Pricing Credit Risk in Commercial Mortgages

By

Charles C. Tu
California State University, Fullerton
Department of Finance
800 N. State College Blvd.
Fullerton, CA 92834
(714) 278-2256
(714) 278-2161 fax
ctu@fullerton.edu

and

Mark J. Eppli
George Washington University
Department of Finance
2023 G St. NW
Washington, DC 20052
(202) 994-7478
(202) 994-5014 fax
meppli@gwu.edu

April 2002

Pricing Credit Risk in Commercial Mortgages

This paper investigates the effect of credit risk on commercial mortgage pricing. It extends the literature by using both contemporaneous Loan-To-Value (LTV) and contemporaneous Debt Service Coverage (DSC) as a double-trigger default event and also by considering the borrower's inability to refinance the mortgage at maturity (balloon risk). Applying the fuzzy boundary default model developed by Riddiough and Thompson (1993), the Monte Carlo simulation results reveal that mortgage pricing models based solely on LTV overestimate the probability and risk premium of default. However, simply adding a DSC-based default trigger underestimates the credit risk premium due to the interaction between term default and balloon risk.

Keywords: Commercial Mortgages; Mortgage Pricing; Default Risk; Balloon Risk

1. Introduction

In this paper we price credit risk in commercial mortgages using a double-trigger default model that considers both term default and balloon risk. The model interactively uses asset prices and property cash flows to estimate default risk, assuming that a borrower's default decision is based on both contemporaneous Loan-To-Value (LTV) and contemporaneous Debt Service Coverage (DSC). Additionally, we explicitly measure the effect of balloon risk on mortgage pricing, as borrowers with a mortgage that is not in default at maturity may not meet the underwriting standards to refinance the loan and therefore have difficulty paying off the debt.

Until recently, most commercial mortgage pricing studies used a contingent-claims, backward pricing framework to estimate default risk premium on commercial mortgages. In this approach, default occurs when property value falls below mortgage value.[1]

[1] The commercial mortgage pricing literature follows that of residential mortgage pricing. For a review of studies using the option-pricing approach of mortgage valuation, see Kau and Keenan (1995) and Vandell (1995). Studies that apply this approach to commercial mortgages include Kau et al. (1987, 1990), Titman and Torous (1989), Childs, Ott and Riddiough (1996a, 1996b), and Ciochetti and Vandell (1999).

However, recent contributions to the commercial mortgage default literature recognize the limitation of using a single-trigger, asset price-only, commercial mortgage default model.[2]

The option-pricing default model assumes borrowers rationally default when the mortgage value exceeds the property value; therefore, a key predictor of default incidence is LTV at loan origination. Archer et al. (2001) argue that LTV at origination is an endogenous risk measure and therefore no empirical relationship between LTV and mortgage default should exist. They investigate 495 multifamily loans and find no relationship between original LTV and mortgage default. However, their results reveal that initial property cash flow (measured by DSC at origination) is a strong predictor of default. Ambrose and Sanders (2001) use a competing risks model to examine default and prepayment behavior using 4,257 commercial loans underlying 33 CMBS deals. They also find no statistical relationship between original LTV and default; in their model, however, no measure of property cash flow is included.

Ciochetti, Gao and Yao (2001) further extend the literature by including contemporaneous measures of LTV and DSC in their empirical analyses. They estimate default and prepayment functions of commercial mortgages using a competing risks proportional hazard model and loan level data, and find that the option-pricing model alone cannot fully explain default incidence. The authors also reveal that both contemporaneous DSC and a binary variable representing balloon year show a strong

[2] The idea of a double-trigger has been discussed in Abraham (1993), Vandell (1995), and Jacob, Hong and Lee (1999), and used in the empirical work of Goldberg and Capone (1998).

impact on default incidence, suggesting the importance of including property income and balloon risk in mortgage pricing.

Goldberg and Capone (2002) propose a theoretical default model that incorporates both property value and property cash flow considerations to predict multifamily mortgage default, and test the model empirically using a dataset of 13,482 multi-family loans. The results show that the double-trigger, joint-probability model is superior to models with a single default trigger (either LTV or DSC). Their findings also reveal a sizable increase in default risk in the balloon year, confirming Ciochetti, Gao and Yao's results.

Consistent with the recent commercial mortgage default literature, we propose a pricing model where borrower default decision is based on both property net equity level and property cash flow.[3] Failure to recognize the importance of both LTV and DSC default triggers may overstate the probability of default and thus introduce significant bias to the commercial mortgage risk premium. Furthermore, we explicitly estimate the impact of balloon risk in our mortgage pricing model. As most commercial mortgages are interest-only or partially amortizing loans, balloon risk arises when the borrower is unable to meet the balloon payment at loan maturity. This situation usually occurs during periods of increasing interest rates and/or slow property appreciation rates, when weaker properties may not qualify for a new loan at maturity.

[3] The option-pricing approach of mortgage valuation assumes that a negative equity position is the sole default trigger event. However, a rational borrower of a non-callable mortgage will not default if property cash flow is sufficient to cover debt service, regardless of how negative the equity level is (Vandell, 1995; and Jacob, Hong and Lee 1999).

Empirical evidence by Ciochetti, Gao and Yao (2001) and Goldberg and Capone (2002) indicate a sizable increase in credit risk in the balloon year. Tu and Eppli (2002) estimate the probability of balloon risk and its associated losses. They find that balloon risk is sensitive to property cash flow volatility and changes with the underwriting standards at loan origination and at maturity.

When a borrower is unable to make the balloon payment, the lender may choose to foreclose on the property or renegotiate the loan contract. While academic research has shown that by renegotiating a discounted loan payoff, borrowers and lenders can eliminate default costs associated with property liquidation or transfer,[4] evidence in practice suggests that renegotiation of maturity is a more common form of workout (Harding and Sirmans, 2001). Harding and Sirmans argue that it is because maturity extension better aligns the incentives of borrowers and lenders than principal negotiation. They find that borrowers who expect lenders to renegotiate loan maturity in the event of default (in lieu of discounted payoff) generally have less incentive to extract cash flow from the property during the term of the mortgage and are less likely to take on additional risk, resulting in lower agency costs.[5] We therefore model balloon risk in the form of maturity extension rather than discounted principal payoff.

The results of our simulation analyses reveal that commercial mortgage credit risk premium is overstated when a single-trigger, property price-only model is used in

[4] See Riddiough and Wyatt (1994).

[5] These two types of agency problem are referred to as underinvestment and overinvestment by Gertner and Scharfstein (1991), and are based on the papers of Myers (1977) and Jensen and Meckling (1976), respectively.

mortgage pricing. Consistent with Goldberg and Capone (2002) and Corcoran (2000), a double-trigger property value and property cash flow model is a better estimator of commercial mortgage default. If property cash flow is not less than (possibly significantly less than) the debt service amount for an extended period of time, the borrower has little incentive to default and thus forgo the value of the put option and the possibility of positive cash flows in the future. While a LTV-only model overestimates the credit risk premium on commercial mortgages, simply adding a second, DSC-based trigger may underestimate the risk premium due to the interaction between term default and balloon risk. We find that term default risk premium falls when using a double-trigger model but balloon risk premium increases, as weaker properties that are able to service the debt may be unable to refinance the property at loan maturity.

The remainder of the paper is organized as follows: The next section discusses our choice of mortgage pricing methodology, which is followed by a discussion of the Monte Carlo double-trigger default model. Section 4 describes the parameters and discusses the simulation results. Section 5 is the conclusion.

2. Mortgage Pricing Methodology

The most popular methodology for pricing mortgages in the literature is the contingent-claims model where the partial differential equation is solved using a backward numeric method.[6] In the model borrowers are assumed to behave rationally when making default decisions, defaulting when the property value drops below the mortgage value. The beauty of this pricing approach is that it recognizes the nature of compound default options and explicitly considers the value of these options.

One problem, however, is that the option-pricing approach largely ignores the borrower's cash flow position (Vandell, 1995). Studies utilizing this approach assume that the borrower will default whenever the equity position falls below a critical level, even if the property generates a positive cash flow (net of debt service). They also assume that borrowers have the ability to finance any cash deficiencies in order to keep the default option alive. If property value and property cash flow are highly (perfectly) correlated, separate modeling of property cash flow is unnecessary. However, past experience tells us that property value and property cash flow are not perfectly correlated,[7] and empirical analyses suggest that property cash flow is a strong predictor of default, even after controlling for the borrower's net equity position (Ciochetti, Gao and Yao, 2001; and Goldberg and Capone, 2002). Therefore, a pricing model that considers only property value (measured with contemporaneous LTV), and excludes property cash flow (measured with contemporaneous DSC) may misestimate the probability of default, and thus incorrectly price the mortgage.

A second problem when using the option-pricing approach to price mortgages is that it disregards the lender's role in affecting default through the decision to foreclose (see

[6] Research on commercial mortgage default that apply this approach include Titman and Torous (1989), Kau et al. (1987, 1990), Childs, Ott and Riddiough (1996a, 1996b), and Ciochetti and Vandell (1999). For a detailed discussion of this pricing approach, see Kau and Keenan (1995).

[7] The correlation between the NCREIF property NOI growth rate and capital return in the period 1978 to 1994 was 0.05. ACLI capitalization rates over the same time period ranged from 8.3% to 13.7% with a standard deviation of 1.24%.

Vandell, 1995). For instance, the lender can eliminate deadweight default costs by renegotiating the loan when the borrower is unable to make scheduled payments (see Riddiough and Wyatt, 1994). At maturity the lender may also choose to extend the loan instead of foreclosure, reducing the borrower's incentive to underinvest or overinvest (see Harding and Sirmans, 2001). When pricing mortgages using the backward numeric method, terminal and boundary conditions must be specified, therefore, workout and renegotiation are not permitted.

In sum, a pricing model that incorporates double default triggers and balloon risk is difficult, if not impossible to solve using the backward approach, therefore we use a forward, Monte Carlo simulation approach.[8] With the forward pricing model, we employ both contemporaneous LTV and DSC as default triggers, and consider the possibility of loan extension if the balloon payment is not met. The Monte Carlo approach is also more flexible than the backward pricing method, as it allows factors such as time to foreclosure, foreclosure costs, property payout rates, among other factors to vary, instead of maintaining a constant value.[9]

A major criticism of the Monte Carlo method is its inability to explicitly measure the value of the borrower's default options. To address this issue, we employ the default probability function developed by Riddiough and Thompson (1993), which replaces the

---

[8] Studies that have adopted the forward mortgage pricing approach include Schwartz and Torous (1989a, 1989b, 1991), and Riddiough and Thompson (1993).

[9] In the event of default, both the investment recovery and the timing of the cash flow are uncertain. Empirical studies find that lender transaction costs associated with foreclosure range from approximately 30% to 36% (see Curry, Blalock and Cole, 1991; Snyderman, 1994; Ciochetti, 1997; Ciochetti and Vandell, 1999). Brown, Ciochetti and Riddiough (2000) find that the time lag between the start and completion of the foreclosure process ranges from six to twenty months.

sharp boundaries in ruthless default models with fuzzy boundaries of default. The Riddiough-Thompson model recognizes the influence of default transaction costs on the borrower's default decision and considers the value of default options implicitly.

3. The Monte Carlo Simulation Model

To investigate the effects of credit risk on commercial mortgage valuation, we first specify the state variables employed in the simulation model. Using the Cox, Ingersoll and Ross (1985) mean-reverting interest rate process, the dynamics of interest rate variation are specified as:

$$dr = k\,(q - r)dt + s_r \sqrt{r}\,dz_r, \qquad (1)$$

where k is the speed of reversion parameter, q is the long-term reverting rate, $s_r \sqrt{r}$ is the standard deviation of changes in the current spot rate, and dz_r is a standard Wiener process. A variety of shapes of the yield curve can be described by using different initial interest rates, r_0.

Property values are assumed to follow a log-normal diffusion process:

$$dP = (a_P - b_P)P\,dt + s_P P dz_P, \qquad (2)$$

where P is property price, a_P is the expected total return on the property, b_P is the continuous property payout rate, s_P is a volatility parameter of property returns, and dz_P is a standard Wiener process. To estimate the credit risk premium of commercial mortgages we apply the risk-neutral valuation principle, where the risk-neutral property price process is specified as:

$$dP = (r - b_P)Pdt + s_P Pdz_P, \tag{3}$$

where r is the riskless spot rate, and there exists an instantaneous correlation between changes in property prices and interest rates, r_{Pr}.

Another stochastic variable that must be specified in the simulation model is property cash flow. Monthly property cash flow is determined by multiplying the property value by the property payout rate, which is assumed to be a linear function of market interest rates.[10]

The Riddiough-Thompson fuzzy boundary default function is adopted to account for the value of borrower default options. To address the issue that borrower transaction costs are unobservable to lenders and are heterogeneous across borrowers, Riddiough and Thompson (1993) develop a mortgage pricing model that replaces sharp default boundaries used in ruthless default models with fuzzy boundaries. In their model, default probability is a function of time to maturity and net equity level (which is the reciprocal of the contemporaneous loan-to-value ratio) where:

[10] Since the data on commercial property payout rates is not available, we estimate the relationship between payout rates and interest rates using property capitalization rate as a proxy. A regression of capitalization rates on mortgage contract rates is estimated using ACLI data. A similar approach is also employed by Goldberg and Capone (1998, 2002).

$$E_t = \frac{P_t}{M_t(r)}. \tag{4}$$

The value of the mortgage at time t (M_t), is stated as a function of the current mortgage rate, r. Based on stylized facts summarized from earlier empirical work, Riddiough and Thompson establish default probability rate bounds at mortgage origination, $f(E_0, 0)$, and at mortgage maturity, $f(E_T, T)$, given different equity levels. These bounds are then used to determine the default probability function during the term of the loan.[11] As a result, the lower the property's net equity level, or the closer to the mortgage's maturity, the higher the probability of default. In Riddiough and Thompson's model, a borrower with a negative equity position is more likely to default, however, a negative equity position is not a necessary condition of mortgage default. Figure 1 graphically illustrates the relationship between default probabilities and a property's net equity level at (1) origination, (2) halfway through the loan term, and (3) maturity.

[Figure 1]

We extend this model to include a second default trigger: property cash flow. If property cash flow is adequate to cover debt service, we presume that a rational borrower will not default and forego the positive cash flow and the time value of the default option. In our double-trigger default model, the borrower must incur a negative cash flow position in addition to an adverse net equity position. Therefore, a cash flow trigger event is a necessary condition of default (i.e. a contemporaneous DSC that is less than one).

Furthermore, a borrower is unlikely to default immediately when the DSC initially drops below parity. The borrower may fund a debt service short fall with reserves or other equity sources until either the borrower becomes illiquid, uses all reserves, or perceives that the negative cash flow is likely to persist and property value is unlikely to recover. Unfortunately, no empirical research has examined the magnitude and length of property cash flow deficiencies before default occurs.[12] In the simulation analysis we consider a series of borrower default criteria related to the contemporaneous DSC while also including a LTV-based default trigger.

In our model, we also explicitly account for balloon risk by examining the possibility that the borrower cannot make the balloon payment even though the mortgage is not in default. At maturity, we estimate the loan amount the borrower is able to refinance based on the contemporaneous property value, property cash flow, and interest rate, as well as the underwriting standards (LTV and DSC). If the justified loan amount at maturity is lower than the balloon payment, the borrower will be unable to pay off the existing mortgage immediately. In this case, the lender and the borrower are likely to negotiate a workout. Consistent with Harding and Sirmans (2001), we assume that the lender will agree to extend the loan maturity, while the borrower continues to make periodic payments. At the end of each extended month, the mortgage may be paid off (if the justified loan amount exceeds the balloon payment), in default (if both LTV and DSC

[11] Riddiough and Thompson use a quadratic weighting system to determine the default probability function for a commercial mortgage. For example, when a loan is half way through its term, the lower bound is weighted 75% ($1 - 0.5^2$) while a 25% weight is placed on the upper bound (0.5^2).

[12] We are currently constructing an industry survey to assess borrower default behavior.

default triggers are satisfied), or extended again (otherwise). It is assumed that the mortgage can be extended for up to two years, and the borrower will be forced to liquidate the property and terminate the existing mortgage if neither default nor payoff occurs during the extension period.

4. Parameters and Simulation Results

Using the simulation model discussed in Section 3, we examine how term default and balloon risk affect the valuation of a ten-year commercial mortgage with a 30-year amortization schedule. To isolate the impact of credit risk on mortgage pricing, we presume a non-callable mortgage.[13] Table 1 describes the mortgage terms, parameters of the interest rate and property value processes, and other variables used the simulations.

[Table 1]

a. Term Default

We first examine the effects of using a double-trigger default model on mortgage value and credit risk premium without considering balloon risk. In the base case we assume a flat yield curve with k = 25.0%, q = 7.5%, and s_r = 8.0%.[14] While the literature generally assumes property price follows a lognormal diffusion process, there is no consensus on the volatility (s_P). We consider a series of standard deviations for the

[13] Commercial mortgage pricing studies have generally presumed non-callable mortgages (see Titman and Torous, 1989; Riddiough and Thompson, 1993; Childs, Ott and Riddiough, 1996). Most commercial mortgages have lockout periods and strict prepayment penalties in the form of defeasance and yield maintenance prepayment penalties.

[14] These parameters are consistent with existing studies on commercial mortgage pricing.

property value process: 12%, 15% and 18%.[15] The property payout rate is defined as a function of long-term interest rate with an initial value of 8.1%.[16]

Table 2 presents the simulated mortgage values and default risk premiums using the double-trigger default model and 10,000 Monte Carlo state variable paths.[17] To highlight how the addition of a second default trigger (DSC) influences the pricing results, we first estimate the mortgage value and default risk premium using a single-trigger, LTV-based, default criterion (Model 1 in Table 2). Across three-property value volatility measures (12%, 15%, and 18%) the default risk premiums range from 127 to 230 basis points. The single-trigger results in Model 1 provide the basis for comparing the results of adding a second, property cash flow trigger to the default model.

[Table 2]

The Riddiough-Thompson fuzzy boundary default function recognizes the unobservable borrower default transaction costs and thus reflects the non-ruthlessness of borrower default behavior. Furthermore, it could have implicitly accounted for the borrower's

---

[15] Property value volatility estimates have varied widely in literature. For example, Titman and Torous (1989) use a series of volatility measures from 15% to 22.5%; Riddiough and Thompson (1993) use 12% and 16%; and Childs, Ott and Riddiough (1996a) use 15% and 20%. Ciochetti and Vandell (1999) compute implied property price volatility using loan level data and find the volatility of various property types ranges from 16% to 18% (with the exception of mixed-use properties). Their findings are consistent with volatilities suggested by Geltner, Graff and Young (1994).

[16] The payout rate is specified as a linear function of long-term interest rate:

$$Payout\ Rate = a + Interest\ \ Rate \times b + e\ .$$

Using ACLI data on mortgage interest rates and property capitalization rates, we have estimated that a = 4.8%, b = 0.45, and the residual standard error is 0.3%. The initial payout rate is generally consistent with the literature.

[17] Additional assumptions made in the base case simulation analysis include: (1) Average lender loss rate of 15% of the mortgage value in foreclosure, with a standard deviation of 5% and minimum of 5%; (2) A

liquidity position.[18] To verify whether their model can adequately reflect the borrower default behavior indicated by a double-trigger model, we start with a less-restrictive DSC-based criterion. In Model 2 of Table 2, a necessary condition of DSC<1 is added. The results reveal a 1 to 5-basis point reduction in the mortgage default premium. Although the difference is small, it is statistically significant. The results indicate that explicitly considering the DSC trigger is necessary to properly measure the credit risk of commercial mortgages.

While the default behavior in Model 2 is plausible, we find it unlikely in that most borrowers have the liquidity to cover small, temporary cash flow deficits to keep the options alive. Generally speaking, when the DSC is slightly lower than 1, the probability that property cash flow again will become positive is relatively high, and the cost of keeping the default option open is relatively small.[19] Therefore, the borrower is unlikely to default immediately when the DSC drops below one. Thus, we consider more restrictive cash flow default triggering conditions. Model 3 in Table 2 considers three consecutive months of cash flow deficits, and reveals slightly lower credit risk premiums (6-10 basis points) over Model 1.

20-month investment recovery lag; and (3) Investor carrying costs between default and foreclosure average 0.5% per month of the outstanding loan balance.

[18] In the Riddiough-Thompson model, default is unlikely but possible when the net equity level is positive; on the other hand, default does not occur immediate when the net equity level turns negative.

[19] For example, the monthly cost of maintaining the option of future cash flows and property appreciation is 0.067% of the loan amount, when DSC drops to 0.9 for a loan with 8.0% mortgage constant. In other words, multiplying the monthly mortgage constant of 0.67% per month by the 0.10 (i.e. 1-DSC) returns a monthly debt service shortfall of 0.067% of the loan amount.

A recent trend in the commercial mortgage underwriting is the requirement of cash reserves or escrows as a cash flow volatility buffer.[20] With a reserve account, borrowers can fund small cash flow deficits. Models 4-6 assume that the borrower has reserves to fund a one-month, three-month, and six-month cumulative debt service shortfall in the previous twelve-month period, where a one-month shortfall is equal to one month's debt service, and so on. The possibility of the borrower funding debt service out of an escrow account for the cumulative amount of one, three, or six month's debt service appears entirely reasonable.[21] Under these conditions, default risk premiums drop, and in some cases drop sharply.

In Model 4 of Table 2, the risk premiums drop to 105-210 basis points, or a 20-22 basis point reduction across the three-property value volatility measures relative to Model 1. When the borrower has the ability to fund a three-month cumulative debt service shortfall, the risk premiums shrink to 64-162 basis points across the 12%, 15%, and 18% property value standard deviation scenarios. Finally, in Model 6, when the borrower can fund a six-month debt service shortfall in the previous twelve-month period, risk premiums fall dramatically. Term default risk premiums range from 9 to 61 basis points.[22]

---

[20] For an example, see the *Fitch Commercial Mortgage Presale Report, GE Capital Commercial Mortgage Corp., Series 2001-3*. The summary statistics on page 4 reveal that 93% of all mortgages in the pool have capital reserve requirements and over 86% have upfront or ongoing expense reserve requirements.

[21] With an 8.0% mortgage constant, the cost of keeping the option open is 4% of the loan amount at origination in the most restrictive six-month cash flow shortfall case.

[22] Over the past three years ACLI commercial mortgage delinquency rates have hovered around 30 basis points, with a December 2001 delinquency rate of 12 basis points. If approximately 30% of delinquent loans default, and the loss rate on default is approximately 35% of the outstanding loan balance (see

Additional simulations are also completed where we adjust the shape of the yield curve, the property payout rate, loan origination terms, among others parameters, and a similar pattern of default risk premium reductions between single-trigger and the double-trigger models are found.

b. Term Default and Balloon Risk

Commercial mortgages have one of three possible outcomes at maturity: 1) Default, 2) Payoff, and 3) Extension. Table 3 presents the term default, payoff, and extension probabilities across the six models presented in Table 2. Note that while the mortgage values and risk premiums are estimated in a risk-neutral framework, default and extension probabilities must be stated in real terms. Hence, an expected property total return (a_P) is necessary.[23] In the simulation we assume a_P = 11.0%. As expected, default probabilities decrease monotonically across both property volatility and default models. While the percent of mortgages that are paid off at maturity is relative stable across default models, the payoff rate decreases as property volatility rises. Interestingly, the risk of a loan being extended increases with the borrower's ability to fund property income shortfalls. In other words, loans that would have otherwise defaulted prior to maturity now reach the balloon payment date; however, these weaker properties are less likely to be refinanced under contemporaneous underwriting standards.

[Table 3]

Ciochetti, 1997), commercial mortgage loss rates should have been less than 10 basis points over the last three years for ALCI loans.

[23] For a discussion of the procedure to calculate default probabilities for mortgages, see Kau, Keenan and Kim (1994).

Table 4 presents the results of commercial mortgage pricing simulations when both term default and balloon risk are considered. In Models 1-3 the results show a similar pattern to those presented in Table 2, although risk premiums are 9-12 basis points higher to account for balloon risk. However, as the borrower is able to fund a one-month to six-month debt service shortfall (Models 4, 5 and 6), the impact of balloon risk on mortgage risk spreads increases dramatically. The increase in balloon risk premium is due to the more restrictive default trigger conditions. In Model 6 risk premiums increase 36-52 basis points across the three property volatility measures, and in two cases more than double the term default risk spread. These results reveal an interesting interaction between term default and balloon risk. Although the double-trigger model is superior to single-trigger model, in the sense that it better models borrower default behavior and thus improves the estimation of default risk premium, simply adding a cash flow-based default trigger without considering its effect on balloon risk may introduce a different kind of bias in mortgage pricing.

[Table 4]

5. Conclusion

In this paper we extend the commercial mortgage pricing literature by using a double-trigger default model that accounts for balloon risk. As found in recent empirical studies of commercial mortgage default (Archer et al., 2001; Ciochetti, Gao and Yao, 2001; and Goldberg and Capone, 2002), contemporaneous DSC is an important predictor of default, in addition to contemporaneous LTV. Adopting the fuzzy boundary default function by Riddiough and Thompson (1993), we develop a Monte Carlo simulation model that uses both property net equity level and property cash flow as default triggers. The results reveal that failure to consider property cash flow significantly overestimates the probability and risk premium of term default.

While most existing research on commercial mortgage pricing assumes that a performing mortgage is immediately paid off at maturity, it is possible that the borrower is unable to make the balloon payment is the property does not meet the contemporaneous underwriting standards. Harding and Sirmans (2001) suggest that it may be in the borrower's and the lender's best interest to negotiate a maturity extension, in lieu of a discounted payoff. We therefore examine the effects of loan extension and possible default during the extension on the mortgage value and the risk premium. The results reveal an interaction between term default and balloon risk. While the double-trigger default criteria (especially in the more restrictive DSC standard cases) generally result in lower term default risk premiums, these low term default risk premiums are partially offset by higher balloon risk premiums.

As a recent trend in commercial mortgage underwriting, property escrows and reserves are often required. These requirements serve as a cash flow volatility buffer and reduce term default risk. However, a property with a depressed value and low property cash flow makes property refinancing at maturity tenuous. As a result, a decrease in term default is likely to lead to an increase in balloon risk, and the reduction in credit risk premium caused by lower term default is partially offset by the higher balloon risk.

References

Abraham, J. 1993. A Cash Flow Model of Property Performance. Unpublished manuscript. McLean, VA: Freddie Mac Corporation.

Ambrose, B. and A.B. Sanders. 2001. Commercial Mortgage-Backed Securities: Prepayment and Default. Working paper: University of Kentucky.

Archer, W.R., P.J. Elmer, D.M. Harrison and D.C. Ling. 2001. Determinants of Multifamily Mortgage Default. Working paper. Washington, DC: FDIC.

Brown, D.T., B.A. Ciochetti and T.J. Riddiough. 2000. Liquidity Provision and Liquidity Transformation in the Resolution of Financial Distress. Working paper. University of North Carolina.

Childs, P.D., S.H. Ott and T.J. Riddiough. 1996a. The Value of Recourse and Cross-Default Clauses in Commercial Mortgage Contracting. *Journal of Banking and Finance* 20: 511-536.

———. 1996b. The Value of Recourse and Cross-Default Clauses in Commercial Mortgage Contracting. *Journal of Banking and Finance* 20: 511-536.

Ciochetti, B.A. 1997. Loss Characteristics of Commercial Mortgage Foreclosures. *Real Estate Finance* 14(1): 53-69.

Ciochetti, B.A., B Gao, and R. Yao. 2001. "The Termination of Lending Relationships through Prepayment and Default in the Commercial Mortgage Markets: A Proportional Hazard Approach with Competing Risks. Working paper. University of North Carolina.

Ciochetti, B.A. and K.D. Vandell. 1999. The Performance of Commercial Mortgages. *Real Estate Economics* 27(1): 27-61.

Corcoran, P. 2000. Stress and Defaults in CMBS Deals: Theory and Evidence. *Real Estate Finance* 17(3): 63-71.

Cox, J.C., J.E. Ingersoll and S.A. Ross. 1985. A Theory of the Term Structure of Interest Rates. *Econometrica* 53: 385-407.

Curry, T., J. Blalock and R. Cole. 1991. Recoveries on Distressed Real Estate and the Relative Efficiency of Public versus Private Management. *AREUEA Journal* 19: 495-515.

Geltner, D., R. Graff and M. Young. 1994. Appraisal Error and Commercial Property Heterogeneity: Evidence from Disaggregate Appraisal-Based Returns. *Journal of Real Estate Research* 4: 403-420.

Gertner, R. and D. Scharfstein. 1991. A Theory of Workouts and the Effects of Reorganization Law. *Journal of Finance* 46(4): 1189-1222.

Goldberg, L. and C.A. Capone. 1998. Multifamily Mortgage Credit Risk: Lessons from Recent History. *Cityscape* 4(1): 93-113.

———. 2002. A Dynamic Double-Trigger Model of Multifamily Mortgage Default. *Real Estate Economics*, forthcoming.

Harding, J.P. and C.F. Sirmans. 2001. Renegotiation of Trouble Debt: The Choice between Discounted Payoff and Maturity Extension. Working paper. University of Connecticut.

Jacob, D.P., T. Hong and L. H. Lee. 1999. An Options Approach to Commercial Mortgages and CMBS Valuation and Risk Analysis. In F.J. Fabozzi and D.P. Jacob (Eds). *The Handbook of Commercial Mortgage-Backed Securities* (pp. 317-346). New Hope, Pennsylvania: Frank J. Fabozzi Associates.

Jensen, M.C. and W.H. Meckling. 1976. Theory of the Firm: Managerial Behavior, Agency Costs and Capital Structure. *Journal of Financial Economics* 3(4): 305-360.

Kau, J.B. and D.C. Keenan. 1995. An Overview of the Option-Theoretic Pricing of Mortgages. *Journal of Housing Research* 6(2): 217-244.

Kau, J.B., D. C Keenan and T. Kim. 1994. Default Probabilities for Mortgages. *Journal of Urban Economics* 35: 278-296.

Kau, J.B., D.C. Keenan, W.S. Muller and J.F. Epperson. 1987. The Valuation and Securitization of Commercial and Multifamily Mortgages. *Journal of Banking and Finance* 11: 525-546.

———. 1990. Pricing Commercial Mortgages and Their Mortgage-Backed Securities. *Journal of Real Estate Finance and Economics* 3: 333-356.

Myers, S.C. 1977. Determinants of Corporate Borrowing. *Journal of Financial Economics* 5(2): 147-175.

Riddiough, T.J. and H.E. Thompson. 1993. Commercial Mortgage Pricing with Unobservable Borrower Default Costs. *AREUEA Journal* 21:265-291

Riddiough, T.J. and S.B. Wyatt. 1994. Strategic Default, Workout, and Commercial Mortgage Valuation. *Journal of Real Estate Finance and Economics* 9: 5-22.

Schwartz, E.S. and W.N. Torous. 1989a. Prepayment and the Valuation of Mortgage-Backed Securities. *Journal of Finance* 44(2): 375-392.

———. 1989b. Valuaing Stripped Mortgage-Backed Securities. *Housing Finance Review* 8(4): 241-251.

———. 1991. Caps on Adjustable Rate Mortgages: Valuation, Insurance, and Hedging. In R.G. Hubbard (Ed.). *Financial Markets and Financial Crises*. Chicago: University of Chicago Press.

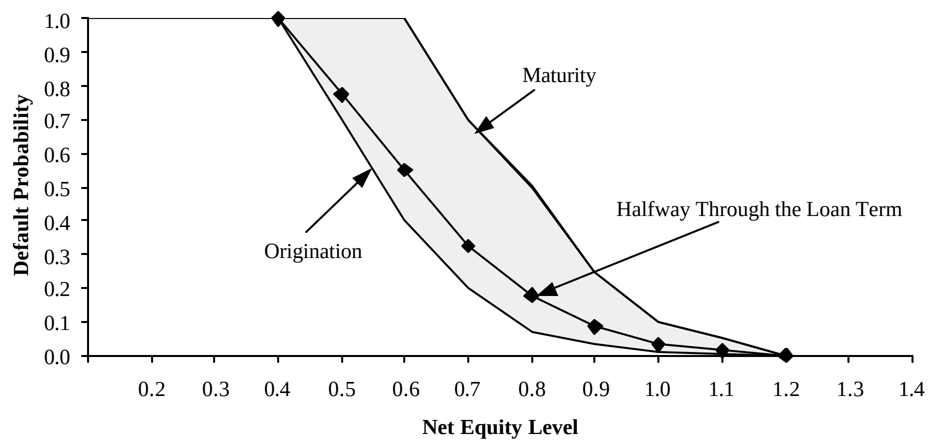
Snyderman, M. 1994. Update on Commercial Mortgage Defaults. *Real Estate Finance* 10(1): 22.

Titman, S. and W. Torous. 1989. Valuing Commercial Mortgages: An Empirical Investigation of the Contingent-Claims Approach to Pricing Risky Debt. *Journal of Finance* 44(2): 345-373.

Tu, C. and M.J. Eppli. 2002. Extension Risk in Commercial Mortgages. *Real Estate Finance,* forthcoming.

Vandell, K.D. 1995. How Ruthless Is Mortgage Default? A Review and Synthesis of the Evidence. *Journal of Housing Research* 6(2): 245-264.

Figure 1. Default Probability as a Function of Property's Net Equity Level



Source: Riddiough and Thompson (1993)

Table 1. Base-Case Parameter Values in the Monte Carlo Simulations

Parameter	Value
Contract Terms	
Mortgage Amount	$1,000,000
Mortgage Term	10 years
Amortization Schedule	30 years
LTV [1]	70%
DSC [1]	1.30
Property Parameters	
Initial Property Value (P_0) [2]	$1,428,571
Initial Payout Rate (b_P) [3]	8.1%
Standard Deviation (s_P)	12.0%, 15.0% and 18.0%
Interest Rate Parameters	
Initial Spot Rate (r_0)	7.5%
Reversion Speed (k)	25.0%
Long-term Reverting Rate (q)	7.5%
Standard Deviation (s_r)	8.0%
Correlation Between r and P (r_{Pr})	0
Number of Iterations	10,000

[1] Mortgage underwriting standards at loan origination and at maturity.
[2] Property value is calculated based on mortgage amount and original LTV.
[3] The payout rate is specified as a function of long-term interest rate:

$$Payout\ Rate = a + Interest\ \ Rate \times b + e \ .$$

Using ACLI data on mortgage interest rates and property capitalization rates, we have estimated that a = 4.8%, b = 0.45, and residual standard error is 0.3%.

Table 2. Mortgage Values and Risk Premiums Assuming Term Default Only

	Property Price Volatility		
Default Criteria	$s_P = 12\%$	$s_P = 15\%$	$s_P = 18\%$
Model 1			
Net Equity Level (NEL) only			
Mortgage Value ($)	919,433	888,242	859,997
Risk Premium (basis points)	127	180	230
Model 2			
NEL & DCR<1			
Mortgage Value ($)	922,072	889,902	860,370
Risk Premium (basis points)	122	177	229
Risk Premium Change from Model 1	-5	-3	-1
Model 3			
NEL & DSC < 1 for Three Consecutive Months			
Mortgage Value ($)	925,096	892,816	863,386
Risk Premium (basis points)	105	172	224
Risk Premium Change from Model 1	-10	-8	-6
Model 4			
NEL & One-Month Cash Flow Deficiency			
Mortgage Value ($)	932,741	900,664	871,208
Risk Premium (basis points)	105	158	210
Risk Premium Change from Model 1	-22	-22	-20
Model 5			
NEL & Three-Month Cash Flow Deficiency			
Mortgage Value ($)	958,558	928,523	896,713
Risk Premium (basis points)	64	112	-162
Risk Premium Change from Model 1	-63	-68	-68
Model 6			
NEL & Six-Month Cash Flow Deficiency			
Mortgage Value ($)	993,817	980,642	960,117
Risk Premium (basis points)	9	29	61
Risk Premium Change from Model 1	-118	-151	-169

This table presents Monte Carlo Simulation results based on the following assumptions and parameters. Loan terms include a $1,000,000 loan amount, 10-year term, 30-year amortization schedule, a 70% LTVR (at origination), and a 1.30 DCR (at origination). Interest rates are modeled with initial spot rate, r_0, of 7.5%, long-term reverting rate, q, of 7.5%, volatility, s_r, of 8.0%, and a reversion speed parameter, ?, of 0.10. Property return volatility, s_p, varies between 12% and 18% as stated above, and correlation between property price and interest rate, $?_{p,r}$, is 0. Property payout rate, b_p is initially set at 8.1%, and then varies as interest rate fluctuates. The mean time to foreclosure is 20 months. The asset recovery rate on is 85% with a standard deviation of 5% and a carrying cost per month of 0.5% of the loan balance. For each parameterization 10,000 Monte Carlo paths were conducted.

Table 3. Probabilities of Term Default, Payoff, and Extension (in percent)

Default Model	Term Default	Payoff	Extension
Panel A: 12% Property Standard Deviation			
Model 1: Net Equity Level (NEL) only	17.8	70.4	11.8
Model 2: NEL & DCR < 1	16.7	71.0	12.3
Model 3: NEL & DSC < 1 for Three Consecutive Months	15.4	71.6	13.0
Model 4: NEL & One-Month Cash Flow Deficiency	11.9	72.5	15.6
Model 5: NEL & Three-Month Cash Flow Deficiency	4.2	73.3	22.5
Model 6: NEL & Six-Month Cash Flow Deficiency	0.3	73.4	26.3
Panel B: 15% Property Standard Deviation			
Model 1: Net Equity Level (NEL) only	28.0	61.9	10.1
Model 2: NEL & DCR < 1	26.9	62.4	10.7
Model 3: NEL & DSC < 1 for Three Consecutive Months	25.3	63.0	11.7
Model 4: NEL & One-Month Cash Flow Deficiency	21.7	64.3	14.0
Model 5: NEL & Three-Month Cash Flow Deficiency	11.5	68.4	22.1
Model 6: NEL & Six-Month Cash Flow Deficiency	1.6	67.0	31.4
Panel C: 18% Property Standard Deviation			
Model 1: Net Equity Level (NEL) only	36.6	53.9	10.5
Model 2: NEL & DCR < 1	35.9	54.3	9.8
Model 3: NEL & DSC < 1 for Three Consecutive Months	34.2	55.1	10.6
Model 4: NEL & One-Month Cash Flow Deficiency	30.6	56.7	12.7
Model 5: NEL & Three-Month Cash Flow Deficiency	19.4	59.7	20.9
Model 6: NEL & Six-Month Cash Flow Deficiency	4.7	60.8	34.5

Probabilities are estimated in the real terms, assuming an expected property total return, a_P, of 11%. Other parameters are the same as the risk-neutral simulation presented in Table 2.

Table 4. Mortgage Values and Risk Premiums with Double Triggers and Balloon Risk

Default Criteria	Property Price Volatility $s_P = 12\%$	$s_P = 15\%$	$s_P = 18\%$
Model 1			
Net Equity Level (NEL) only			
Mortgage Value ($)	913,736	883,234	855,003
Risk Premium (basis points)	136	189	239
Risk Premium Change from Term Default-Only Model	9	9	9
Model 2			
NEL & DSC < 1			
Mortgage Value ($)	915,859	884,364	854946
Risk Premium (basis points)	133	187	239
Risk Premium Change from Model 1	-3	-2	0
Risk Premium Change from Term Default-Only Model	11	10	10
Model 3			
NEL & DSC < 1 for Three Consecutive Months			
Mortgage Value ($)	918,263	886,508	856,949
Risk Premium (basis points)	129	183	236
Risk Premium Change from Model 1	-7	-6	-3
Risk Premium Change from Term Default-Only Model	12	11	12
Model 4			
NEL & One-Month Cash Flow Deficiency			
Mortgage Value ($)	923,786	892,204	863,061
Risk Premium (basis points)	120	173	225
Risk Premium Change from Model 1	-13	-16	-14
Risk Premium Change from Term Default-Only Model	15	11	15
Model 5			
NEL & Three-Month Cash Flow Deficiency			
Mortgage Value ($)	945,157	913,187	883,987
Risk Premium (basis points)	85	137	187
Risk Premium Change from Model 1	-51	-52	-52
Risk Premium Change from Term Default-Only Model	19	25	25
Model 6			
NEL & Six-Month Cash Flow Deficiency			
Mortgage Value ($)	970,370	950,506	927,753
Risk Premium (basis points)	45	76	113
Risk Premium Change from Model 1	-91	-113	-126
Risk Premium Change from Term Default-Only Model	36	47	52

This table presents Monte Carlo Simulation results considering both double default trigger and balloon risk. Borrower default behavior during the term of the mortgage is the same as in Table 2. At maturity, a justified refinance loan amount is calculated based on the contemporaneous property value, property cash, and interest rate, as well as the underwriting standards (LTV and DSC). If the justified loan amount is lower than the balloon payment, the mortgage will be extended and the borrower continues to make periodic payments. At the end of each extended month, the mortgage may be paid off (if the justified loan amount exceeds the balloon payment), defaulted (if both LTV and DSC triggers are met), or extended again (otherwise). It is assumed that the mortgage can be extended for up to two years, and the borrower will be forced to liquidate the property and terminate the existing mortgage if neither default nor payoff occurs during the extension period.