Forthcoming in Real Estate Economics

Estimating Returns on Commercial Real Estate: A New Methodology Using Latent Variable Models

by

*David C. Ling, Andy Naranjo, and M. Nimalendran**

*University of Florida
Warrington College of Business Administration
Department of Finance, Insurance and Real Estate
321 Stuzin Hall
P.O. Box 117168
Gainesville, FL 32611-7168
Phone: (352) 392-0153
Fax: (352) 392-0301

December 1998
Latest Revision: July 1999

We thank David Downs, David Geltner, Michael Giliberto, William Goetzmann, the participants at the 1999 Real Estate Research Institute (RERI) annual conference, and two anonymous referees for valuable comments and suggestions. Naranjo and Nimalendran also thank RERI for providing funding support for this project.

Estimating Returns on Commercial Real Estate:
A New Methodology Using Latent Variable Models

Abstract

Despite their widespread use as benchmarks of U.S. commercial real estate returns, indices produced by the National Council of Real Estate Investment Fiduciaries (NCREIF) are subject to measurement problems that severely impair their ability to capture the true risk/return characteristics—especially volatility—of privately-held commercial real estate. We utilize latent variable statistical methods to estimate an alternative index of privately-held (unsecuritized) commercial real estate returns. Latent variable methods have been extensively applied in the behavioral sciences literature and, more recently, in the finance and economics literature. Unlike factor analysis or other unconditional statistical approaches, latent variable models allow us to extract interpretable common information about unobserved private real estate returns using the information contained in various competing measures of returns that are measured with error. We find that our latent variable real estate return series is approximately twice as volatile as the aggregate NCREIF total return index, but less than half as volatile as the NAREIT equity index. Overall, our results strongly support the use of latent variable statistical models in the construction of return series for commercial real estate.

Estimating Returns on Commercial Real Estate: A New Methodology Using Latent Variable Models

1. Introduction

The return performance of both publicly-traded (securitized) and privately-held (unsecuritized) commercial real estate, and their proper roles in mixed-asset portfolios have received considerable attention over the past decade. Critical inputs into the evaluation of return performance and the specification of input parameters in mean-variance asset allocation decisions include historical means and variances, as well as covariances among the returns from the various asset classes potentially includable in the mixed-asset portfolio. For publicly traded stocks, including real estate investment trusts (REITs), historical return measurement poses little problem, as an abundance of performance data are available for both individual assets and portfolios. However, less than five percent of investible U.S. commercial real estate is publicly traded. And, unfortunately, transaction-based return information on privately-held commercial properties is virtually nonexistent due to the relative infrequency of sales of the same property. In the absence of transaction data, the private market uses periodic property appraisals to assess changes in values. This process creates smoothing and inertia in unsecuritized valuation series; a problem that has been extensively documented and discussed in the professional and academic literature (see, for instance, Giliberto, 1988, and Geltner, 1991).

In this paper, we use latent variable statistical methods to estimate an alternative index of returns on privately-held commercial real estate. Latent variable methods have been extensively applied in the behavioral sciences literature, where they have allowed researchers to analyze such abstract concepts as intelligence and motivation from proxies that are measured with error (see, for example, Joreskog and Sorbom, 1979). These same methods also have recently been applied in the finance and economics literature to measure unobservable macroeconomic risk factors, the determinants of corporate capital structure choice, and the effects of earnings surprises on stock prices.[1] Unlike factor analysis or other unconditional statistical approaches, latent variable models allow us to extract interpretable common information about unobserved private real estate returns using the information contained in various competing measures of returns that are measured with error.

The results from applying latent variable techniques to estimate a return series for unsecuritized commercial real estate are encouraging. Using quarterly data from 1978 - 1997, we estimate a latent real estate return series that is about twice as volatile as the aggregate NCREIF total return index, but less than half as volatile as the NAREIT Equity Index. Furthermore, the latent real estate returns generally move with returns on publicly traded real estate companies (correlation of 0.62), although with much less volatility. The latent real estate returns also exhibit a high correlation with NCREIF returns (0.61), but they are significantly more volatile and often lead NCREIF returns. The high correlation and relative volatility of our latent real estate return series with both the public and private benchmarks are intuitively appealing. Moreover, unlike several traditional "adjustments" to NCREIF returns, no ad-hoc assumptions or adjustments are required. Rather, our estimated latent real estate return series is derived from a sound

[1] See, for instance, Ferson (1990), Titman and Wessels (1988), and Maddala and Nimalendran (1995). Mei and Liu (1994) also use a variation of latent variable techniques in an asset-pricing framework.

theoretical foundation and widely available proxies for real estate returns. Overall, our results strongly support the use of latent variable statistical models in the construction of return series for commercial real estate.

The following section provides some background on the difficulties associated with measuring periodic total returns on (infrequently traded) commercial real estate. Section 3 presents our latent variable framework, estimation procedures, and test statistics. Section 4 discusses the data employed in the estimation, and Section 5 contains our results along with specification and robustness tests. We conclude with a summary in Section 6.

2. Background

Two types of real estate performance data are commonly used as proxies for the unobservable (latent) returns on privately-held commercial real estate: (1) indices of returns on privately-held, investment grade real estate, and (2) indices of returns on publicly-traded real estate investment trusts (REITs). As measures of unsecuritized commercial real estate returns, both data sources suffer from well known deficiencies.

The most widely used indices of privately-held, institutional-quality real estate returns are produced quarterly by the National Council of Real Estate Investment Fiduciaries (NCREIF). A significant shortcoming of the NCREIF total return index is the way in which changes in the values of the properties that comprise the index are estimated. At least once a year, independent fee appraisals are obtained by the owner on all index properties. These appraisals are then up-dated in house during the intervening quarters. If a property in the index happens to sell in a particular quarter, then only is a transaction price used to calculate the property's appreciation. Thus, the appreciation component of the total return index return is largely estimated from changes in appraised values.

The use of appraisals in determining quarterly changes in property values has received substantial criticism because both theory and empirical evidence strongly suggests that the use of appraisal-based value estimates significantly "smoothes" indicated changes in underlying property values. This smoothing is thought to result from the tendency of appraisals to lag (i.e., only partially adjust to) true value changes. The appraisal-based smoothing causes downward biased estimates of total return volatility.[2] Geltner (1991) and others have developed statistical techniques for "unsmoothing" appraisal-based return series. In effect, these unsmoothing techniques attempt to "put back" the true volatility of the portfolio's return over time. These techniques significantly increase the estimated volatility of NCREIF total returns. However, the process of unsmoothing is somewhat ad hoc and potentially subject to numerous problems (see, for example, Lai and Wang, 1998).

Indices of returns on publicly-traded REITs are produced by a number of firms and organizations, including the National Association of Real Estate Investment Trusts (NAREIT) and WILSHIRE Associates. Because these return indices are based on actual transactions of publicly traded shares, they provide a precise measure of REIT portfolio returns.[3] However, Giliberto (1993) and others have argued that REIT returns and prices

[2] For more on the effect of appraisal-based returns on return volatility, see Geltner (1991), Fisher, Geltner, and Webb (1994) and Giliberto (1988). Geltner (1991) also shows that such "appraisal smoothing" may bias the mean return.

[3] For more information on the construction of these and other REIT-based indices, see Ziering and Taylor (1998) and Giliberto and Sidoroff (1995).

may reflect factors not inherently attributable to the underlying properties, such as general stock price movements, the use of financial leverage at the REIT level, and changes in the market's opinion of the REIT's management team. Thus, REIT performance patterns may not be representative of the true risk and returns associated with investing directly in the underlying real estate.

Several techniques have been proposed to address the potential deficiencies of using REITs to measure underlying real estate returns. For instance, techniques are available for removing the effects of REIT leverage on returns. However, these "unlevering" methods are imprecise and sensitive to a number of assumptions. Furthermore, to strip REIT portfolio returns of the excess volatility associated with the general stock market, Giliberto (1993) has developed a procedure to "smooth" or "market-adjust" REIT returns. Giliberto's technique decomposes periodic changes in, say, the NAREIT price index into the portion explained by movements in the general stock market (S&P 500) and the idiosyncratic (or unexplained) portion. In calculating the appreciation component of the underlying real estate return, only the idiosyncratic portion of the NAREIT return is used. Although useful, this technique also has its limitations. Most importantly, it assumes that underlying commercial real estate returns are orthogonal to general stock price movements. However, because some of the macroeconomic risk factors that move the general stock market (changes in interest rates, unexpected inflation, etc.) have been shown to affect commercial real estate values, we believe this technique provides an estimate of real estate return volatility that is biased downward.[4]

In short, despite numerous suggestions for correcting errors in real estate return measurement (unsmoothing, unlevering, etc.), both appraisal-based and REIT-based indices of underlying real estate returns are measured with substantial error. However, these and other commercial real estate return measures do contain elements of "truth." The power of the latent variable framework is its ability to extract underlying commercial real estate returns from the co-movements of these various return indices.

3. Research Methodology

Numerous models in financial economics are formulated in terms of theoretical or hypothetical concepts that are not directly observable or measurable (i.e., latent variables). However, several indicators or proxies for these unobservable variables are often available, although these proxy variables measure the unobservable variable with error. Thus, their use as regressors in regression models leads to standard errors-in-variables problems such as biased estimates and inconsistent standard errors. Moreover, when there are multiple indicators of multiple latent attributes, a single equation regression framework is not capable of identifying the independent attributes. However, if a single unobservable (or latent) variable occurs in different equations as an explanatory variable (multiple indicators of a latent variable), the latent variable/structural equation methodology can produce consistent estimates of the

[4] For evidence that common macroeconomic risk factors are important determinants of real estate returns, see Ling and Naranjo (1996) and Karolyi and Sanders (1998).

coefficients on the unobservable variable. Although many problems in real estate and finance fall into this category, there have been relatively few applications of these models to these areas.[5]

3.1 A General Formulation of Latent Variable Models with Measurement Errors

The latent variable models used in this study are discussed in Zellner (1970), Goldberger (1972), Griliches (1974), Joreskog and Goldberger (1975), and popularized by the LISREL program of Joreskog and Sorbom (1989).[6] Suppose that **Y** is a px1 vector of observable indicator variables, and **Z** is a qx1 vector of unobservable latent variables. Furthermore, assume that **Y** is linearly related to **Z** through the parameter matrix $\boldsymbol{\beta}$ plus an error term $\boldsymbol{\varepsilon}$ with variance-covariance matrix $\boldsymbol{\Omega}$. The multiple indicator/multiple latent variable model can then be formulated as follows:

(1) $$\mathbf{Y} = \boldsymbol{\beta}\mathbf{Z} + \boldsymbol{\varepsilon}\ .$$

The model depicted in equation (1) can be estimated using several techniques including maximum likelihood and weighted least squares (both are described in detail in the Appendix). This model has several advantages over traditional regression procedures used to measure underlying commercial real estate returns. First, the information from several proxies is used efficiently. Second, the errors in the structural equation and any measurement errors in the proxy variables are explicitly modeled. Third, the model allows for correlation among the error terms and latent variables. Finally, under general

---

[5] Notable exceptions in corporate finance are models estimated by Titman and Wessels (1988) and Maddala and Nimalendran (1995).

[6] The models are also referred to as linear structural models with measurement errors, analysis of covariance structures, path analysis, causal models, and content variable models. Bentler and Bonett (1980) and Bollen (1989) provide an excellent introduction to the subject.

conditions, the latent variable estimation technique provides consistent estimates, allowing researchers and practitioners to make correct inferences.[7]

3.2 A Simple Three Indicator/One Latent Variable Model

Suppose there are three indicator variables (y_1, y_2, y_3) that measure one latent variable (z) with mutually independent errors (ε_1, ε_2, ε_3) that have zero means and variances, $\sigma_i^2, i = 1,2,3.$ The indicator variables and the latent variable are assumed to have zero means. Furthermore, assume the indicator variables are correlated through the latent variable z, and that z is uncorrelated with the errors. This leads to the following latent variable model:

$$(2.1-2.3)\qquad \begin{aligned} y_1 &= \beta_1 z + \varepsilon_1, \\ y_2 &= \beta_2 z + \varepsilon_2, \\ y_3 &= \beta_3 z + \varepsilon_3. \end{aligned}$$

The parameters, β's, and error variances can be consistently estimated using the information in the sample variance-covariance matrix of the observable indicator variables (the y's). Let the sample variance-covariance matrix for the y's be:

$$S = Cov\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ & s_{22} & s_{23} \\ & & s_{33} \end{bmatrix}.$$

The variance-covariance structure implied by the system of equations (2.1-2.3) is given by:

$$\Sigma = \begin{bmatrix} \beta_1^2\sigma_z^2 + \sigma_1^2 & \beta_1\beta_2\sigma_z^2 & \beta_1\beta_3\sigma_z^2 \\ & \beta_2^2\sigma_z^2 + \sigma_2^2 & \beta_2\beta_3\sigma_z^2 \\ & & \beta_3^2\sigma_z^2 + \sigma_3^2 \end{bmatrix}.$$

[7] Maddala and Nimalendran (1995) examine the effects of unexpected earnings on market microstructure and show that measurement errors can substantially bias the OLS estimates leading to incorrect inferences. Israel, Ofer, and Siegel (1990) provide simulation results to make this point and propose an estimator to partially correct these biases.

If we assume the errors and the latent variable are uncorrelated and that $\sigma_z^2=1$, the sample moments (s_{ij}'s) provide six pieces of information to consistently estimate the six parameters (β_i and $\sigma_i^2, i = 1,2,3$). For this simple model, the method of moments provides a unique solution for the six unknown parameters in terms of the six sample moments:

$$\beta_1 = \left[\frac{s_{12}s_{13}}{s_{23}}\right]^{1/2},$$

$$\beta_2 = \left[\frac{s_{12}s_{23}}{s_{13}}\right]^{1/2},$$

$$\beta_3 = \left[\frac{s_{13}s_{23}}{s_{12}}\right]^{1/2},$$

$$\sigma_i^2 = s_{ii} - \beta_i^2, i = 1,2,3.$$

Note that it is not necessary to observe the latent variable in order to estimate the parameters of the model. The moments of the sample data contain sufficient information to identify the structural parameters. For the model represented by equations (2.1-2.3), the solution is unique, and hence the estimates are also maximum likelihood estimates.

If the number of indicator variables exceeds three and the number of latent variables is greater than one, then the model may not be exactly identified, and it is necessary to impose some parameter restrictions. Maximum likelihood and weighted least squares techniques can be used to obtain consistent estimates. Furthermore, one can obtain standard errors for the parameter estimates through the Hessian matrix.[8]

[8] Both LISREL and SAS provide ML estimates and their standard errors as well as the asymptotically distribution free WLS estimates.

Given the parameter estimates, we can measure the underlying latent variable conditional on the observed indicator variables. In particular, the underlying latent variable is given by:

$$\hat{z} = E[z \mid y] = E[z] + Cov(z, y)' Var[y]^{-1}(y - E[y]).$$

Given the model represented by equations (2.1-2.3), the above conditional value is given by:

$$\hat{z} = \sum_{i=1}^{3} \lambda_i y_i,$$

where,

$$\lambda_i = \frac{\beta_i / \sigma_i^2}{\left[1 + \sum_{i=1}^{3} \beta_i^2 / \sigma_i^2\right]}.$$

From above, we can see that the latent variable is an optimally weighted sum of the observed indicator variables, where the weights are related to the error variance associated with the indicator variable and the loadings on the latent variable.

3.3 A Latent Variable Model of Real Estate Returns

Suppose that there are four noisy proxies that measure underlying commercial real estate returns: NCREIF, NAREIT, WILSHIRE, and ACLI. The NCREIF and NAREIT indices were discussed earlier. The WILSHIRE Real Estate Securities Index (WILSHIRE) is a market capitalization weighted index of publicly traded real estate securities including REITs, real estate operating companies (REOCs), and public partnerships.[9]

[9] For a detailed description of the WILSHIRE index and a comparison to the NAREIT index, see Giliberto and Sidoroff (1995).

The final proxy for commercial real estate returns, ACLI, is computed by combining NCREIF data on income returns with a time series of capitalization rates from the ACLI (American Council of Life Insurance Companies). The capitalization rate for a sold property is defined as net operating income divided by its selling price. Quarterly capitalization rates on commercial loans issued by large life insurance companies are gathered by survey and published in the ACLI publication *Mortgage Commitments on Multifamily and Nonresidential Properties*. These capitalization rates are averages from life insurance companies that provided long-term (over one year) mortgage financing on commercial properties during the preceding quarter. Our imputed ACLI price index is constructed by dividing the NCREIF income component of the total quarterly return by the average ACLI capitalization rate. A total return index is then calculated by combining this price index with the NCREIF index of income returns. The estimated prices that result from dividing NCREIF NOI data by ACLI capitalization rates do not represent specific market transactions. However, Liu, et al. (1990), Fisher, et al. (1994), and others have argued that ACLI cap rate changes provide an indicator of variations in actual sale prices over time because they are averages computed from specific market transactions.[10]

Using the total return proxies described above (i.e., NCREIF, NAREIT, WILSHIRE, and ACLI) along with a structural equation latent variable model, we can

[10] Although estimated from actual transactions, ACLI cap rates represent estimates of stabilized NOI divided by the lender's assessment (appraisal) of market value—which may differ from the contract price. In addition, the properties in the ACLI sample vary considerably. Some properties likely have above market lease rates, some below. Some have below market mortgage financing associated with them and some not. Even for properties with market rate leases and financing, we would expect cap rates to vary depending on the specific lease provisions. To some extent, however, these same criticisms also apply to the NCREIF return data.

estimate a more accurate series of underlying commercial real estate returns. In particular, consider the following latent variable structural equation model for total returns on underlying commercial real estate:

$$
\begin{aligned}
&\text{NCREIF} &&= \alpha_1 + \beta_1 \text{LRE} + \varepsilon_1, \\
&\text{ACLI} &&= \alpha_2 + \beta_2 \text{LRE} + \varepsilon_2, \\
&\text{NAREIT} &&= \alpha_3 + \beta_3 \text{LRE} + \delta_3 \text{LMKT} + \varepsilon_3, \\
(3.1 - 3.6) \quad &\text{WILSHIRE} &&= \alpha_4 + \beta_4 \text{LRE} + \delta_4 \text{LMKT} + \varepsilon_4, \\
&\text{S\&P 500} &&= \alpha_5 \qquad\qquad\quad + \delta_5 \text{LMKT} + \varepsilon_5, \\
&\text{WILSHIRE(-1)} &&= \alpha_6 + \beta_5 \text{LRE} + \varepsilon_6,
\end{aligned}
$$

where LRE is the unobservable commercial real estate return and LMKT is the underlying stock market return. We include LMKT in the NAREIT, WILSHIRE, and S&P 500 equations to control for the excess volatility in these returns attributable to general stock market effects. Conceptually, the inclusion of LMKT in these equations is similar to Giliberto's attempt to control for market-induced volatility that may affect REIT returns. We also include lagged WILSHIRE as an additional indicator variable, because several studies have found that lagged values of exchange-traded real estate returns possess contemporaneous real estate return information (e.g., Gyourko and Keim, 1992, and Ling, Naranjo and Ryngaert, 1999).[11]

The system of equations (3.1 - 3.6) represent a two-latent variable model.[12] In this specification, all the dependent variables are measured as deviations from their means. Because the latent variables LRE and LMKT are unobserved, we fix their units

[11] Similar results are obtained if we use lagged NAREIT, but the model fit is somewhat compromised.

[12] We also experimented with several alternative model specifications. We found the specification represented by (3.1)-(3.6) provided the best model fit. In section 5.2, we further discuss some of the alternative specifications that we employed. Furthermore, although our focus is not on multifactor asset-pricing models and accompanying pricing relations, our formulation is also consistent with a multifactor asset-pricing framework. In particular, if one assumes that the estimates of the average risk premiums for bearing a unit of each respective beta are estimated by the mean levels of the factors, one obtains our

of measurement by setting their variances equal to one. Furthermore, the error terms are assumed to be uncorrelated with LRE, LMKT, and with each other. Lastly, to identify the system, we restrict β_5, δ_1, δ_2, and δ_6 equal to zero. In section 5.2, we perform several specification tests to verify the uncorrelated error assumption and parameter restrictions.

3.4 Creating the Real Estate Index from the Model Estimates

Given the parameter estimates, we can then measure the latent real estate and market returns, Z = {LRE, LMKT}, conditional on the observed indicator variables, Y ={NCREIF, ACLI, NAREIT, WILSHIRE, WILSHIRE(-1), S&P 500}. The conditional mean of Z is given by:

$$E[Z \mid Y] = E[Z] + Cov[Y,Z]'[Var[Y]]^{-1}(Y - E[Y]).$$

Since we standardize the variables to have zero means and normalize the variances of the latent variables to one, the above equation reduces to:

$$E[Z \mid Y] = Cov[Y,Z]'[Var[Y]]^{-1}(Y),$$

where,

$$Var[Y] = \beta\beta' + \Omega,$$

and

$$Cov[Y,Z] = \beta.$$

Therefore,

$$E[Z \mid Y] = \hat{Z} = \beta'[\beta\beta' + \Omega]^{-1}Y.$$

Finally, inserting the parameter estimates from the model, the unobservable latent variables are given by:

$$\hat{Z} = \hat{\beta}'[\hat{\beta}\hat{\beta}' + \hat{\Omega}]^{-1}\tilde{Y},$$

where, $\tilde{Y}$ are the non-standardized variables.

3.5 Estimation Procedures and Test Statistics

---

specification. Fama and French (1996), Naranjo, Nimalendran, and Ryngaert (1998), and others also

To estimate our latent variable model, we utilize the information in the sample variance-covariance matrix of the observed total real estate return proxies. The parameters requiring estimation are given by the vector $\theta = \{\sigma_i^2$, i=1 to 6, β_1, β_2, β_3, β_4, β_6, δ_3, δ_4, $\delta_5\}$, where σ_i^2 is the variance of the error term, ε_i.[13]

The statistical model and the assumptions imply a covariance structure $\Sigma(\theta)$ for the observable random variables that is estimated by the sample variance covariance matrix S. The parameters of this model are estimated by minimizing a fit function $F[S,\Sigma(\theta)]$ of S and $\Sigma(\theta)$. We employ both the maximum likelihood (ML) and the Weighted Least Squares (WLS) fit functions in our analysis. The ML approach provides an estimator that is consistent, asymptotically efficient, and scale invariant. Furthermore, through the Hessian matrix, we obtain standard errors for the parameter estimates. However, these standard errors are consistent only under the assumption that observed variables are multivariate normal. If the observed variables have significant excess kurtosis, the asymptotic covariance matrix, standard errors, and the χ^2 statistic based on the estimator are incorrect (even though the estimator is still consistent). Under these circumstances, WLS provides consistent estimates when an estimate of the asymptotic covariance matrix of the sample variance-covariance matrix is used as the weighting function.[14] The ML and WLS fit functions (see Appendix) are described in detail by Bollen (1989) and Joreskog and Sorbom (1989).

---

employ this reasonable assumption to derive their multifactor asset-pricing models.

[13] Recall that the parameters $\{\alpha_i\}$, i=1 to 6, are set to zero because all variables are defined as deviations from their means. The variances of the latent variables, LRE and LMKT, are normalized to 1.

[14] Browne (1982, 1984) shows that if the weighting matrix is chosen to be equal or to be a consistent estimator of the variance-covariance matrix of S, then WLS is asymptotically efficient and provides the correct standard errors and test statistic under general distributional assumptions.

Let $\hat{\theta}$ be the value of θ that minimizes the fit function F[S,Σ(θ)] for the given sample variance-covariance matrix S. One can then evaluate the model by utilizing the test statistic c= (N-1) F[S,Σ($\hat{\theta}$)], which is asymptotically distributed χ^2 with (s-t) degrees of freedom, where s is equal to the number of unique pieces of information in the variance covariance matrix, and t is equal to the number of parameters to be estimated. Note that s=(k(k+1)) / 2, where k equals the number of variables.

Having established the validity of the model, we then test alternative models by placing restrictions on the original model. In addition to the χ^2 statistic, we also use the Akaike Information Criterion (Akaike, 1974) to choose among competing models. This measure is given by: $AIC = c + 2t$, where c is estimated for the particular model under consideration and t is the number of estimated parameters. The model that has the lower AIC value is considered to be a better fit. This measure considers parsimony in selecting models by penalizing models with fewer restrictions. We also check for the robustness of the model specification by utilizing modification indices (Sorbom 1989)). Finally, it is also important that the final model specification be economically sound.

4. Data

The top panel of Table 1 reports sample statistics (means, standard deviations, auto-correlations, skewness, and kurtosis) for the four quarterly real estate return proxies and the S&P 500, while the bottom panel of Table 1 reports return correlations. Over the 1978-1997 sample period, the NAREIT, WILSHIRE, imputed ACLI, and S&P 500 indices have produced similar mean returns; approximately 4 percent (16% annually). The average return on the appraisal-based NCREIF index is a substantially lower, 2.15

percent per quarter. The standard deviations of the NAREIT and S&P 500 return indices are 6.84 percent (13.7% annually) and 7.09 percent (14.2 % annually), respectively. Thus, the NAREIT and S&P 500 indices display very similar risk/return characteristics over our sample period. The correlation coefficient of 0.66 between these two return series is also quite high. Although the WILSHIRE index has an average return similar to the NAREIT and the S&P 500 indices, its 9.23 percent standard deviation is noticeably higher than NAREIT or the S&P 500.

As expected, the appraisal-based NCREIF index exhibits substantially less volatility than the REIT-based NAREIT and WILSHIRE indices; 1.84 percent quarterly, 3.7 percent annually. The volatility of the imputed ACLI index is 3.06 (6.1% annually); less than half the volatility of NAREIT and the S&P 500, but 66 percent more volatile than NCREIF. As previously discussed, most market participants posit that the "true" volatility of commercial real estate returns lies between NAREIT and NCREIF. NCREIF returns display significant autocorrelation; the other return series do not. Moreover, the contemporaneous NCREIF total return index has an extremely low correlation with the NAREIT, S&P 500, WILSHIRE, and imputed ACLI returns.[15]

Table 1 also reports the skewness and excess kurtosis for each of the return series. Because NCREIF, S&P 500 and WILSHIRE exhibit significant excess kurtosis, the standard errors of the maximum likelihood (ML) estimators may be incorrect.[16] Therefore, we also use distribution free WLS estimators to obtain correct standard errors. Our final estimates of the latent real estate return series are based on the WLS estimates.

[15] The 0.02 correlation between NCREIF and imputed ACLI is somewhat surprising given that the income portion of the imputed ACLI total return is measured by the NCREIF income component.

[16] Note that the standard error of the excess kurtosis is $(24/T)^{0.5}$, where T = 79.

5. Empirical Results

5.1 Model Estimates

Table 2 summarizes the results from estimating the latent variable system described

in Section 2 (equations 3.1 – 3.6).[17] We focus on the results using the WLS method, due to the potential problems discussed earlier with the ML estimation technique. As shown in Table 2, there is a significant positive relationship between the real estate latent variable, LRE, and NCREIF, imputed ACLI, WILSHIRE, and lagged WILSHIRE, with t-statistics ranging from 3.24 to 6.65. The coefficient of 0.06 on NAREIT is not significant.[18] Note that the 0.49 coefficient on LRE in the lagged WILSHIRE equation indicates that exchange-traded real estate is predictive of returns on underlying properties. Finally, the χ^2 (7) statistic of 13.92 indicates that the model fits well.

The importance of controlling for the effects of general stock price movements when using exchange-traded real estate as a proxy for underlying returns is made clear by the estimates of the NAREIT and WILSHIRE equations. The coefficients on LMKT in the NAREIT and WILSHIRE equations are 0.93 and 0.95, respectively, with corresponding t-statistics of 36.66 and 32.06. Because LRE and LMKT are orthogonal

---

[17] Because we standardize each variable to have a unit variance and a mean of zero, it is possible to compare the proxy variables with the latent variables that also have unit variances and means of zero. Furthermore, the square of the standardized coefficient that we obtain is the R^2 for that particular latent variable. Also recall that the latent real estate variable is constructed using the non-standardized variables.

[18] Although we find that NAREIT has a small loading on LRE, similar to Mei and Lee (1994) we do find that there is a real estate factor in both exchange traded and non-exchange traded real estate as shown by the significant coefficients on WILSHIRE, NCREIF, and ACLI.

and have unit variance, the coefficients (loadings) on LRE of 0.06 and 0.30 in the NAREIT and WILSHIRE equations capture the marginal relationship between these exchange-traded return series and the latent real estate return variable.

The validity of our return proxies can be judged along several dimensions. For example, the R^2 of each equation measures the strength of the relationship and the informativeness of the variable. As indicated by the high R^2's, the model is well specified. For example, the R^2 for the WILSHIRE equation is 0.99. This indicates that by explicitly modeling the information structure, one can obtain a significant improvement in the explanatory power of the model. We also find that the R^2 for the NCREIF equation is 0.72, indicating the unobservable latent real estate return series explains 72 percent of the variation in NCREIF returns.

Table 3 provides the estimated latent variable real estate return series over the 1978-1997 sample period, along with the NCREIF, WILSHIRE, NAREIT, and S&P 500 total return series. Summary statistics are provided at the bottom of the table. The mean annualized return on LRE is 9.32 percent, or 72 basis points higher than NCREIF. However, this 9.32 percent mean return is 612 and 672 basis points, respectively, below the returns on WILSHIRE and NAREIT. The annualized standard deviation of LRE returns is 6.56 percent, which exceeds the standard deviation of NCREIF returns by a factor of 1.78. However, the LRE standard deviation is below that of WILSHIRE and NAREIT by a factor of 0.36 and 0.48, respectively. Thus, returns on unsecuritized commercial real estate over the sample period are, approximately, twice as volatile as NCREIF returns and half as volatile as NAREIT returns. The quarterly correlation coefficient for LRE and NCREIF returns is 0.61; for LRE and WILSHIRE, 0.62; and for LRE and NAREIT, 0.36.

In terms of quarterly returns per unit of risk (standard deviation), LRE's ratio of 0.71 exceeds the corresponding ratios for WILSHIRE, NAREIT, and S&P 500, but is far below the 1.17 NCREIF ratio. This suggests that the use of LRE returns in an ex post, mean-variance, portfolio allocation model would produce much lower allocations to commercial real estate than the use of raw NCREIF, but higher allocations than suggested by WILSHIRE and NAREIT.

Figure 1 provides a plot of the latent variable real estate return series versus the NCREIF and WILSHIRE real estate return series, while Figure 2 plots the LRE series against NCREIF and NAREIT returns. These plots also are encouraging and shed additional light on the quarterly return correlations. As shown in Figure 1, LRE returns generally move with WILSHIRE returns (correlation of 0.62), although with much less volatility. Although LRE and NCREIF returns exhibit an almost identical correlation (0.61), LRE exhibits significantly more volatility and often leads NCREIF returns. Figure 2 provides similar results using NAREIT returns in place of the WILSHIRE returns.

5.2 Specification and Robustness Tests

The two primary sources of potential specification problems are the presence of measurement errors that are correlated and the omission of relevant variables. These problems will manifest themselves in the stochastic error term in each equation, leading to a violation of our assumption that the errors are mutually independent. To examine whether the assumption of uncorrelated errors is reasonable, we analyze the modification indices associated with the model. The modification index gives the approximate change in the model χ^2 value when a constrained parameter is allowed to vary (Sorbom, 1989). The index gives an approximate χ^2 with 1 degree of freedom to test the restriction. The critical value of χ^2 (1) at the 5% level is equal to 3.84.

In Panel A of Table 4, we report the modification indices for the loading on the LRE variable in the S&P 500 equation. The modification index of 0.42 suggests that the latent real estate return variable has no explanatory power in the S&P 500 equation. Similarly, we estimated the indices for the loading on the LMKT variable in the NCREIF, ACLI, and WILSHIRE(-1) equations. The highest modification index was only 2.29, indicating that these variables are not related to the LMKT latent variable. In Panel B of Table 4, we also report the modification indexes for the error covariances. As shown in Panel B, four of the 12 modification indices exceed the χ^2 (1) critical value, indicating some correlation in the error structure. However, when we account for these correlated errors in our estimation, the fit improvement is insignificant and the model has difficulty converging.

As an additional specification test, we also augmented our latent variable model to include a bond market factor as in Liu and Mei (1992). The final results are again substantially unchanged. In particular, we find that the model fit does not significantly improve (i.e., the χ^2 difference test for model improvement is not statistically significant), and more importantly, the LRE index is not significantly different (mean difference in the annual return of only 0.05%). We also employed three other proxies for the market portfolio: the Russell 2000; Ibbotson and Associates small stock portfolio; and CRSP's value-weighted market portfolio. In each case, we obtain similar results. Overall, the evidence suggests that our model is appropriately specified.

To check for the robustness of our estimation technique, we compare the WLS estimates to the corresponding ML estimates. Even though the ML estimates are consistent in the presence of significant kurtosis, the standard errors are incorrect, as noted earlier. An examination of our ML estimates (reported in Table 2) indicates that the parameter estimates, standard errors, R^2 and the model χ^2 are substantially unchanged. This analysis suggests that our model estimates are robust.

As a final robustness check, we also test the stability of our results with respect to time and potentially influential outliers. In particular, when we estimate the model over sub-samples, we obtain very similar results to the full sample estimates. Furthermore, when we eliminate potentially influential outliers, we also obtain very similar results.

6. Conclusion

Indices produced by the National Council of Real Estate Investment Fiduciaries (NCREIF) are the most widely used benchmark of privately-held commercial real estate returns in the U.S. Despite their widespread use, their reliance on periodic appraisals to assess changes in property values severely impairs their ability to capture the true risk/return characteristics of privately-held commercial real estate.

This paper presents a new methodology for estimating returns on privately-held (i.e., unsecuritized), commercial real estate. In particular, we utilize latent variable statistical models to estimate an alternative index of commercial returns. The latent variable system provides a framework to explicitly model the presence of multiple attributes and measurement errors associated with real estate return proxies.

The mean annualized return of our estimated commercial return series is 9.3 percent, or 72 basis points higher than corresponding NCREIF returns. This estimated return is 612 and 672 basis points, respectively, below the mean returns on the WILSHIRE Real Estate Securities Index and NAREIT Equity Index. The annualized standard deviation of our latent return series is 6.6 percent, which exceeds the 3.7 percent standard deviation of NCREIF returns by a factor of 1.78. However, it is below that of the WILSHIRE and NAREIT indices by a factor of 0.36 and 0.48, respectively. Thus, underlying returns on commercial real estate over the sample period are, approximately, twice as volatile as NCREIF returns and half as volatile as NAREIT returns. The quarterly correlation coefficient for our latent return series with NCREIF, WILSHIRE, and NAREIT are 0.61, 0.62, and 0.36, respectively.

In terms of average quarterly return per unit of risk, our latent real estate return series has a ratio of 0.71, which exceeds the corresponding ratios for WILSHIRE, NAREIT, and S&P 500, but is far below the 1.17 NCREIF ratio. This suggests that the use of our latent return series in an ex post, mean-variance, portfolio allocation model would produce much lower allocations to commercial real estate than the use of raw NCREIF, but higher allocations than suggested by WILSHIRE and NAREIT. Latent real estate returns generally move with WILSHIRE returns (correlation coefficient of 0.62), although with much less volatility. Although the latent real estate returns and NCREIF returns exhibit an almost identical correlation (0.61), our latent return series exhibits significantly more volatility and often leads NCREIF returns. These volatility and correlation results are intuitively appealing. Moreover, no ad-hoc assumptions or adjustments are required. Rather, the latent real estate return series is based on a sound theoretical foundation and is derived solely from widely available proxies for real estate returns. Overall, our results strongly support the use of latent variable statistical models in the construction of return series for commercial real estate.

Appendix

A.1 Maximum Likelihood Estimators (ML)

Let Z′ = (NCREIF, ACLI, NAREIT, WILSHIRE, S&P 500) be a p-dimensional vector of the observed variables as a deviation from their sample means, and let **S** be consistent estimate of the sample variance-covariance matrix of Z. Furthermore, let $\Sigma(\theta)$ be the covariance matrix written as a function of the free model parameters $\theta = \{\sigma_i^2$, i=1 to 6, β_1, β_2, β_3, β_4, β_6, δ_3, δ_4, $\delta_5\}$, based on the model described in Equation (3.1 - 3.6).

We assume that there are N independent and identically distributed observations $\{\mathbf{Z} = Z_i$, i=1,...,N$\}$. Furthermore, it is assumed that the Z_i are multivariate normal. Under these assumptions, the joint density of the Z_i's is the product of the marginal densities, and the logarithm of the likelihood function is given by:

$$\mathrm{L}(\theta) = \mathrm{logl}(\theta) = \text{constant} - \left(\frac{\mathrm{N}}{2}\right)\left[\log|\Sigma(\theta)| + \mathrm{tr}[\mathrm{S}\Sigma^{-1}(\theta)\right].$$

The parameters are estimated by numerically optimizing the likelihood function with respect to the parameter vector θ. The necessary and sufficient conditions are that the first partial derivatives of the likelihood function with respect to the parameter vector θ be zero and the matrix of second partial derivatives evaluated at the optimum be negative semi-definite.

The asymptotic covariance matrix of the ML estimator is given by:

$$AsyCov(\hat{\theta}) = \left\{-E\left[\frac{\partial^2 L(\theta)}{\partial\theta\partial\theta'}\right]\right\}^{-1}_{\theta=\hat{\theta}},$$

which is the inverse of the information matrix evaluated at the optimum.

A.2 Weighted Least Squares Estimators (WLS)

In the WLS method, the objective function that is minimized is given by:

$$F_{WLS}(\theta) = [s \ \sigma(\theta)]^{T} W^{1}[s \ \sigma(\theta)],$$

where s is a vector of order p(p+1) of all the non-duplicated elements of the sample variance covariance matrix and σ(θ) is the corresponding vector for the model variance covariance matrix Σ(θ). The matrix W^{-1} is a [.5p(p+1)] x [.5p(p+1)] positive definite weight matrix. The elements of the weighting matrix W are chosen to be the fourth order central sample moments. Using this weight matrix gives an "asymptotically distribution free best GLS estimator," and Browne (1982, 1984) shows that for this estimator one can obtain asymptotically correct standard errors and χ^2 statistics.

References

Akaike, H., 1974, "A New Look at Statistical Model Identification," *IEEE transactions on Automatic Control*, 18, 716-723.

Anderson, T., 1984, *An Introduction to Multivariate Statistical Analysis*, Second Edition, John Wiley and Sons.

Bentler, P. M. and D. G. Bonnett, 1980, "Significance Tests and Goodness of Fit in the Analysis of Covariance Structures," *Psychological Bulletin*, 85, 688-606.

Bollen, K., 1989, *Structural Equations with Latent Variables*, New York, Wiley.

Browne, M., 1982, "Covariance Structures" in *Topics in Applied Multivariate Analysis*, D.M. Hawkins (ed.), Cambridge University Press.

Browne, M., 1984, "Asymptotically Distribution-Free Methods for the Analysis of Covariance Structures," *British Journal of Mathematical and Statistical Psychology*, 37, 62-83.

Ferson, W., 1990, "Are Latent Variables in Time-Varying Expected Returns Compensation for Consumption Risk?," *Journal of Finance*, 45, 397-430.

Fisher, J., D. Geltner and R.B. Webb, 1994, "Value Indices of Commercial Real Estate: A Comparison of Index Construction Methods," *Journal of Real Estate Finance and Economics*, 9, 137-164.

Geltner, D., 1991, "Smoothing in Appraisal-Based Returns," *Journal of Real Estate Finance and Economics*, 4, 327-345.

Giliberto, M., 1988, "A Note on the Use of Appraisal Data in Indexes of Performance Measurement," *Journal of the American Real Estate and Urban Economics Association*, 16, 77-83.

Giliberto, M., 1993, "Measuring Real Estate Returns: The Hedged REIT Index," *Journal of Portfolio Management*, Spring: 94-99.

Giliberto, M. and F. N. Sidoroff, 1995, "Real Estate Stock Indexes," *Real Estate Finance*, 12 (2), 56-62.

Goldberger, A. S., 1972, "Structural Equation Methods in Social Sciences," *Econometrica*, 40, 979-1001.

Greene, W., 1990, *Econometric Analysis*, MacMillan: New York.

Griliches, 1974, "Errors in Variables and Other Observables," *Econometrica*, 42, 971-998.

Gyourko, J. and D. B. Kein, 1992, "What Does the Stock Market Tell us About Real Estate Returns?" *Journal of the American Real Estate and Urban Economics Association*, 20, 457-485.

Israel, R., A. Ofer, and D. Siegel, 1990, "The Use of Changes in Equity Value as a Measure of the Information Content of Announcements of Changes in Financial Policy," *Journal of Business & Economic Statistics*, 8, 209-216.

Joreskog, K. G. and A. S. Goldberger, 1975, "Estimation of Models with Multiple Indicators and Multiple Causes of a Single Latent Variable," *Journal. of American Statistical Association*, 70, 631-639.

Joreskog, K. and D. Sorbom, 1989, *LISREL 7 - A Guide to the Program and Applications*, Second Edition. Chicago. SPSS Publications.

Joreskog, K., and D. Sorbom, 1979, *Advances in Factor Analysis and Structural Equation Models*, Abt. Books.

Karolyi, A. and A. B. Sanders, 1998, "The Variation of Economic Risk Premiums in Real Estate Returns," *Journal of Real Estate Finance and Economics*, 17, 245-262.

Lai, T. and K. Wang, 1998, "Appraisal Smoothing: The Other Side of the Story," *Real Estate Economics*, 26, 511-536.

Ling, D. C. and A. Naranjo, 1996, "Economic Risk Factors and Commercial Real Estate Returns," *Journal of Real Estate Finance and Economics*, 14, 283-307.

Ling, D.C., A. Naranjo, and M. Ryngaert, 1999, "The Predictability of Equity REIT Returns," *Journal of Real Estate Finance and Economics*, forthcoming.

Liu, C., D. Hartzell, W. Greig, and T. Grissom, 1990, "The Integration of the Real Estate Market and the Stock Market: Some Preliminary Evidence," *Journal of Real Estate Finance and Economics*, 3, 261-282.

Liu, C. and J. P. Mei, 1992, "The Predictability of Returns on Equity REITs and Their Co-Movement with Other Assets," *Journal of Real Estate Finance and Economics*, 5, 401-418.

Maddala, G. S. and M. Nimalendran, 1995, "An Unobservable Component Panel Data Model to Study the Effects of Earnings Surprises on Stock Prices, Volume of

Trading and Bid-Ask Spreads," *Journal of Econometrics*, 68, July.

Mei, J. P. and A. Lee, 1994, "Is There a Real Estate Factor Premium," *Journal of Real Estate Finance and Economics*, 9, 113-126.

Mei, J. P. and C. Liu, 1994, "The Predictability of Real Estate Returns and Market Timing," *Journal of Real Estate Finance and Economics*, 8, 115-135.

Naranjo, A., M. Nimalendran, and M. Ryngaert, 1998, "Stock Returns, Dividend Yields, and Taxes," *Journal of Finance*, 53, 2029-2057.

Sorbom, D., 1989, "Model Modification," *Psychometrika*, 54, 371-384.

Titman, S., and R. Wessels, 1988, "The Determinants of Capital Structure Choice," *Journal of Finance*, 43, 1-19.

Zellner, A., 1970, "Estimation of Regression Relationships Containing Unobservable Independent Variables," *International Economic Review*, 11, 441-454.

Ziering, B. and L. Taylor, 1998, "Real Estate Benchmarks and the Public Market Real Estate Dynamics Shaping Them," *Real Estate Finance*, 15, 2 (Summer), 7-21.

Figure 1

Plot of Latent Real Estate Returns versus NCREIF and WILSHIRE Returns: 1978:2 – 1997:4 (79 Quarters)

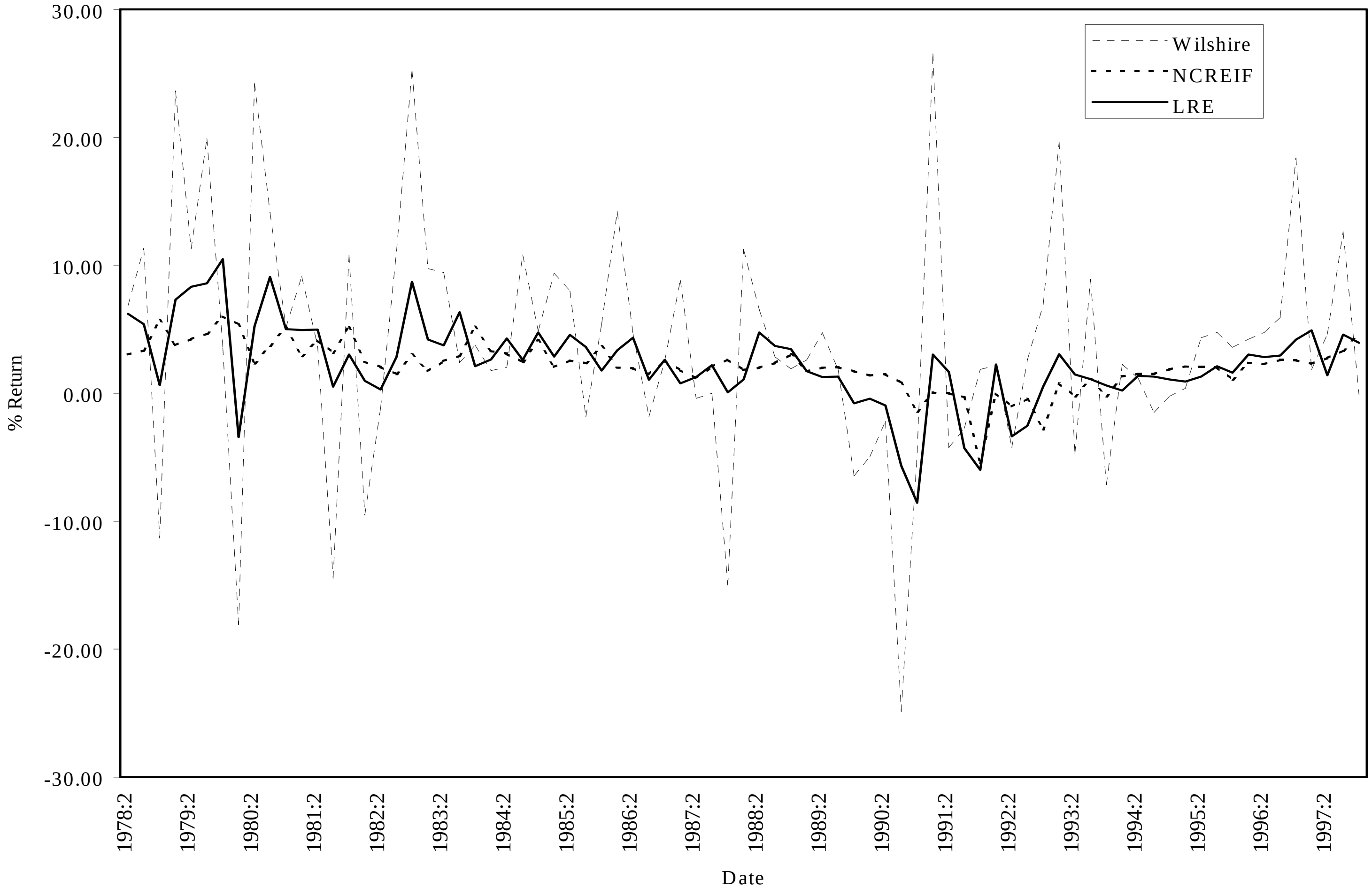

Figure 2

Plot of Latent Real Estate Returns versus NCREIF and NAREIT Returns: 1978:2 – 1997:4 (79 Quarters)

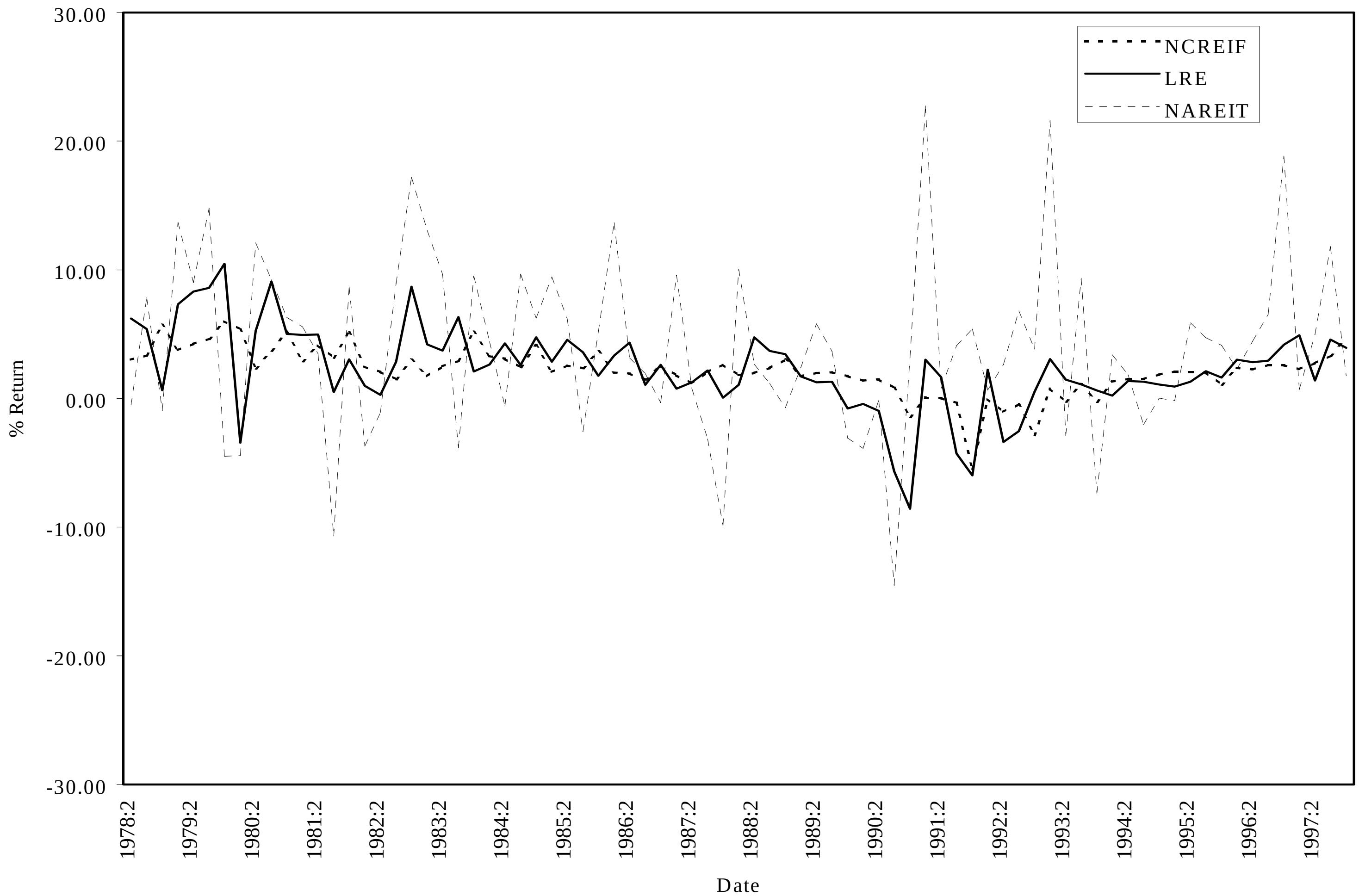

Table 1

**Summary Statistics for the Commercial Real Estate Return Proxies:
1978:2 – 1997:4 (79 Quarters)**

	Mean	Standard Deviation	Auto-Correlation	Skewness	Kurtosis
NAREIT	4.01	6.84	-0.01	0.22	0.75
WILSHIRE	3.86	9.23	-0.01	-0.09	1.31
NCREIF	2.15	1.84	0.67	-0.95	3.40
ACLI (imputed)	3.83	3.06	-0.03	-0.30	0.62
S&P 500	4.29	7.09	0.03	-0.68	2.18

	Correlation Matrix				
	NAREIT	WILSHIRE	NCREIF	ACLI	S&P 500
NAREIT	1.00				
WILSHIRE	0.90	1.00			
NCREIF	0.03	0.11	1.00		
ACLI	0.09	0.18	0.02	1.00	
S&P 500	0.66	0.68	-0.07	0.17	1.00

All variables are in percentage terms. NAREIT is the quarterly return on the NAREIT index. WILSHIRE is the quarterly return on the WILSHIRE Real Estate Securities Index. NCREIF is the quarterly return on the total NCREIF index. The imputed ACLI quarterly returns are computed with imputed sale prices by combining NCREIF NOI data with ACLI capitalization rate data. S&P 500 is the quarterly return on the S&P 500 stock market index. The returns on NCREIF and ACLI are continuously compounded.

Table 2

Latent Variable Model Parameter Estimates:
1978:2 – 1997:4 (79 Quarters)

	Weighted Least Squares				Maximum Likelihood			
Equation	Coeff. on LRE	Coeff. on LMKT	$\hat{\sigma}_i^2$	R^2	Coeff. on LRE	Coeff. on LMKT	$\hat{\sigma}_i^2$	R^2
NCREIF	0.85 (6.65)	——	0.28 (1.29)	0.72	0.51 (2.41)	——	0.74 (3.34)	0.26
ACLI	0.33 (3.24)	——	0.89 (13.03)	0.11	0.25 (1.45)	——	0.94 (5.74)	0.06
NAREIT	0.06 (0.83)	0.93 (36.66)	0.13 (2.71)	0.87	0.06 (0.47)	0.92 (10.28)	0.15 (2.90)	0.85
WILSHIRE	0.30 (3.74)	0.95 (32.06)	0.01 (0.13)	0.99	0.19 (1.52)	0.96 (11.17)	0.04 (0.70)	0.96
S&P 500	——	0.70 (14.71)	0.51 (7.56)	0.49	——	0.72 (7.27)	0.48 (5.77)	0.52
Wilshire(-1)	0.49 (4.13)	——	0.76 (6.59)	0.24	0.46 (2.32)	——	0.79 (3.94)	0.21
	Model Fit							
χ^2 (7 d.f.)	13.92				10.96			
AIC	-0.08				-3.04			

t-statistics in parentheses

The estimates are based on the following latent variable system:

$$\begin{aligned}
\text{NCREIF} &= \alpha_1 + \beta_1\text{LRE} + \varepsilon_1, \\
\text{ACLI} &= \alpha_2 + \beta_2\text{LRE} + \varepsilon_2, \\
\text{NAREIT} &= \alpha_3 + \beta_3\text{LRE} + \delta_3\text{LMKT} + \varepsilon_3, \\
\text{WILSHIRE} &= \alpha_4 + \beta_4\text{LRE} + \delta_4\text{LMKT} + \varepsilon_4, \qquad (3.1 – 3.6) \\
\text{S\&P 500} &= \alpha_5 + \delta_5\text{LMKT} + \varepsilon_5, \\
\text{WILSHIRE(-1)} &= \alpha_6 + \beta_6\text{LRE} + \varepsilon_6,
\end{aligned}$$

where LRE and LMKT are the latent real estate and stock market returns. NAREIT is the quarterly return on the NAREIT Equity Index. WILSHIRE is the quarterly return on the WILSHIRE Real Estate Securities Index. NCREIF is the quarterly return on the aggregate NCREIF index. The imputed ACLI returns are constructed with imputed sale prices by combining NCREIF income return data with ACLI capitalization rate data. S&P 500 is the quarterly return on the S&P 500 stock market index. The returns on NCREIF and ACLI are continuously compounded. The additional estimation restrictions imposed are: Var(LRE)=Var(LMKT)=1; $Cov(\varepsilon_i, \varepsilon_j) = \sigma_i^2$ if i=j, 0 otherwise (i, j=1...6); and $Cov(LRE, \varepsilon_i)=Cov(LMKT, \varepsilon_i)=0$ (i=1...6).

Table 3

Latent Real Estate Returns (LRE) versus NCREIF, NAREIT, WILSHIRE and S&P 500 Return Series: 1978:2 – 1997:4 (79 Quarters)

Panel A: Real Estate Return Series

Date	LRE	NCREIF	WILSHIRE	NAREIT	S&P 500
1978:2	6.22	3.02	6.85	-0.51	8.5
1978:3	5.39	3.33	11.35	7.83	8.67
1978:4	0.64	5.71	-11.33	-0.94	-4.93
1979:1	7.31	3.75	23.63	13.71	7.07
1979:2	8.32	4.23	11.26	9.01	2.72
1979:3	8.59	4.64	19.92	14.77	7.55
1979:4	10.47	6.01	3.55	-4.50	0.13
1980:1	-3.44	5.38	-18.10	-4.44	-4.08
1980:2	5.25	2.33	24.23	12.09	13.41
1980:3	9.09	3.72	14.12	9.24	11.2
1980:4	5.02	5.18	5.14	6.29	9.46
1981:1	4.93	2.91	9.13	5.57	1.32
1981:2	4.97	4.14	3.70	3.47	-2.31
1981:3	0.51	3.17	-14.50	-10.70	-10.22
1981:4	3.03	5.15	10.77	8.67	7.01
1982:1	0.98	2.46	-9.52	-3.69	-7.23
1982:2	0.29	2.05	-1.15	-1.09	-0.62
1982:3	2.85	1.51	11.11	8.92	11.46
1982:4	8.70	3.00	25.25	17.20	18.14
1983:1	4.21	1.73	9.75	13.07	10.05
1983:2	3.73	2.51	9.44	9.66	11.11
1983:3	6.34	2.92	2.40	-3.83	-0.14
1983:4	2.10	5.17	3.76	9.54	0.34
1984:1	2.64	3.30	1.77	4.43	-2.27
1984:2	4.29	3.10	2.02	-0.60	-2.58
1984:3	2.60	2.43	10.82	9.67	9.68
1984:4	4.76	4.13	4.85	6.23	1.76
1985:1	2.86	2.06	9.38	9.45	9.35
1985:2	4.56	2.57	8.01	6.20	7.49
1985:3	3.60	2.35	-1.85	-2.58	-4.05
1985:4	1.76	3.66	5.39	5.18	17.18
1986:1	3.34	2.01	14.20	13.68	14.07
1986:2	4.34	1.94	4.65	3.16	5.91
1986:3	1.06	1.49	-1.83	1.93	-6.97
1986:4	2.62	2.54	2.54	-0.32	5.4
1987:1	0.77	1.81	8.84	9.59	21.33
1987:2	1.27	1.18	-0.41	0.75	5.14
1987:3	2.18	2.07	0.01	-3.14	6.62
1987:4	0.06	2.63	-15.00	-9.90	-22.63
1988:1	1.07	1.82	11.23	10.06	5.87
1988:2	4.75	1.98	6.53	2.66	6.6
1988:3	3.69	2.36	2.84	1.16	0.39
1988:4	3.44	3.02	1.91	-0.71	3.1
1989:1	1.72	1.73	2.60	2.38	7.03
1989:2	1.26	1.98	4.73	5.80	8.8
1989:3	1.30	2.03	1.84	3.67	10.65
1989:4	-0.78	1.73	-6.45	-3.08	2.05
Date	LRE	NCREIF	WILSHIRE	NAREIT	S&P 500

1990:1	-0.42	1.37	-4.97	-3.87	-3.02
1990:2	-0.96	1.51	-2.19	-0.12	6.29
1990:3	-5.66	0.84	-24.91	-14.55	-13.78
1990:4	-8.57	-1.44	-4.67	3.17	8.95
1991:1	3.02	0.05	26.53	22.74	14.56
1991:2	1.66	0.01	-4.24	0.78	-0.21
1991:3	-4.28	-0.33	-2.75	4.09	5.38
1991:4	-5.98	-5.48	1.86	5.39	8.36
1992:1	2.25	-0.03	2.17	0.66	-2.55
1992:2	-3.37	-1.04	-4.24	2.64	1.97
1992:3	-2.54	-0.44	2.61	6.82	3.1
1992:4	0.52	-2.85	6.93	3.83	5.1
1993:1	3.05	0.77	19.65	21.64	4.28
1993:2	1.47	-0.24	-4.72	-2.87	0.51
1993:3	1.10	1.09	8.89	9.35	2.56
1993:4	0.62	-0.25	-7.17	-7.38	2.31
1994:1	0.21	1.30	2.25	3.40	-3.81
1994:2	1.37	1.53	1.19	1.84	0.41
1994:3	1.31	1.50	-1.51	-2.04	4.92
1994:4	1.07	1.86	-0.25	0.02	-0.03
1995:1	0.92	2.09	0.38	-0.17	9.74
1995:2	1.31	2.06	4.35	5.88	9.49
1995:3	2.12	2.04	4.74	4.71	7.95
1995:4	1.61	1.08	3.59	4.14	5.96
1996:1	3.02	2.37	4.22	2.27	5.44
1996:2	2.82	2.26	4.74	4.45	4.51
1996:3	2.95	2.60	5.91	6.54	3.06
1996:4	4.18	2.58	18.39	18.85	8.37
1997:1	4.92	2.28	1.84	0.70	2.62
1997:2	1.39	2.74	4.60	4.97	17.49
1997:3	4.58	3.32	12.63	11.82	7.52
1997:4	3.94	4.65	-0.15	1.75	2.87

Panel B: Descriptive Statistics on Latent Real Estate Return Series

	LRE	NCREIF	WILSHIRE	NAREIT	S&P 500
Mean Return (annualized)	9.32%	8.60%	15.44%	16.04%	17.16%
Standard Deviation (annualized)	6.56%	3.68%	18.46%	13.68%	14.18%
Quarterly Return Per Unit of Risk	0.71	1.17	0.42	0.59	0.61
Correlation with LRE	1.00	0.61	0.62	0.36	0.23

All variables are in percentage terms. LRE is the latent real estate return series estimated from the model in Table 2. NAREIT is the quarterly return on the NAREIT index. WILSHIRE is the quarterly return on the WILSHIRE Real Estate Securities Index. NCREIF is the continuously compounded quarterly return on the total NCREIF index. S&P 500 is the quarterly return on the S&P 500 stock market index. Quarterly Return Per Unit of Risk is measured by dividing the mean quarterly return of each series by its quarterly standard deviation.

Table 4

Modification Indices: Expected Changes in Parameter Estimates

Panel A: Modification Indices for Latent Variables LRE and LMKT

Equation #	LRE	LMKT
3.1	—	0.03 (-0.02)
3.2	—	1.94 (0.15)
3.3	—	—
3.4	—	—
3.5	0.42 (-0.06)	—
3.6	—	2.29 (-0.14)

Panel B: Modification Indices for the Error Covariances

	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
ε_1	-					
ε_2	7.35 (-0.48)	-				
ε_3	0.17 (-0.04)	1.94 (-0.06)	-			
ε_4	4.06 (0.31)	1.94 (0.15)	N.A.*	-		
ε_5	0.76 (-0.06)	1.68 (0.09)	N.A.*	N.A.*	-	
ε_6	1.19 (0.38)	5.56 (0.30)	2.08 (0.07)	6.90 (0.19)	0.56 (-0.04)	-

The modification index gives the approximate change in the model χ^2 value when a particular parameter that is constrained is allowed to vary (Sorbom, 1989). Thus, it is equal to the difference in χ^2 between two models, one in which a parameter is constrained to be zero and the other in which the parameter is allowed to vary. The modification index gives an approximate χ^2 with 1 degree of freedom to test the restriction. The number in the parentheses in each cell provides an estimate of the change in the parameter estimate.

* Modification indices not applicable due to the violation of non-negativity constraints.