equation, which describes how the state variables s_t evolve over time, and equation (25) is the observation equation that links the state variables s_t to the observable variables y_t. The state space model is estimated using Kalman filter (see Durbin and Koopman 2001 for more details).

5. Data and Results

We compile a comprehensive dataset using data from the National Council of Real Estate Investment Fiduciaries (NCREIF), Real Estate Research Corporation (RERC), Real Capital Analytics (RCA) and the Federal Reserve.

NCREIF compiles the NCREIF Property Index (NPI) which is a quarterly index tracking the performance of core institutional property markets in the U.S. It also reports cap rates, NOI growth and vacancy rates. We obtain the quarterly series of its current value cap rates, which is from properties that were revalued but not limited to those sold during the quarter. The data starts from the first quarter of 1978. Cap rates for all properties at the national level as well as cap rates for four major property types (apartment, office, retail and industrial) are collected, although the latter ones have a shorter span only covering 1990 to the present. Return data are also from 1978 to the most recent and is also measured quarterly, except that apartment return data only starts from 1984 Q1. We are interested in the total return, which includes both price appreciation and income return. NCREIF also reports NOI growth rates, which are changes in net operating income (NOI) from quarter to quarter for properties that are in the NPI index at the beginning and end of the respective quarter. As mentioned in section 4, due to accounting lag or different treatments of capital expenditures, this data does not well reflect real NOI growth or NOI growth expectations. We use the four quarter moving average in our model estimation.

NOI growth data starts from 1978 Q2. Vacancy data series generally starts in 1983, although those for apartments and industrial properties start at a later time.

We also collect transaction cap rates from Real Capital Analytics (RCA). The data series only starts from 2001, however they are measured monthly. RERC maintains a quarterly institutional survey on cap rates and other market indicators. The cap rate data series we get start from 1993, and are reported for 9 specific property types. Since this data is directly from the survey and thus reflects investors' expectation about cap rates, we will use it as direct observations of the *ex-ante* cap rate $\tilde{c}_t$ and will compare model estimation results using this *ex-ante* cap rate with those using NCREIF *ex-post* data. Risk-free interest rate is obtained from the Federal Reserve.

Table 1 provides a list of the variables we use in model estimations with variable definitions, data source, frequency and length of the data as well as whether the data is for all properties or by property type/region. Notice that in our model, cap rate, return and rental growth are all in log forms, so we re-calculate some of the variables to obtain log property return, log risk free rate and log rental growth and take logs on cap rate.

We plot cap rate, property total return and NIO growth data in figure 2, figure 3 and figure 6 respectively. There's substantial time variation in cap rates and the overall trends tends to be cyclical. Further, we see four trend regimes for cap rates: the extended, moderately downward sloping era of 1979 - 1990; the sharply upward sloping period of 1990 - 1996; the moderately downward sloping trend of 1996 - 2001; and the extremely steep downward trend of 2002 - 2008. Correspondingly, we see from figure 3 that property returns are downward sloping during 1979-1990, upward sloping during the period of 1993-1998, downward sloping again during 1998-2002 and then upward sloping during 2002-2008. Generally, the trend of cap rate tends to match the trend of property returns, showing a positive relationship between contemporaneous cap rate and property return. However, there are several periods when cap rate and return diverge. For example, from 1990 to 1993, cap rate tends to climb up while property returns are in their historical lows. NOI growths shown in figure 6 probably help explain this to some extend:

in 1991 and 1993, NOI growths are mainly in the negative territory, which offsets the low property returns' impact on cap rate; during 2002 and 2003, cap rates show an upward trend and stays at a rather high level, although property returns are low. Again this may be explained by the low NOI growth during 2002 and 2003. Overall, both return and rental growth tends to have impacts on cap rate.

Figures 4 and 5 presents the 3-month Treasury bill rates and commercial property vacancy rates used in our analysis. Figures 7 through 10 plot NCREIF cap rate, return, vacancy and NOI growth by property type. There are some cross-sectional variations in those variables across property types. However, the overall trends of those measures for different property types follow each other. For returns, we see a significant plunge during the early 1990s' commercial real estate market crisis. The 2001-2002 recession also affects commercial real estate return significantly.

Figure 11 presents the RERC surveyed cap rate by nine property types. These are *ex-ante* cap rates and apparently they are much less volatile than the NCREIF cap rates presented in figure 7.

Table 2 reports sample statistics of the three measurement variables for all properties, as well as for the four major property types. NCREIF cap rates for all property types have an average of 7.7 percent over the 30 year period. The highest cap rate is 9.8 percent and the lowest is 5.4 percent. Property return is reported in log form and is quarterly. The average quarterly log return for all properties is 1.1 percent, which is lower than the simple return, which is 2.5 percent. There is substantial variation in property return over time with a high of 2.6 percent and a low of -2.4 percent. Across property types, office properties see the highest average return and industrial properties have the lowest average log return. The average vacancy rate during 1983 and 2008 for all property types is 8.6 percent, with a historical low of 4 percent and historical high of 14.2 percent. Apartment properties are generally the most highly occupied. The average vacancy rate in the past 20 years for apartment buildings is 6.5 percent. Office properties see the highest average vacancy rate, although industrial properties have the highest variance in vacancy rates.

In table 3, we report sample statistics of the RERC *ex-ante* cap rates. The standard deviations are significantly lower than those of the NCREIF cap rates.

Table 4 presents our maximum likelihood estimates of our structural time series model with Kalman filter. From equations (16), (18) and (22), we see that the best estimates of the long term means of property return and NOI growth rate are just the long term means of observed property returns and growth rates. Therefore, we calculate the sample means of those two variables and use them as the estimates of $\bar{r}$ and $\bar{g}$. This helps reduce our estimation dimensions and greatly saves computation time.

There are a number of interesting results from this table. First, the system works well as most of the parameter estimates are significant at 99 percent significance level[6], and the parameter estimates are meaningful. For example, the expected property return shows long term mean-reverting with a speed between 0.19 and 0.50 depending on property type (κ parameter); commercial properties have a risk premium over the Treasury of 0.0037-0.0051 (θ parameter), which can be converted into a simple return premium of 148 to 205 bps; occupancy growth is positively related to NOI growth (β parameter). Second, estimation results show that cap rate is significantly related to both future expected return and expected rental growth (γ parameter). In fact, putting the parameter back into equation (9), we have $c_t \simeq E_t \sum_{j=0}^{\infty} 0.9977^j \left(r_{t+j} - g_{t+j} \right)$ for all properties. Expected return is positively related to cap rate while rental growth is negatively related to cap rate, consistent with the common wisdom that low cap rates imply expectations of higher rent and/or diminishing returns. Also, investors weigh expectations about return and rental growth in different future periods differently. They place more weights on the nearer future. These findings suggest that overlooking either return or rental growth will lead to a biased assessment regarding cap rate movements, and that taking a multi-period dynamic approach is more appropriate than taking a static approach. Third, expected

[6] The asymptotic covariance is calculated with the information matrix using the BHHH method.

return is significantly mean-reverting according to our estimation. This reflects the fact that commercial real estate market is cyclical but supply/demand adjustments always force the market towards equilibrium in the long run. An implication of this result is: because of the mean-reverting property in expected return and because of the positive relationship between cap rate and expected return, investors should adopt upward sloping cap rates if the current and past returns are low. Fourth, the volatilities of expected return and rental growth (0.0041 and 0.0048) are substantially smaller than those of the observed return and rental growth (0.0070 and 0.0068). This could be because investors are sluggish in adjusting their expectations or they have rational expectations about the cyclicality of the real estate market and thus smooth their expectations about return and rental growth.

We plot the original data series, the filtered state variables, within sample prediction errors and prediction variances in figures 12 through 20. For example, the heavy dark line in figure 12 represents the filtered expected property return. It is extracted from the time series information in the *ex-post* property return (blue line) and risk-free rate (red line). The prediction variance is high during the first period because we use a diffuse initialization in our Kalman filter. It soon converges to a constant value. A very interesting observation from these figures is that prediction errors of NOI growth and cap rate within our model decrease over time from 1980s to the most present, suggesting that NOI growth and cap rates are becoming more predictable.

We report our estimates of the dynamic cap rate model with *ex-ante* cap rate data from RERC in table 5. We see that the results are very similar to those presented in table 4.

We also compare our structural estimates with those from a reduced-form VAR model, which is reported in table 6. A VAR estimation by Shilling and Sing (2007) shows that cap rate is positively related to *past* property excessive return, which contradicts the theoretical prediction. We thus estimate a VAR model with the information set we use in our structural model estimation. Our estimates show a negative relationship between cap rate and property return. Given that property return is again shown to be mean-reverting, we infer a positive relationship between cap rate and *future* property return, consistent with the theory. However, the VAR estimates show that there is no significant relationship between cap rate and NOI growth, either statistically or economically. This result is actually similar to the reduced-form estimate by Plazzi, Torous and Valkanov (2008), which finds that cap rate is not useful in predicting NOI growth. The bottom line here is our structural model captures the relationship between cap rate and NOI growth that is not captured by reduced-form models.

6. Conclusions and Discussions

Capitalization rate is used by investors and appraisers to convert net rental income of a property into property value. Therefore, to understand how investors form their beliefs of cap rates is important. A static Gordon (1962) model shows that cap rate is just the difference between required return and rental growth. Therefore, a low cap rate implies expectations of higher rent and diminishing returns. Conversely, high cap rates imply the expectation of weakening rents and high required rate of return. Although qualitatively right, the aforementioned relationship in the static Gordon model fails empirical tests. A simple plot shows that $r - g$ does not coincide with cap rate c even in terms of time trends, not to mention the levels.

The purpose of this study is to conduct an anatomy of cap rate. We intend to tease out the implied expectations in cap rate about property return and rental growth with a structural model. The objective is to find a model that more accurately describes the interactions between cap rate, and expectations about return and rental growth. Further, we model dynamics of those expectations and structures of interactions between cap rate and those expectations so that reversely investors can use our model to predict future cap rates based on past information on return and rental growth.

We build a dynamic cap rate model following Campbell and Shiller (1989), which yields a structural model that links cap rate to multiple periods expected returns and rental growths. In our model, cap rate is the weighted average of all future “growth-adjusted discount rates" and those "growth-adjusted discount rates" can have substantial variations over time. We further estimate our structural model with Kalman filter.

Our estimation results reveal that cap rate is significantly related to both future expected return and expected rental growth. This finding suggests that overlooking either component will lead to a biased assessment regarding cap rate movements. Expected return is positively related to cap rate while rental growth is negatively related to cap rate, consistent with the common wisdom that low cap rates imply expectations of higher rent and/or diminishing returns. Second, investors weigh expectations about return and rental growth in different future periods differently. They place more weights on the nearer future. Third, expected return is significantly mean-reverting according to our estimation. This reflects the fact that commercial real estate market is cyclical but supply/demand adjustments always force the market towards equilibrium in the long run. An implication of this result is: because of this mean-reverting property in expected return and because of the positive relationship between cap rate and expected return, investors should look for upward sloping cap rates if the current and past returns are low. Fourth, the volatilities of expected return and rental growth are substantially smaller than those of the observed return and rental growth. This could be because investors are sluggish in adjusting their expectations or they have rational expectations about the cyclicality of the real estate market and thus smooth their expectations about return and rental growth.

Our study contributes to the literature in significant ways. First, different from most studies that are based on the static Gordon (1962) model, we develop a dynamic cap rate model that fully incorporates investors' expectations about multi-period future returns and rental growths that are time-varying in forming cap rates. Second, with advanced time series techniques we are able to estimate our model based on the structural form of our theoretical model. This enables us to capture the relationship between cap rate and NOI growth that is not captured by reduced-form models including single equation linear regressions and VARs. Last but not least, we estimate the dynamics of expected return and rental growth based on a mean-reverting structure together with the aforementioned relationship between cap rate, return and rental growth. This enables us to present a complete dynamic cap rate model that is useful in cap rate predictions.

Future research can go several directions. First, applying our model to different data sources may be valuable. In fact, we haven't utilized the RCA data we collected in model estimations. Estimating the model with different data will further help evaluate the validity of our model and provide better understanding of cap rate dynamics. Second, refinements of the constant volatility mean-reverting structures imposed on expectations of returns and rental growth may be desirable. As we see from the data plots, there are several regimes in property return and cap rate. New models on return and rental growth expectations may produce better "fit" and deeper understanding of those different regimes. Finally, to evaluate the out-of-sample predicting power of our dynamic cap rate model and compare it with those of other reduced-form models is an important task.

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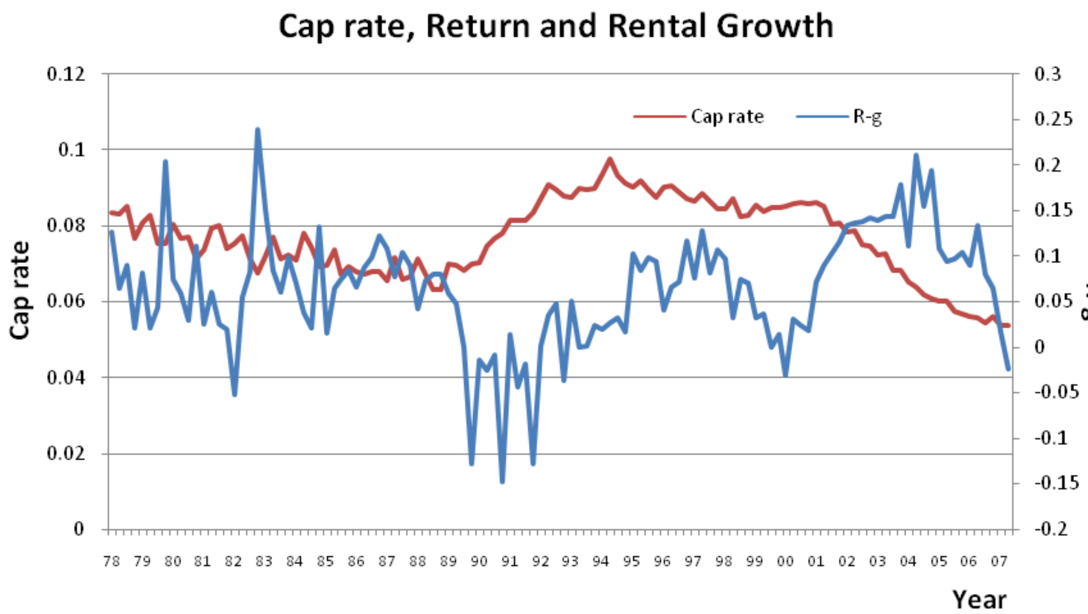

Figure 1: Commercial Real Estate Cap Rate, Return and Rental Growth

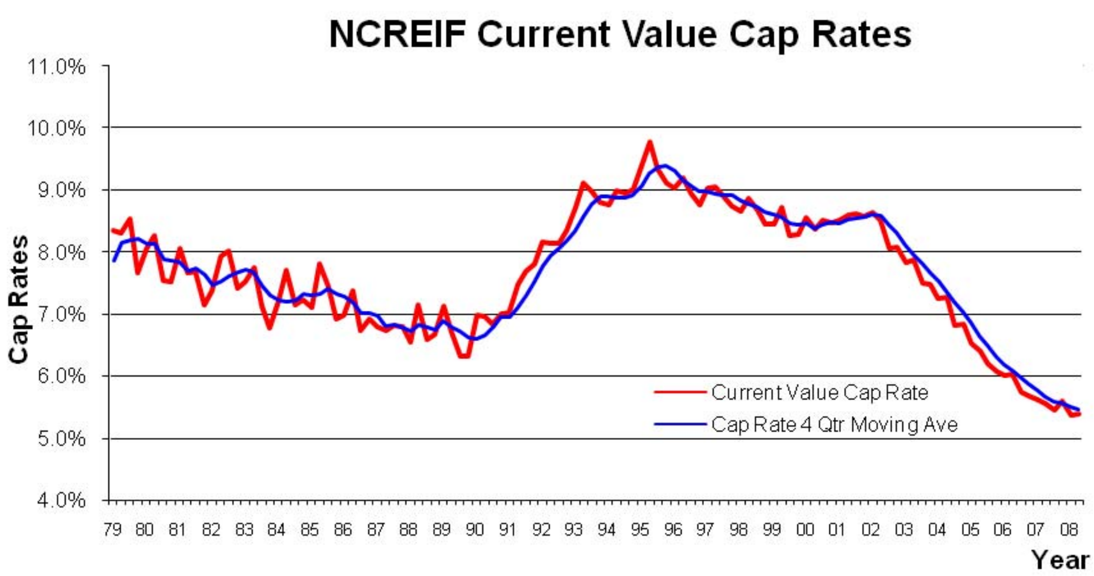

Figure 2: Commercial Real Estate Cap Rate 1978-2008, All Property Types

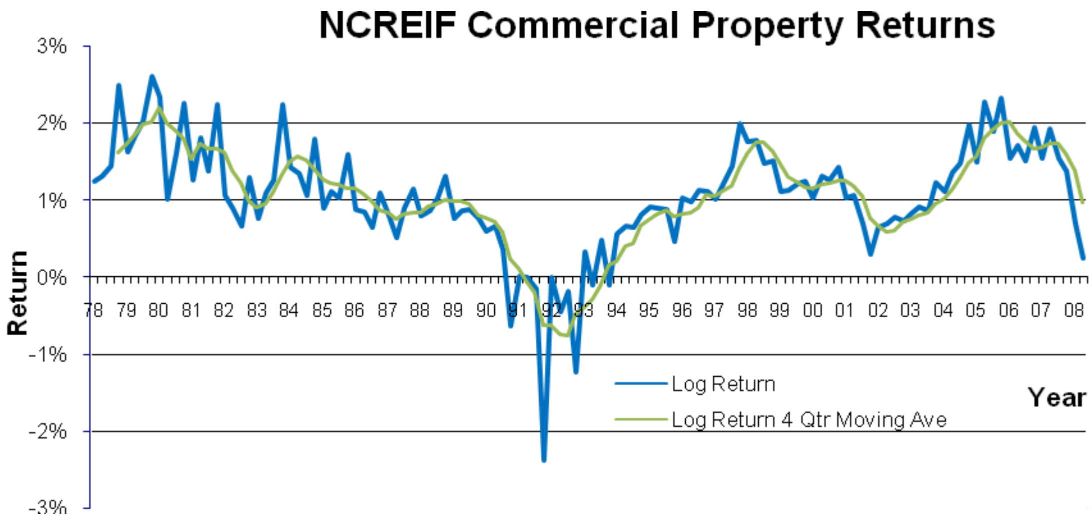

Figure 3: Commercial Real Estate Quarterly Total Return 1978-2008, All Property Types

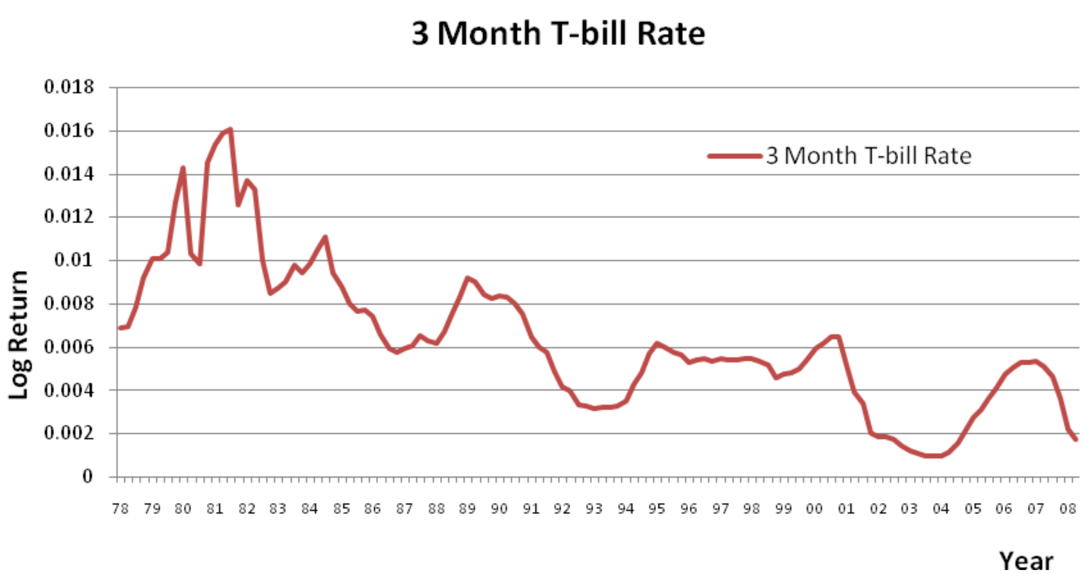

Figure 4: Three-month US Treasury Bill Rates 1978-2008

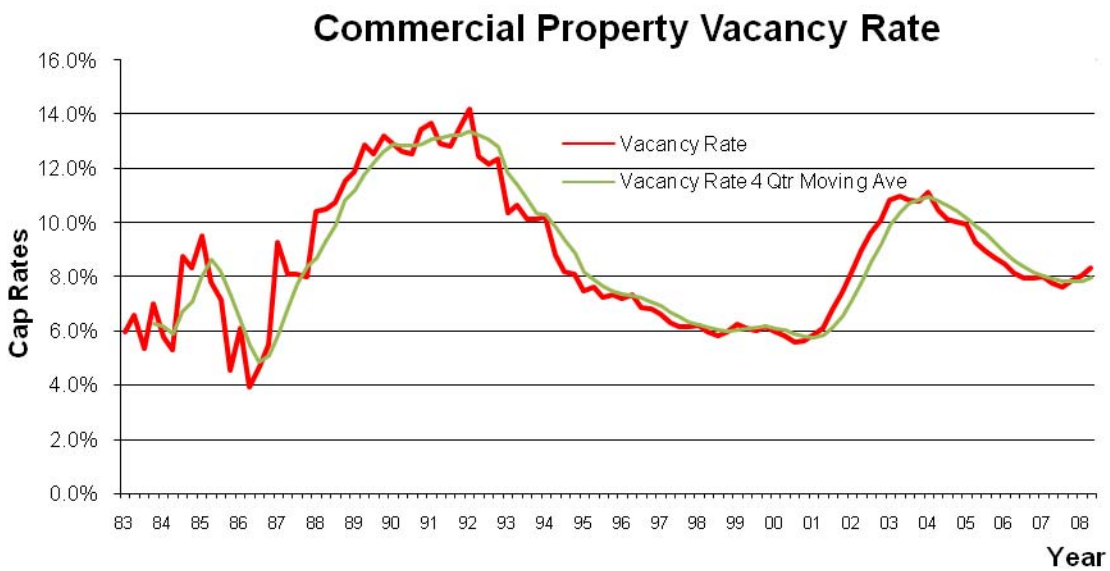

Figure 5: Commercial Property Vacancy Rate 1983-2008, All Property Types

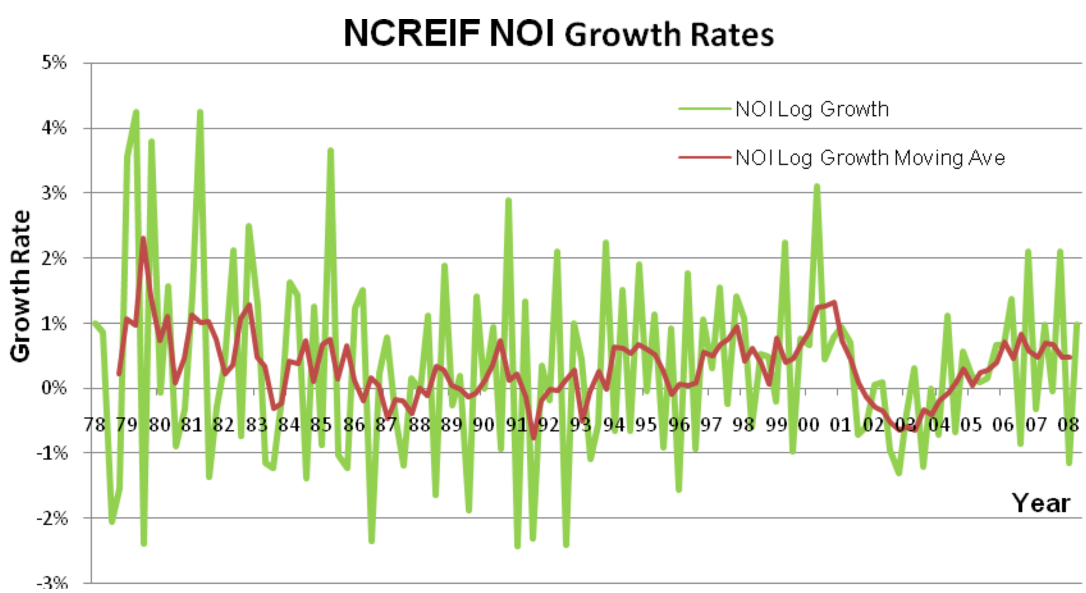

Figure 6: Commercial Real Estate NOI Growth Rates 1978-2008, All Property Types

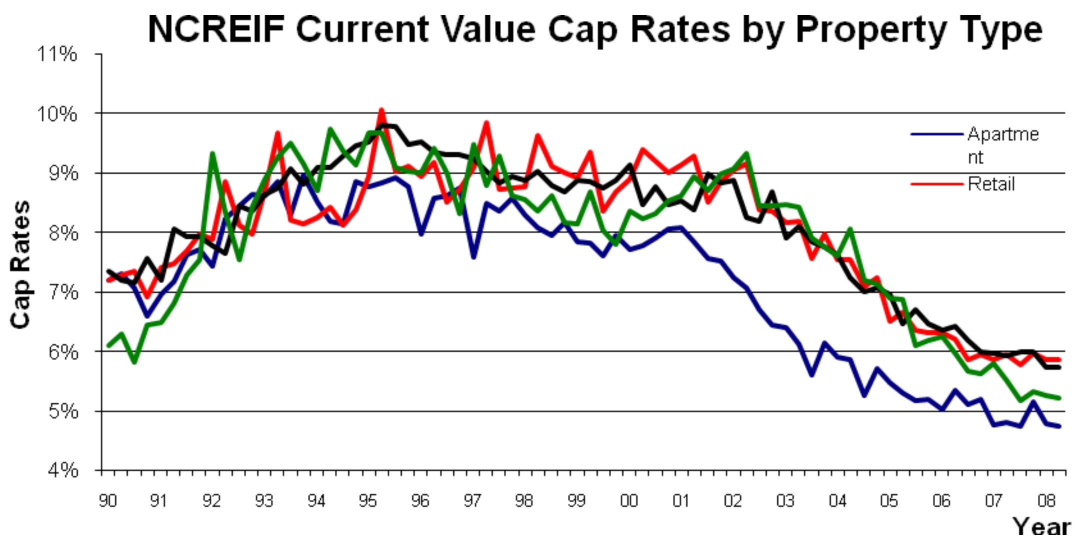

Figure 7: Cap Rates by Property Type

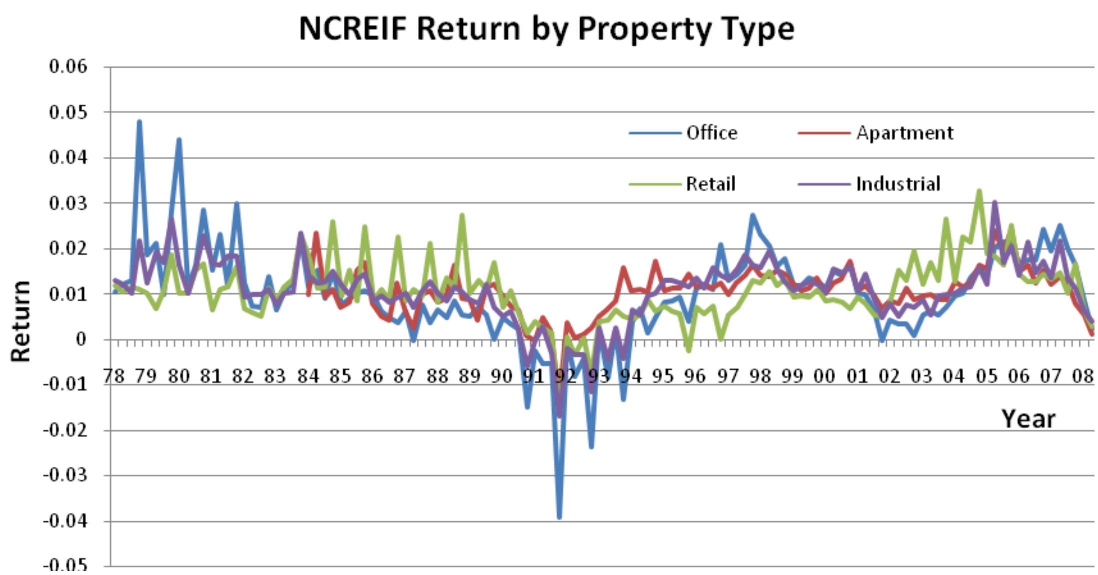

Figure 8: Total Returns by Property Type

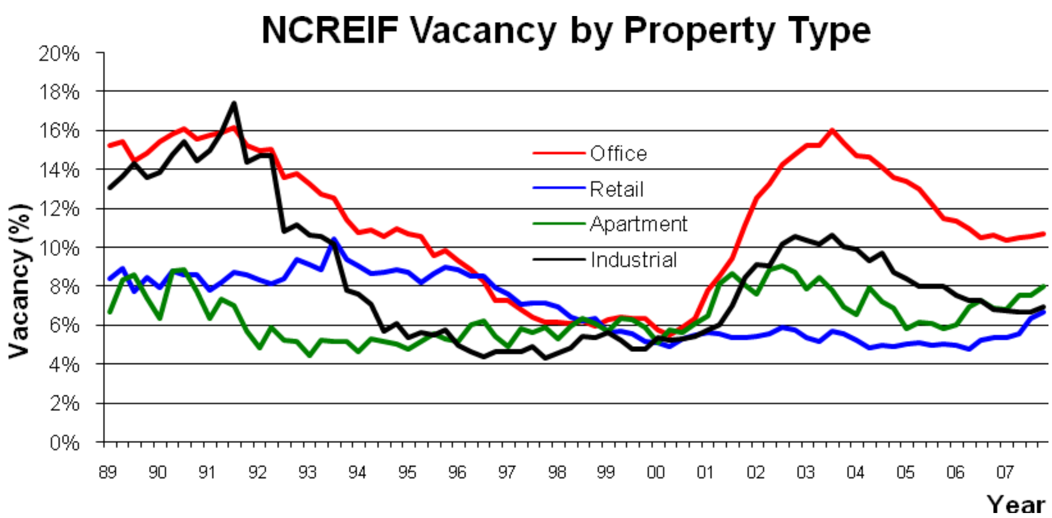

Figure 9: Vacancy Rate by Property Type

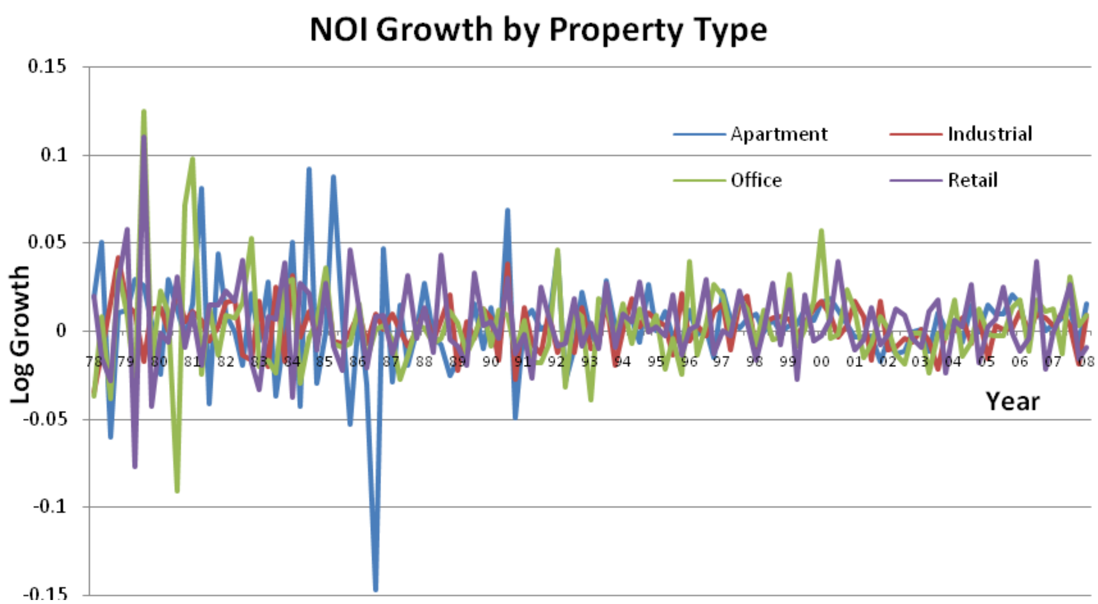

Figure 10: NOI Growth by Property Type

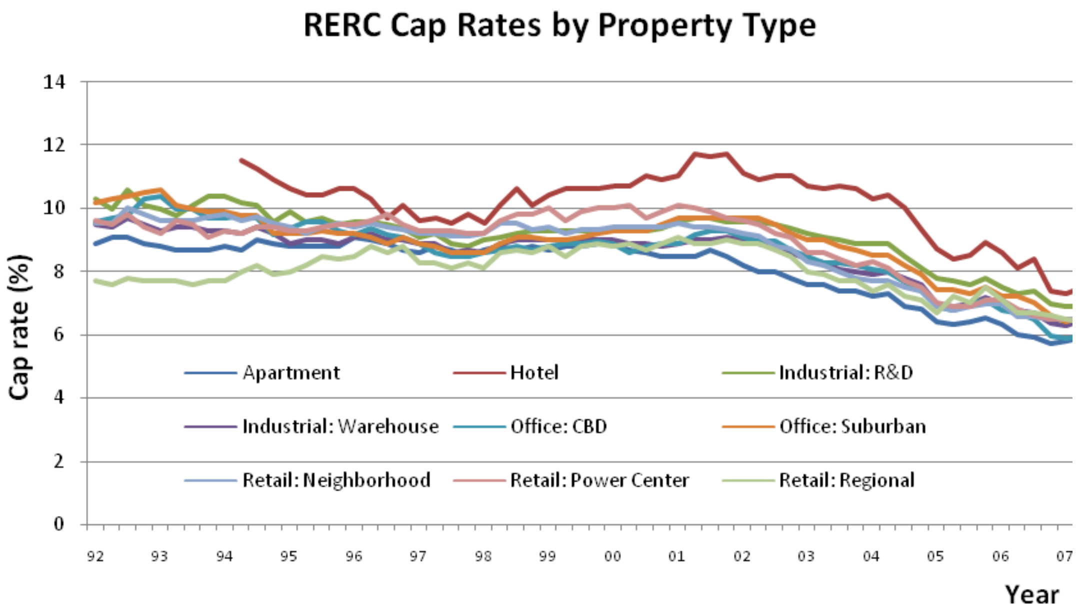

Figure 11: Surveyed Cap Rates by Property Type

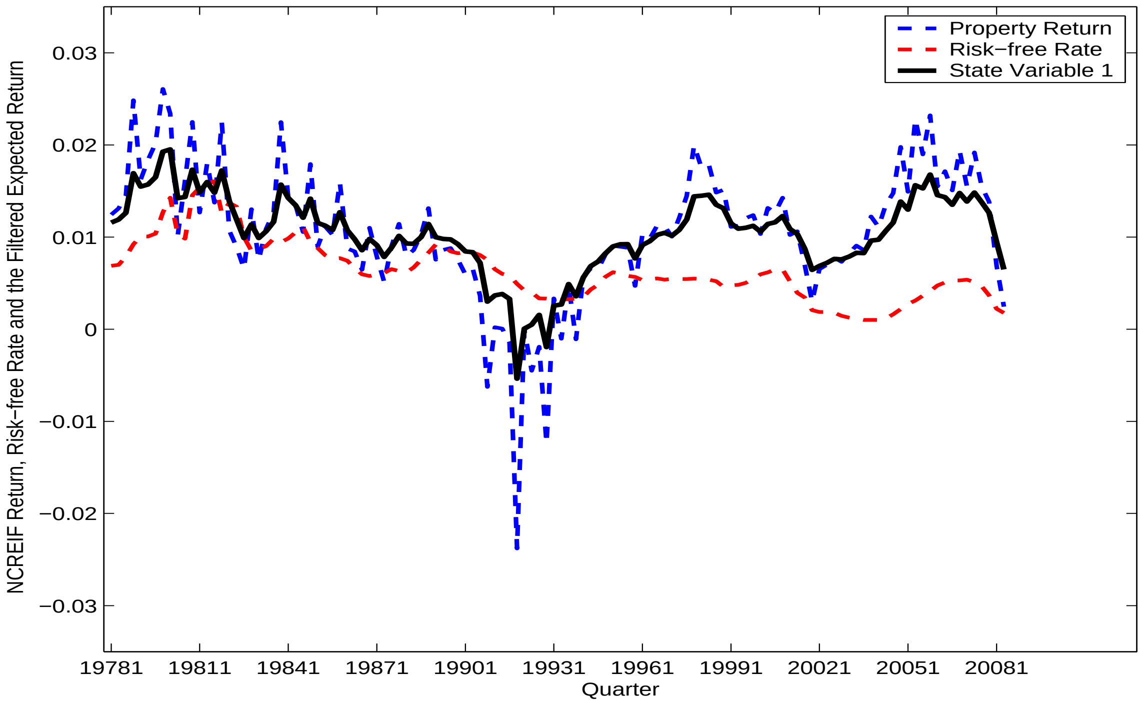

Figure 12: NCREIF Return, Risk-free Rate and the Filtered Expected Return

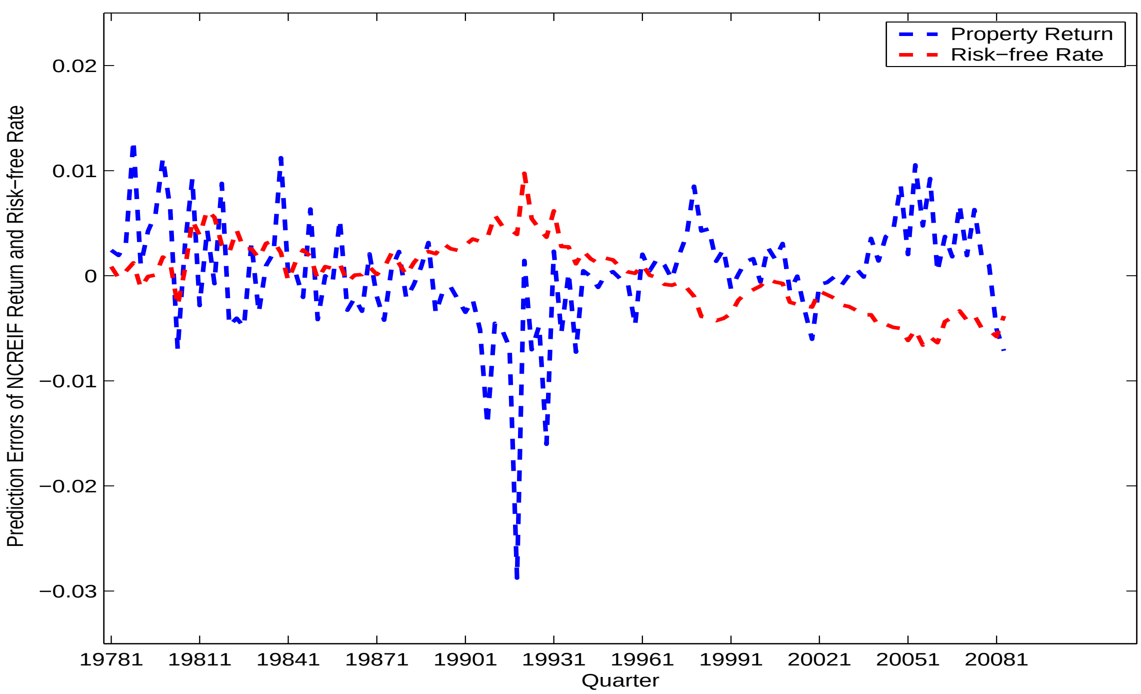

Figure 13: Prediction Errors of NCREIF Return and Risk-free Rate

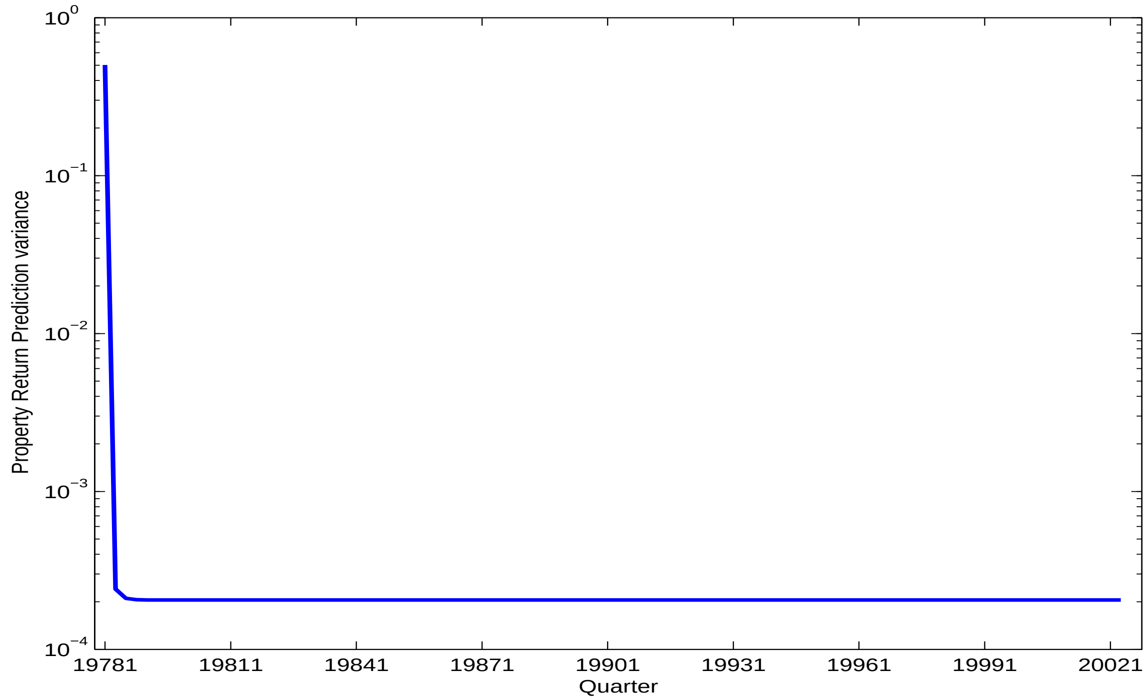

Figure 14: NCREIF Return Prediction Variance

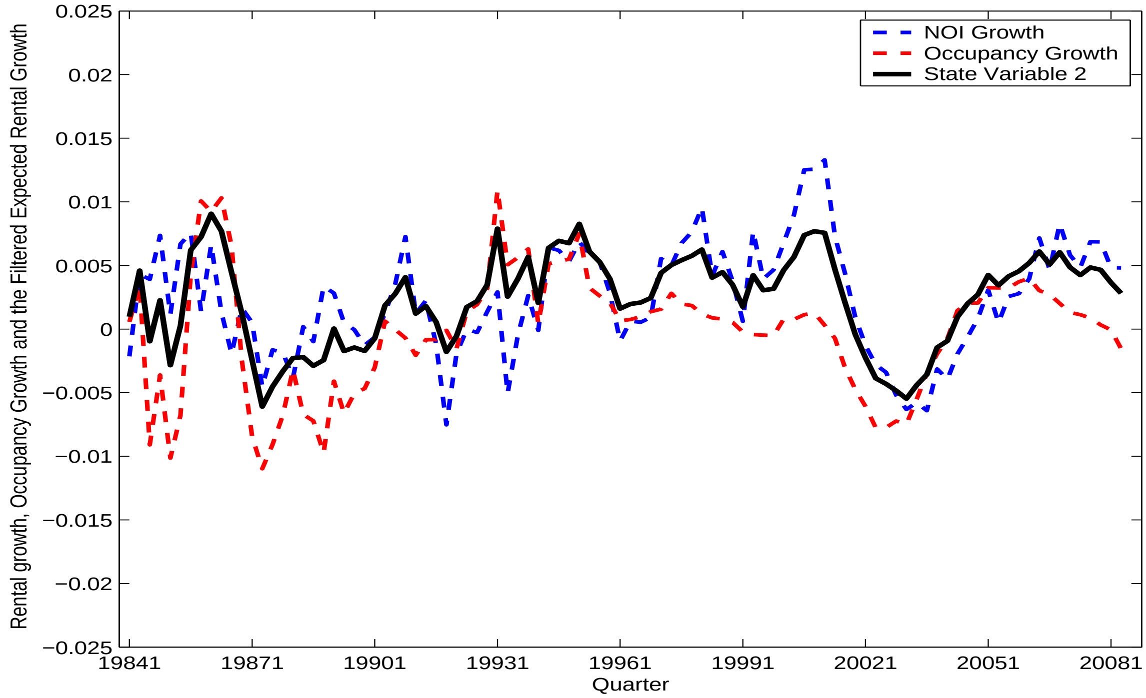

Figure 15: NOI Growth, Occupancy Growth and the Filtered Expected NOI Growth

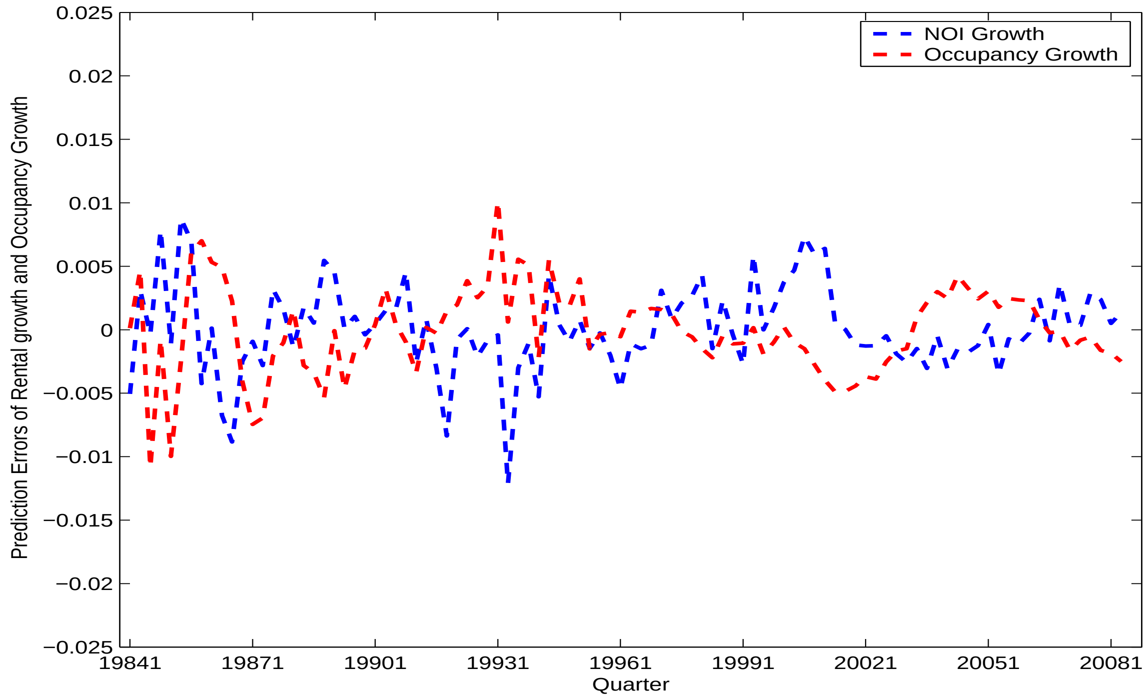

Figure 16: Prediction Errors of NOI Growth and Occupancy Growth

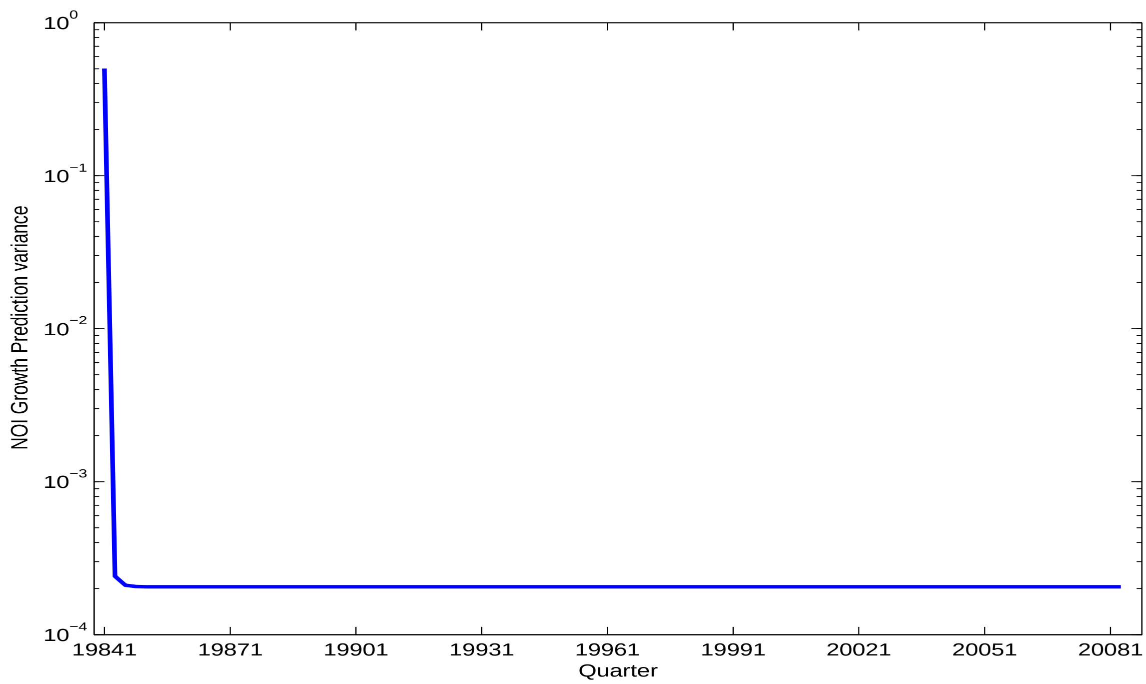

Figure 17: NOI Growth Prediction Variance

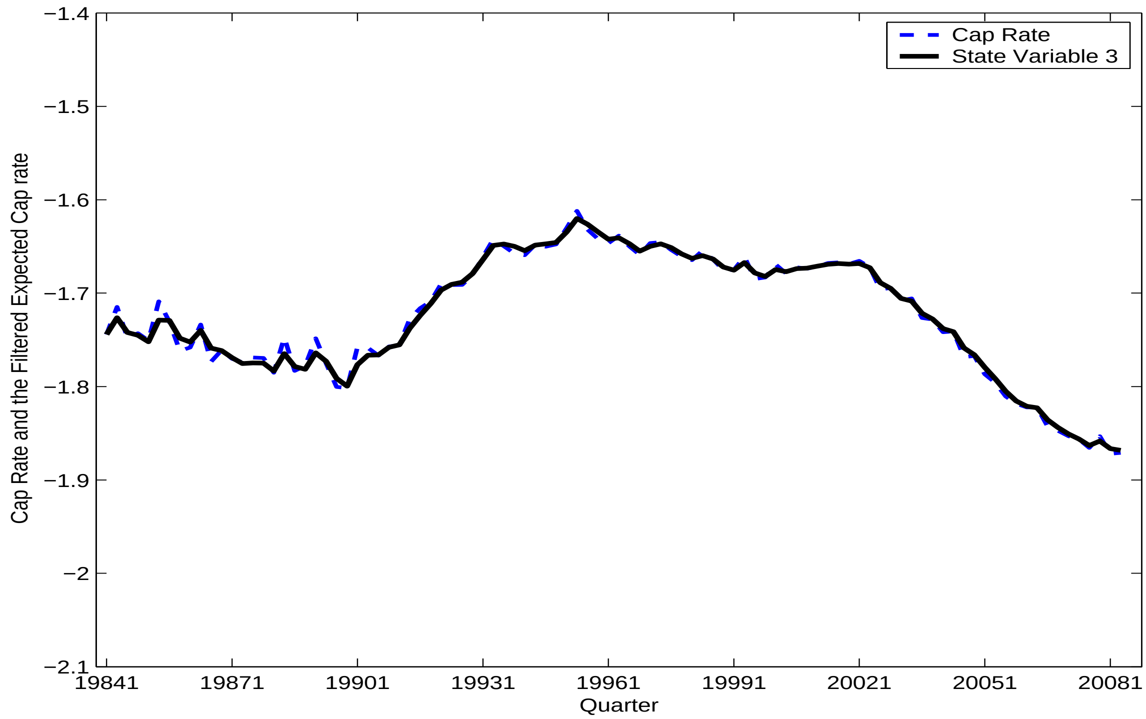

Figure 18: NCREIF Cap Rate and the Filtered Expected Cap Rate

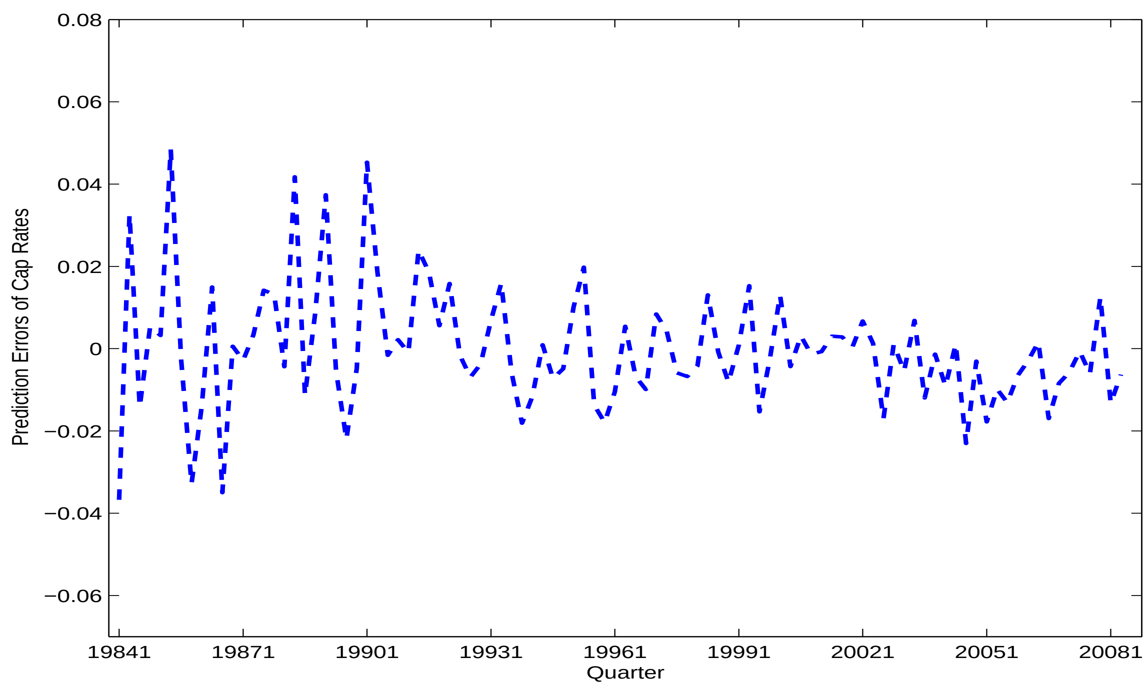

Figure 19: Prediction Errors of Cap Rate

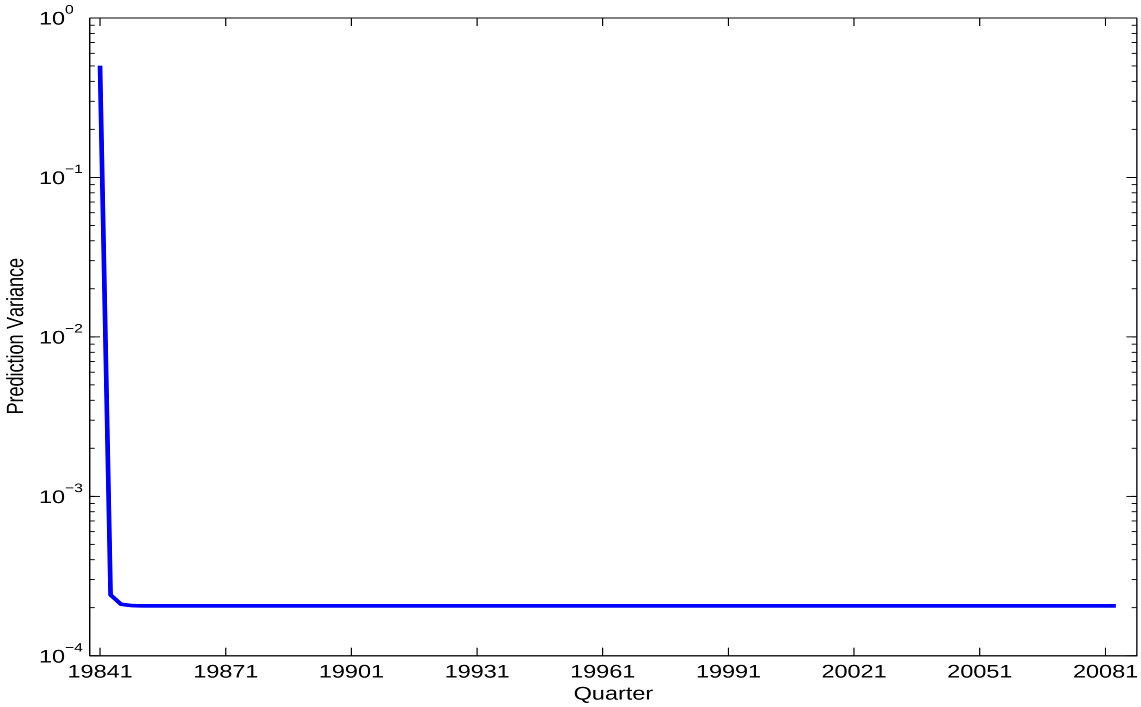

Figure 20: Cap Rate Prediction Variance

Table 1: List of Variables and Descriptions

Variable Name	**Symbol**	**Description**	**Break Down**	**Data source**	**Frequency and Time Span**
NCREIF cap rate	c_t	The NCREIF cap rate is the current value cap rate derived from properties that were revalued (not limited to those sold) during the quarter. Ex-post data. We use log term in the model.	On a national basis and by the 4 NCREIF regions and 4 property types.	NCREIF	Quarterly, 1978 Q1-2008 Q2
RCA cap rate	c_t	The RCA cap rate is the transaction cap rate collected by RCA (Real Capital Analytics). Ex-post data. We use log term in the model.	On a national basis and by 5 property types.	RCA	Monthly, 2001M1-2008M8
RERC cap rate	$\tilde{c}_t$	It is the surveyed cap rate from RERC's quarterly survey of institutional investors. Ex-ante data. We use log term in the model.	By 9 property types.	RERC	Quarterly, 1993 Q1-2008 Q2
Property return	R_t	Log return calculated based on NCREIF reported property value, NOI, capital expenditure and adjustments during the quarter. Ex-post data.	On a national basis and by the 4 NCREIF regions and 4 property types.	NCREIF	Quarterly, 1978 Q1-2008 Q2
Risk free rate	r_t^f	Log return calculated based on three month US Treasury Bill rate. Ex-post data.		Federal Reserve	Monthly, 1978 M1-2008 M6
NOI growth	g_t	Log growth of NOI for properties owned by NCREIF members. Ex-post data. We use the moving average of 4 quarter growths in our model.	On a national basis and by the 4 NCREIF regions and 4 property types.	NCREIF	Quarterly, 1978 Q2-2008 Q2
Vacancy rate	v_t	Vacancy rate for properties owned by NCREIF members. Vacancy is equal weighted by property. Ex-post data. We use the moving average of 4 quarter growths of occupancy rate (1-vacancy rate) in our model.	On a national basis and by the 4 NCREIF regions and 4 property types.	NCREIF	Quarterly, 1983 Q1-2008 Q2

Table 2: Descriptive Statistics of the NCREIF Data

	Variable	**Mean**	**STD**	**Min**	**Max**
All properties	NCREIF cap rate	0.077	0.010	0.054	0.098
	Property return	0.011	0.007	-0.024	0.026
	Vacancy rate	0.086	0.025	0.040	0.142
Apartment	NCREIF cap rate	0.072	0.013	0.047	0.090
	Property return	0.017	0.009	-0.009	0.048
	Vacancy rate	0.065	0.013	0.044	0.094
Office	NCREIF cap rate	0.079	0.013	0.052	0.098
	Property return	0.019	0.014	-0.008	0.049
	Vacancy rate	0.111	0.037	0.038	0.178
Retail	NCREIF cap rate	0.080	0.012	0.058	0.101
	Property return	0.017	0.013	-0.008	0.058
	Vacancy rate	0.068	0.015	0.038	0.104
Industrial	NCREIF cap rate	0.081	0.011	0.057	0.098
	Property return	0.016	0.011	-0.003	0.059
	Vacancy rate	0.080	0.038	0.009	0.174

Notes: 1) Cap rates for all properties are from 1978Q1 to 2008Q2. Cap rates for separate property types run from 1990Q1 to 2008Q2. 2) Property returns are log returns and are quarterly. The returns for all properties are from 1978Q1 to 2008Q2, while those for separate property types are from 1996Q1 to 2008Q2. 3) Vacancy rates for all property run from 1981Q1 to 2008Q2. Vacancy rates for office properties are from 1983Q2 to 2008Q2, for retail properties are from 1983Q1 to 2008Q2, for apartment properties are from 1987Q1 to 2008Q2, and for industrial properties are from 1984Q4 to 2008Q2.

Table 3: Descriptive Statistics of the RERC Surveyed Cap Rates

Property type	Sub-type	Mean	STD	Min	Max
Apartment	--	0.081	0.010	0.057	0.091
Hotel	--	0.101	0.011	0.073	0.117
Industrial	R & D	0.091	0.009	0.069	0.106
	Warehouse	0.086	0.009	0.063	0.097
Office	CBD	0.086	0.011	0.059	0.104
	Suburban	0.089	0.010	0.064	0.106
Retail	Neighborhood	0.088	0.011	0.065	0.100
	Power Center	0.089	0.011	0.064	0.101
	Regional	0.080	0.007	0.064	0.091

Notes: 1) Data from the Real Estate Research Corporation (RERC); 2) Cap rate series are from 1992Q3 to 2007Q4.

Table 4: Estimates of the Dynamic Cap Rate Model with the NCREIF Cap Rate

Coefficient Description	**Expected log return mean-reverting speed**	**Long term mean of expected log return**	**Volatility of expected log return**	**Expected rental growth mean-reverting speed**	**Long term mean of expected rental growth**	**Volatility of expected rental growth**	**Expected cap rate auto-regression parameter**	**Volatility of expected cap rate**
Symbol	κ	$\bar{r}$	$\sigma_{\tilde{r}}$	λ	$\bar{g}$	$\sigma_{\tilde{g}}$	γ	$\sigma_{\tilde{c}}$
All properties	0.2481 (0.0988)	0.0100 (--)	0.0041 (0.0006)	0.1507 (0.1091)	0.0024 (--)	0.0048 (0.0007)	1.0023 (0.0039)	0.0074 (0.0015)
Apartment	0.5037 (0.0884)	0.0100 (--)	0.0043 (0.0004)	0.1188 (0.1288)	0.0024 (--)	0.0066 (0.0016)	1.0153 (0.0076)	0.0057 (0.0018)
Office	0.2960 (0.0773)	0.0100 (--)	0.0054 (0.0010)	0.1478 (0.1123)	0.0015 (--)	0.0087 (0.0011)	0.9938 (0.0181)	0.0128 (0.0024)
Retail	0.3964 (0.1637)	0.0100 (--)	0.0045 (0.0007)	0.2701 (0.1524)	0.0045 (--)	0.0040 (0.0008)	1.0014 (0.0170)	0.0069 (0.0022)
Industrial	0.1939 (0.1243)	0.0100 (--)	0.0048 (0.0005)	0.2404 (0.1482)	0.0016 (--)	0.0043 (0.0011)	1.0068 (0.0170)	0.0077 (0.0017)

Cap rate model constant	**Cap rate shock volatility**	**Log return shock volatility**	**Log rental growth shock volatility**	**Commercial property risk premium**	**Commercial property risk premium volatility**	**Occupancy growth shock volatility**	**Occupancy rate model constant**	**Occupancy-rental growth loading**
k	η_c	η_R	η_g	θ	η_{r^f}	η_o	ϕ	β
0.0097 (0.0068)	0.0098 (0.0012)	0.0039 (0.0014)	0.0001 (0.0000)	0.0040 (0.0008)	0.0071 (0.0009)	0.0001 (0.0000)	-0.0024 (0.0011)	0.9641 (0.1807)
0.0218 (0.0085)	0.0143 (0.0015)	0.0000 (0.0891)	0.0001 (0.0001)	0.0051 (0.0009)	0.0052 (0.0011)	0.0001 (0.0001)	-0.0025 (0.0012)	0.7342 (0.2036)
-0.0014 (0.0206)	0.0150 (0.0025)	0.0059 (0.0012)	0.0001 (0.0000)	0.0037 (0.0013)	0.0121 (0.0015)	0.0001 (0.0001)	-0.0009 (0.0014)	0.8249 (0.1718)

0.0055 (0.0185)	0.0167 (0.0020)	0.0037 (0.0020)	0.0014 (0.0033)	0.0039 (0.0009)	0.0095 (0.0019)	0.0009 (0.0021)	-0.0044 (0.0016)	1.1016 (0.2582)
0.0131 (0.0185)	0.0087 (0.0012)	0.0035 (0.0013)	0.0035 (0.0024)	0.0046 (0.0011)	0.0046 (0.0011)	0.0006 (0.0079)	-0.0038 (0.0019)	1.9358 (0.6427)

Notes: 1) These are Maximum Likelihood Estimates with Kalman Filter. 2) Standard errors are calculated using the BHHH method.

Table 5: Estimates of the Dynamic Cap Rate Model with the RERC Surveyed Cap Rate

Coefficient Description	**Symbol**	**Apartment**	**Office: CBD**	**Office: Suburban**	**Retail: Neighborhood**	**Retail: Power Center**	**Retail: Regional**	**Industrial: R & D**	**Industrial: Warehouse**
Expected cap rate auto-regression parameter	γ	0.9996 (0.0128)	1.0077 (0.0195)	1.0045 (0.0181)	1.0049 (0.0113)	1.0047 (0.0177)	1.0028 (0.0127)	1.0129 (0.0198)	1.0083 (0.0180)
Volatility of expected cap rate	$\sigma_{\tilde{c}}$	0.0086 (0.0008)	0.0111 (0.0012)	0.0100 (0.0009)	0.0080 (0.0007)	0.0098 (0.0009)	0.0094 (0.0011)	0.0099 (0.0010)	0.0090 (0.0007)
Cap rate model constant	k	0.0043 (0.0221)	0.0123 (0.0207)	0.0119 (0.0300)	0.0116 (0.0118)	0.0110 (0.0291)	0.0129 (0.0218)	0.0264 (0.0319)	0.0191 (0.0300)

Notes: 1) These are Maximum Likelihood Estimates with Kalman Filter. 2) Standard errors are calculated using the BHHH method.

Table 6: Estimates of a Reduced-form VAR Cap Rate Model with the NCREIF Data

	Independent variable	**Constant**	**Cap rate lag**	**Property return lag**	**Risk-free rate lag**	**NOI growth lag**	**Occupancy rate lag**	
Dependent variable	Symbol	α	c_{t-1}	R_{t-1}	r_{t-1}^{f}	g_{t-1}	v_{t-1}	*Adjusted R-square*
Cap rate	c_t	-0.0315 (-0.7038)	0.9820 (36.7830)	-0.6587 (-2.5177)	1.0640 (1.3717)	-0.0765 (-0.1638)	0.0545 (0.1453)	*0.9455*
Property return	R_t	0.0027 (0.1857)	-0.0004 (-0.0446)	0.7204 (8.5573)	-0.1656 (-0.6637)	0.0123 (0.0819)	-0.0068 (-0.0561)	*0.4937*
Risk-free rate	r_t^f	0.0021 (1.5506)	0.0014 (1.712)	0.0330 (4.1754)	1.005 (42.8234)	-0.0568 (-4.0206)	0.0219 (1.9306)	*0.9569*
NOI growth	g_t	0.0021 (0.2477)	0.0020 (0.4072)	0.0951 (1.9507)	0.3078 (2.1289)	0.5807 (6.6702)	0.1040 (1.4869)	*0.5567*
Occupancy rate	v_t	0.0028 (0.3256)	0.0013 (0.2501)	-0.0211 (-0.4269)	-0.1052 (-0.7189)	0.04399 (0.4991)	0.7729 (10.9163)	*0.6127*