

# The Anatomy of Vacancy Behavior in the Real Estate Market\*

Yongping Liang<sup>†</sup>

March 2006

---

\*This research is supported by the *Real Estate Research Institute*. I am grateful for the guidance of Kerry D. Vandell. I also give many thanks to Hugh Kelly, my mentor from RERI, who not only played a key role in helping me get the data from Grubb & Ellis, but also provided detailed and insightful comments on this research. I also appreciate valuable comments by James E. Hodder, Stephen Malpezzi, François Ortalo-Magné, Tim Riddiough, James D. Shilling, Eric Rosenblatt, Geng Deng as well as participants in the doctoral session at the 2005 ARES annual conference in Santa Fe, the doctoral seminar in the department of Real Estate and Urban Land Economics at the University of Wisconsin Madison, and the seminar in Credit Finance Department of Fannie Mae. Robert Bach of Grubb & Ellis generously provided data. All shortcomings are my own.

<sup>†</sup>Department of Real Estate and Urban Land Economics, School of Business, University of Wisconsin Madison. E-mail: yliang4@wisc.edu

## Abstract

A model of vacancy formation with the strategic interaction of tenants, landlords and potential builders in an uncertain dynamic environment is proposed in this paper. Previous literature finds that both search frictions and demand uncertainty yield greater vacancies in equilibrium. In our model, the same results apply. Moreover, we find that interaction of above two factors negatively affect the vacancy rate; supply-side restrictions and demand uncertainty interactively reduce the vacancy rate. By a proprietary data set with detailed rental market information about 50 metro areas over 40 quarters, complemented by some public data sources and some supply-side measures, we estimate equilibrium vacancy rates by measures of search frictions, demand uncertainty, supply-side constraints and their interactions. We find that coefficients on all key explanatory variables and the interactions have the same signs as predicted by the model. Interaction terms have high explanatory power.

## 1 Introduction

A vacancy rate serves as an important indicator of the health of a real estate market. But we cannot draw sound inferences about a market just by observing the rate alone because many factors contribute to a vacancy rate. The same rate may tell different stories, and different rates may tell the same story. For example, in the fourth quarter of 2004, office vacancy rates of Houston and San Francisco were both about 20%.<sup>1</sup> But this number has very different implications: When we observed the historical pattern of the vacancy rates in these two areas, we found that for San Francisco, the vacancy rate is quite sensitive to the change of unemployment rate. Our guess is, the high vacancy rate in San Francisco may mainly because of the loss of jobs. For Houston, we find that the vacancy rate responses less to the change of unemployment rate. Therefore the high vacancy rate in Houston should be explained differently. In this paper we will explore the different reasons behind these rates. Another example is the difference in vacancy rates across property types. In 2004 the

---

<sup>1</sup>Grubb & Ellis, local office market vacancy rates.

U.S. national average vacancy rate for the office market was 15.4%, and 9.5% for the industrial market.<sup>2</sup> Such differences can be explained partly by the comparatively longer construction time and shorter turnover rate of the office market. Analysis of the mechanism of vacancy formation is thus important because different underlying contributors of vacancy rates may have different implications for market participants and also for government intervention.

Numerous studies have documented many possible contributing factors to vacancy formation. Important studies include the following: frictions in the search and matching by Arnott (1989), Wheaton (1990), Igarashi (1991), and etc.; option values embedded in vacant units by Grenadier (1995a); expected growth in demand by Shilling, Sirmans, and Corgel (1987); overbuilding created by the strategic decisions of competing developers by Wang and Zhou (2000), Grenadier (1996) and Grenadier (1999); overbuilding because of incomplete information by Childs, Ott, and Riddiough (2002a) and Childs, Ott, and Riddiough (2002b) (COR thereafter); effects of government policies on the demand and supply sides of the market by Vandell (2003), and etc.

But most of the above studies do not take into account the interdependence of search-based vacancy rates and development activities. New developments decisions are affected by how difficult the search and matching is for the current leasing market; on the other hand, landlords and potential tenants also consider possible new completions when they decide whether or not to set a match. Grenadier (1995a) and Buttner and Ott (2004) are two exceptions: they consider both the current leasing market and potential developments in their models. However Grenadier only considers vacancies due to the embedded option values, not due to search friction; Buttner and Ott model the rent level as an exogenous stochastic process and therefore ignore the important effect of vacancy rate on rent level adjustment. Our study follows this strand of research and constructs a model with the search and matching among the landlords and potential tenants, while potential developers choose the proper time to

---

<sup>2</sup>The 24th edition of Comparative Statistics of Industrial and Office Real Estate Markets, published by *The Society of Industrial and Office Realtors*<sup>®</sup> (SIOR)

enter the market. Equilibrium vacancy rate and rent level are simultaneously determined as the result of the interactions of landlords, potential tenants, and potential developers. In contrast to Grenadier, our study considers vacancies caused by frictional search. Unlike Buttner and Ott, our model generates the equilibrium asking rent and reservation rent endogenously rather than exogenously. Our model generates conclusions about the interactive effects of frictional search and future development on vacancies, which has not been fully studied. Our model explains a mechanism of vacancy formation that can explain data better than the implications from previous studies.

In the next section, we briefly review previous studies and discuss the ones closely related to our research. In Section 3, we set up a simple model that studies the interactive decisions related to the search and the presence of new development in a dynamic uncertain environment. Analytical comparative statics and numerical simulations show that search and development decisions affect vacancy rates interactively. That is, higher demand uncertainty and search friction provoke higher vacancy rates; but at the same time, frictional search mitigates the effects of demand uncertainty; supply-side constraints interact with demand uncertainty to result in lower vacancy rates. In Section 4, using a panel data set at metropolitan level provided by Grubb & Ellis, we estimate the equilibrium vacancy rates following the classic method developed by Voith and Crone (1988). Based on the estimated structural vacancy rates, we tested whether the factors documented in our model are empirically significant. By a unique data set generated from Grubb & Ellis, a proprietary data resource, merged with data from Bureau of Labor Statistics, and the supply-side information about metropolitan areas constructed by Malpezzi, our test is able to include explanatory variables that are the most comprehensive of any empirical study on office market vacancy rates to date. Our empirical results strongly support most of our model's implications: Particularly, demand uncertainty and search friction cause higher vacancy rates; in addition, the coefficients of interactions between frictional search and demand uncertainty have a negative sign on equilibrium vacancy rate; supply-side constraints are significant in explaining the equilibrium vacancy rate; and interact with

demand uncertainty to induce lower vacancy rates. In Section 5, we summarize our paper and discuss extensions of this research.

Our findings have important practical implications to the commercial rental market. It helps to explain and predict the different market responses when facing an uncertain environment. In the reality local markets may face all kinds of random shocks in the demand side, and the market players react to such uncertainties differently depending on more fundamental conditions of the local market and therefore the vacancy rates also react to such shocks differently: when the search is more difficult, then the vacancy rate responses less to the uncertainties; Similarly when the supply-side of the market is more constrained, the vacancy rate also responses less to the uncertainties. For example the different patterns of vacancy rates in Houston and San Francisco over time can be explained by our study: we check the data and find that the search is more frictional in Houston than in San Francisco. We can draw further prediction that the high vacancy rate in San Francisco is more like a transient phenomenon that can be reduced when the economy becomes more stable. But for Dallas the vacancy rate is more possible to keep at high level for a long time.

## 2 Related Studies

The literature on the relationship between rental adjustment and vacancies began with empirical studies. For example, the very early research by Blank and Winnick (1953) and ?) tested the relationship between the observed vacancy rate and rental adjustment. They did not get consistent relationship between the observed vacancy rate and the rental adjustment. The seminal work of Rosen and Smith (1983) disentangles the vacancy rate into its natural and temporary components. Thereafter Rosen and Smith, Wheaton (1987), Voith and Crone (1988), Glascock, Jahanian, and Sirmans (1990), Grenadier (1995b), Hendershott, MacGregor, and Tse (2002), and a special issue of AREUEA Journal (vol. 16, 4) provided more empirical evidences of the relationship between rental adjustments and vacancy rates. Maybe due to the data availability, more studies are on the residential market and less are on the

commercial side of the market.

The formal theoretical studies on vacancy rates start with the analogy from labor-economic models of search and unemployment. Most of them study the search and matching in a static situation. Examples of this include Arnott (1989), Wheaton (1990), Read (1988), Read (1991) and Igarashi (1991). Williams (1995) considered new perspective of search behavior: search with demand uncertainty. But the vacancy rate in his model is a constant. All these search-based models have simple assumptions about the supply-side behaviors. They either assume a fixed supply (Wheaton (1990), Read (1988), Read (1991), Williams (1995)), or assume a state of long-term equilibrium in which the profit of supply side is zero (Arnott (1989)). Igarashi (1991) considers both of these two cases. None incorporates decisions regarding development. Wheaton rationalized the assumption of fixed stock in his 1990 paper: “The assumption in matching models of fixed jobs and workers, . . . , nicely fits the housing market’s ‘stock-flow’ character, where prices adjust in the short run to equate demand to a fixed stock.” This statement is true for a static market. But when we study vacancy formation in a dynamic context, the supply side should be set as adjustable. Moreover, we should allow the new development to have feedback effects on the current matching decisions.

When deciding whether to set up a match with a landlord, a potential tenant considers the probability of finding a different match (at a different price or with a different unit) in the next period. For example, if more completions occur in the next period, it will be more possible for a tenant to find a better match at lower price. Therefore, the potential tenant tends to be choosier in the current period. For a landlord, it also makes sense to consider future competitors or potential developers in setting an asking price. Vacant units serve as a link between the current and the future market. Through this link, new completions of the next period affect the search and matching decisions in the current market. Current search and matching decisions, in the other direction, also affect new development decisions. If the searchers are choosier, and the current vacancy rate is higher, developers are benefited by waiting for better market conditions before they enter the market. When considering the

possible uncertainties in the future, the decisions of all market players are obviously complicated. The effect of uncertainties on development has been extensively studied, starting from the seminal work of Titman (1985). He adopted the simple binomial tree model to illustrate the option value of vacant land. His model has the virtue of simplicity with rich insights. Later studies explored the effects of different types of market frictions on the development option. For example, Grenadier (1999) and COR (2002a,b) studied the effects of incomplete information on the exercise of development options. They showed that the value of a development option is affected by the revelation of information over time. The only paper that studied the effects of costly search on development is Williams (1995). He found that costly search of developing opportunity reduces the value of a development option. But in his paper vacancy rates are set as constant over time. The current vacancy rate does not have feedback effects on future development.

Based on the real option framework, Grenadier (1995) explores three stages of a property: raw land development, construction, and leasing-out stage. The down-sloping lease price is determined by the stochastic demand for space. In the leasing-out stage, a certain lease price level triggers a change in vacancy status. In the construction and development stages, lease price determines development decisions because when the development is completed, the developers will become landlords and join the rental market. Because of demand uncertainty, the vacancy rate at completion is a variable and the probabilities of overbuilding are comparatively higher than in a deterministic case. Grenadier's research was the first to explain the persistence of vacancy rate by real option model.

Buttimer and Ott (2004) extended Grenadier's framework but assumed the lease price and reservation price of tenants are governed by two correlated exogenous stochastic processes. The arrival and departure rates of potential tenants are affected by these two processes. Because of such assumptions, the effect of new development on leasing price is ignored (individual property owners are price takers). Buttimer and Ott provided a good example of the interaction between the current rental market and further development by using a framework in which speculative buildings are

developed concurrently with the development of preleasing space.

Both Grenadier and Buttimer and Ott studied the interdependence of development and leasing decisions. But Grenadier considered only the vacancy explained by the embedded option value, not those that result from frictions in the search and matching. The drawback with Buttimer and Ott is their assumption of exogenous price processes.

Our research follows the lead of Grenadier and Buttimer and Ott, i.e. we study the interaction of the search and matching within the current rental market and the development in the market of the next period. In our model we consider both search-induced vacancies and option-induced vacancies. Vacancies are the result of previous development and leasing decisions, and will affect the development and leasing decisions in the next stage. Price is endogenously generated in the process of search and matching. Therefore our model has the virtue of modelling the dynamic process of vacancy formation and its effects, considering the sequential decisions of developers, landlords and potential tenants. Since in our model, the decisions of different parties are entangled together, we split the time horizon into discrete periods and study the process period by period. Under such discrete-time set-up, the role of vacancy rate – the link between the current market and the future market, is more significant. We adopt the simplified modelling method similar to Titman (1985) method. We solve a two-period model, which is enough to describe the market operation process as well as to keep the study simple.

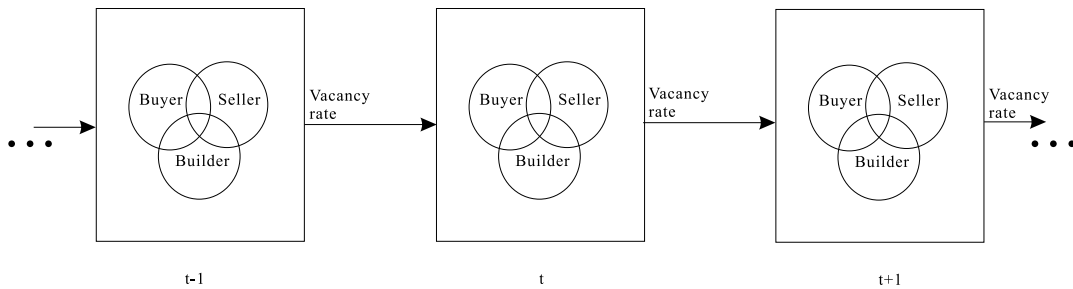
### **3 Model**

We model vacancy formation through three parties playing games strategically: landlords (i.e. sellers), potential tenants (i.e. buyers), and developers (i.e. builders). Each party's decision-making fits into the framework of discrete-time dynamic optimal control. In a general dynamic optimal control system, the control policy is determined by the current state and will affect the state in the next period. There might be random shocks affecting the transitions of the state. In our context, each party of



the market, landlords, potential tenants and developers, have their own optimization problems while strategically interacting with one another. We set the vacancy rate, and also the numbers of buyers and sellers in each period as the state variables; control policies are the asking price of the sellers, the reservation price of buyers, and the new development from potential developers. The change in demand for rental space acts as random shocks to this system. In each period, different parties make decisions, taking into account the possible reactions of other market players: landlords prefer a high asking price but must make a trade-off with a low matching probability. Potential tenants prefer a low price, but they also must risk a less likelihood of a match. Developers must decide whether to enter the market based on the expected demand for space at the project completion. At equilibrium, the asking price, the reservation price, and the number of new developments are determined optimally, and an equilibrium vacancy rate is generated. The current vacant units become part of the inventories for the next period. The interactive relationship of these three market players is illustrated in Figure 1. Our model provides explanations for the vacancy formation documented in previous empirical studies and also guide our empirical tests presented in Section 4.

Figure 1: An interaction system of three parties: Builders, Buyers and Sellers. Each party makes his or her optimal decision.



### 3.1 Model Set Up

We considered a rental real estate market in which units are infinitely durable and do not depreciate. Each landlord holds one unit. Each tenant searches for one unit. All

potential tenants have the same original wealth but have idiosyncratically different preferences. A potential tenant searches for a unit that fits his or her need. All units cost the same to construct and maintain; however, units are idiosyncratically heterogeneous in characteristics. Such heterogeneities could be: the color of walls, the direction of windows, or some other characteristics. Because of such multidimensional heterogeneity, the real estate market is thin. In this thin market, a potential tenant searches for a vacant unit and has a certain probability of finding one. When a vacant unit is found, the potential tenant investigates and gets insight into the characteristics of the unit. Then he or she decides whether to set up a match with this specific unit based on his or her own preference. A landlord has to offer the same price to all potential tenants because tenants do not reveal their true preferences for a specific unit but only reveal a decision to accept or reject the unit at the current asking price. In this case landlords cannot charge tenants any extra premium beyond the asking price. Before doing any search a tenant has a “reservation price” for a unit, determined by the endowment of the tenant and the benefits/costs from renting a unit; the reservation price is thus more of a psychological benchmark than an observable price on the market. Since we assume that all tenants have equal amount of original wealth and suffer the same loss when they can not find a match, reservation prices are the same for all potential tenants. A match is possible only when a tenant’s reservation price is higher than a landlord’s asking price. But a match may not occur even when this condition is satisfied because the potential tenant may find that the characteristics of the specific unit unsuitable. In summary, there are two important reasons contributing to the results of unmatched unit: the price is not profitable, or the characteristics of the unit are not favorable.

The above descriptions about search and matching are modeled mathematically in the following way. For a given time  $t$ , there are  $S_t$  vacant units and  $B_t$  potential tenants on the market.  $q_t$  is the number of new developments in period  $t$ . If we assume that new development takes one period to finish, then  $S_t$  is given by the vacancies

left over from time  $t - 1$  plus the new developments at time  $t - 1$ ,<sup>3</sup>

$$S_t = S_{t-1}v_{t-1} + q_{t-1}, \quad (1)$$

where  $v_{t-1}$  is the vacancy rate in the previous period.

On the demand side, we assume that in each period  $t$ , the growth rate of the demand for space is exogenously determined by a random variable  $\theta_t$  that denotes the random shocks generated by the general economic environment. Therefore, the demand for space at time  $t$  is

$$B_t = B_{t-1}\theta_t. \quad (2)$$

Before period  $t$ , the distribution of  $\theta_t$  is known by all market participants, whose optimal decisions are based on such knowledge.

By comparing the transition of state variable  $B_t$  and  $S_t$  in (1) and (2), one may notice that we do not consider the leftover demand from the last period. This assumption is consistent with the fact that the demand side of a rental space market adjusts much faster than the supply side. Real estate lasts for a long time; as a result, the vacancies in the current period do not disappear instantly. They have substantial effects on the market in the next period. But on the demand side, random shocks are often generated by factors outside of the real estate market that nevertheless have an instant effect on the demand for rental space. An example is the effect of the crash of the dotcom economy on the office market.

Both the demand and supply sides of the market are competitive. Therefore all landlords set up a uniform asking price and all potential tenants have an identical reservation prices. Let  $p_t$  denote the asking price of the landlords and  $\xi_t$  the reservation price of the tenants at time  $t$ . Sellers make a take-it-or-leave-it offer to the buyers: if they match with each other, the buyers pay the asking price  $p_t$ . The opportunities for a landlord and a tenant to meet with each other in the market are

---

<sup>3</sup>In our model turnovers are ignored. It is easy to consider a fixed rate of turnover, or a fraction of unit turnover in each period. Since such a complication cannot add many more implications to our model, we will ignore this consideration for the time being. A good example that considers the turnover rate is in Stokey, Lucas, and Prescott (1989), chapter 10.8.

generated by a Poisson process. Let  $\lambda_t$  denote the rate of this Poisson process,

$$\lambda_t = (B_t)^\alpha (S_t)^\beta, \text{ where } \alpha > 0 \text{ and } \beta > 0.$$

This expression suggests that as the number of buyers and sellers grows, they have a higher probability of meeting.  $\alpha$  and  $\beta$  denote, respectively, the effects of the number of buyers and sellers. Because a landlord competes with other landlords, and also a potential tenants competes with other tenants, we set  $\alpha < 1$  and  $\beta < 1$ . We model the rate at which a landlord and a tenant match as

$$(\xi_t - p_t)\lambda_t/h.$$

Here  $h$  is an index denoting the extent of market heterogeneity with  $h \geq 0$ , where  $h = 0$  is the most homogenous case.  $h$  is explained as the heterogeneity of units' characteristics, or other market conditions that enlarge the search friction. The higher the value of  $h$ , the more difficult the search. <sup>4</sup>  $h$  is assumed to be invariant over time.

A few characteristics of search and matching behavior are implied: when the buyers and sellers meet with each other more frequently, the rate of match is faster; when the asking price is higher, the rate of match is slower; when the buyers' reservation price is higher, the buyers are more tolerant to high prices and the rate of match is higher; when the market is more heterogenous, the rate of matching is lower. According to the density function of a Poisson distribution, the probability that a buyer and a seller meet in one discrete period is

$$\begin{cases} 1 - e^{(p_t - \xi_t)\lambda_t/h}, & \text{when } p_t < \xi_t, \\ 1, & \text{when } p_t \geq \xi_t. \end{cases}$$

---

<sup>4</sup>“Heterogeneity” is an ambiguous expression but a convention in search literature. It is true, as Hugh Kelly pointed out, that when the characteristics of goods and buyers are more heterogeneous, a potential buyer has more choices. But in this case, will the match be easier or more difficult? It is an empirical question. Here we assume that when the market is more heterogeneous, it is more difficult to find a match. In fact we deem  $h$  as a measure for general search friction. Many factors that affect such frictions can be considered as part of  $h$ ; for example, higher rental dispersion or less accessibility of rental information, etc. induce a higher  $h$ .

Therefore, the probability that a representative sets up a matching with a suitable unit with a unit is

$$1 - e^{(p_t - \xi_t) \frac{\lambda_t}{B_t h}}, \quad \text{when } p_t < \xi_t,$$

while the probability that a representative seller leases out a unit is

$$1 - e^{(p_t - \xi_t) \frac{\lambda_t}{S_t h}}, \quad \text{when } p_t < \xi_t.$$

As we pointed out before, the sellers ask the same price to all potential buyers, and all buyers have the same reservation price; accordingly, we study the decisions of a representative seller and buyer. From now on, we will only consider the case that  $p_t < \xi_t$ , because when  $p_t \geq \xi_t$  no transaction will occur. The objective functions for a representative landlord and a representative potential tenant in period  $t$  should be in the form of Bellman's equation,

$$\Pi(t) = \max_{p_t} (1 - e^{(p_t - \xi_t) \frac{\lambda_t}{S_t h}}) p_t + e^{(p_t - \xi_t) \frac{\lambda_t}{S_t h}} (l_s + \rho \Pi(t + 1)), \quad (3)$$

$$C(t) = \min_{\xi_t} (1 - e^{(p_t - \xi_t) \frac{\lambda_t}{B_t h}}) p_t + e^{(p_t - \xi_t) \frac{\lambda_t}{B_t h}} (l_b + \rho C(t + 1)). \quad (4)$$

Here  $\Pi(t)$  is the maximized profit function of a representative landlord and  $C(t)$  is the minimized cost function of a representative potential tenant.  $l_b$  is the cost for a buyer who fails to find a satisfactory match.  $l_s$  is the payoff of a seller who fails to find a satisfactory match.  $\rho \in [0, 1]$  is an exogenously fixed discount factor.

In equilibrium<sup>5</sup>, vacancy rate in period  $t$  is

$$v_t = e^{(p_t^* - \xi_t^*) \frac{\lambda_t}{S_t h}}, \quad (5)$$

where  $p_t^*$  and  $\xi_t^*$  are the optimal asking and reservation prices derived from (3) and (4). This equation implies that vacancy rate in a specific period is equivalent to the failing probability that a seller finds a match.

We now turn to the decisions of potential developers (builders). We assume that builders enter the market sequentially. They have the same information set and they

---

<sup>5</sup>It is not zero as long as there are frictions in the search process, i.e.  $h > 0$ , or the buyers are very tolerant, i.e.  $\xi \rightarrow \infty$ .

compete with each other. Such assumption indicates that when a builder decides to build, he or she is aware of the existence of other undergoing constructions, which will compete with his or her project in the next period (we assume new construction takes one period to finish). Our assumptions preclude the following “irrational” construction decisions: when the market needs only one new unit, ten developers all want to take this opportunity and therefore the market is flooded with ten new units.

<sup>6</sup> The objective function of the builders is constrained by the land market condition. We study the two extreme cases of the land supply: first, the land-supply curve is flat, i.e. the land supply is infinitely elastic and extra land supply does not drive up the land price. In the second case the land supply is constrained at a fixed level.

*Case I: Land supply is infinitely elastic*

The optimal number of new units to be developed is given by the total number of developments by the potential developers. If the land supply is infinitely elastic, then based on the expectation for the future market condition, the developers keep entering the market until there is not excess profit available. Such development decisions can be written as:

$$\begin{aligned} \max_{q_t} \quad & q_t \\ \text{s.t.} \quad & E_{\theta_{t+1}}[\Pi(t+1|q_t)] \geq CC \\ & q_t \geq 0 \end{aligned} \tag{6}$$

where  $q_t$  is the total number of units to be developed at time  $t$ ;  $E_{\theta_{t+1}}[\Pi(t+1|q_t)]$  is the expected optimal profit of a landlord in the next period, conditional on the current number of new developments as  $q_t$ .  $CC$  is the construction cost per unit, which is set as a constant value. This expression proposes that potential developers keep entering the market until no more expected excess profit is available in the next period, given that the units developed in this period will be completed in the next period.

*Case II: Land supply is totally inelastic*

Assume the density of construction is fixed. The maximum number of total new units is then set as  $N$ . We assume that land price is not driven up by the increase

---

<sup>6</sup>Of course such irrational development happen in real market. But it is not the focus of the current research.

in the demand for space. This assumption simplifies our model by isolating it from the price adjustments in the land market. Such simplicity is achieved at a cost: The equilibrium in our model may not represent an equilibrium state in the land market.

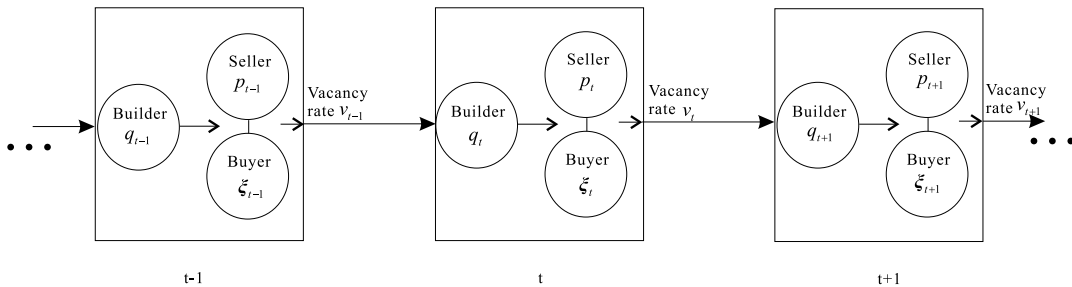
The optimal number of new developments is given by

$$\begin{aligned}
& \max_{q_t} && q_t \\
& \text{s.t.} && E_{\theta_{t+1}}[\Pi(t+1|q_t)] \geq CC \\
& && q_t \geq 0 \\
& && E_{\theta_{t+1}}[\Pi(t+1|q_t)] \geq E_{\theta_{t+2}}[\Pi(t+2|q_{t+1})] \\
& && \sum_{i=t}^{\infty} q_i \leq N - \sum_{i=1}^{t-1} q_i.
\end{aligned} \tag{7}$$

Where  $q_1, \dots, q_{t-1}$  are numbers of units developed in previous periods. The builders choose the number of developments such that: (1) development opportunities are not wasted; (2) expected profit can cover the construction cost; (3) allocation of new developments over time maximizes the expected lifetime profit of potential developers; (4) the constraint of land supply is satisfied.

The dynamic optimization problem for builders illuminates the important fact a builder's decision is based on the awareness of the behaviors and tactics of the other two parties. The flows of decisions of the players in the market over time and the role played by the vacancy rate are demonstrated in Figure 2.

Figure 2: An Illustration of Decision Flows in Our Model



### 3.2 Two-Stage Solutions

The above system of optimizations is dramatically simplified in a straightforward but meaningful two-period case. All the involved periods are  $t = 0, 1, 2$ .  $t = 0$  is

an initial period at which no transaction occurs.  $S_0$  and  $B_0$  are given as the initial number of vacant units and potential tenants. The builders choose the number of new developments  $q_0$ , according to their expected profit. Market participants act in analogy to the general case. At the final period  $t = 2$ , the objective functions are

$$\Pi(t = 2) = \max_{p_2} (1 - e^{-(p_2 - \xi_2) \frac{\lambda_2}{S_2 h}}) p_2 + e^{-(p_2 - \xi_2) \frac{\lambda_2}{S_2 h}} L_s, \quad (8)$$

$$C(t = 2) = \min_{\xi_2} (1 - e^{-(p_2 - \xi_2) \frac{\lambda_2}{B_2 h}}) p_2 + e^{-(p_2 - \xi_2) \frac{\lambda_2}{B_2 h}} L_b, \quad (9)$$

for sellers and buyers. A seller who fails to find a match gets a final payoff  $L_s$ ; A buyer does not find a match has a final cost  $L_b$ . We assume that  $L_b > L_s$ , because when  $L_b \leq L_s$ , neither the potential tenants nor landlords have an incentive to trade. It is rational that the asking price  $p_t$  and reservation price  $\xi_t$  in each period must be located between  $L_s$  and  $L_b$ . The builders do not build any units at  $t = 2$ , because  $t = 2$  is the final period. Therefore, even if there are three periods involved, each party in the market is only active in two periods. Thus, this simplified model is deemed to be a two-period model.

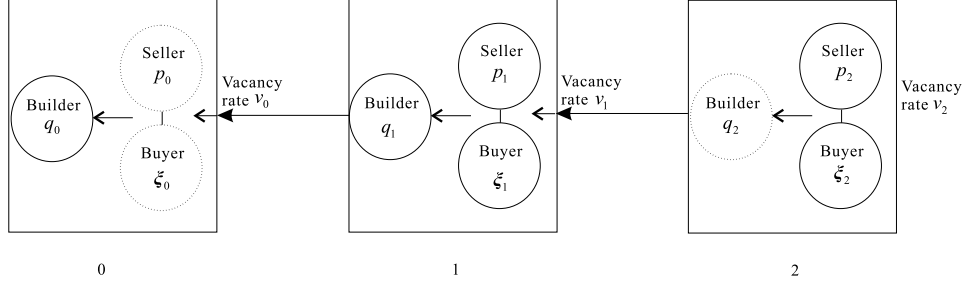
Now we are ready to solve this two-period problem. The classic Dynamic Programming algorithm suggests that an optimal policy can be constructed in piecemeal fashion, first by constructing an optimal policy for the “tail subproblem” involving the last period, then by extending the optimal policy to the “tail subproblem” involving the last two periods. We continue in this manner until an optimal policy for the entire problem is constructed. Subsequently, we start from  $t = 2$  and solve backward recursively. Figure 3 demonstrates the application of the Dynamic Programming method in our two-period model.

In the market, the sellers and buyers choose their optimal “policy” by analyzing opponents’ strategic options, preferences and reactions. Here, given a reservation price  $\xi_2$  of buyers, the sellers choose optimal  $p_2$  and vice versa. Equilibrium is obtained when no party can improve his or her payoff unilaterally; i.e. the optimal solutions  $p_2^*$  and  $\xi_2^*$  solve Bellman’s equations (8) and (9) simultaneously.

At period  $t = 2$ ,  $S_2$  and  $B_2$  are determined. The first order conditions of (8) with



Figure 3: Solving the Two-stage Model Recursively (Variables in the dotted circles are not considered.)



respect to  $p_2$  is:

$$\frac{d\Pi(t=2)}{dp_2} = -\frac{\lambda_2}{S_2 h} e^{(p_2 - \xi_2) \frac{\lambda_2}{S_2 h}} p_2 + 1 - e^{(p_2 - \xi_2) \frac{\lambda_2}{S_2 h}} + \frac{\lambda_2}{S_2 h} e^{(p_2 - \xi_2) \frac{\lambda_2}{S_2 h}} L_s = 0. \quad (10)$$

This expression conveys that for a given  $\xi_2$ , sellers choose optimal  $p_2$ , which implies an implicit functional relationship between  $p_2$  and  $\xi_2$ . If we deem  $p_2$  as a function of  $\xi_2$ ,

$$p_2 = p_2(\xi_2),$$

the first order information of  $dp_2/d\xi_2$  is calculated as:

$$\frac{dp_2}{d\xi_2} = \frac{-\lambda_2/S_2 \cdot p_2 - h + \lambda_2/S_2 \cdot L_s}{-\lambda_2/S_2 \cdot p_2 - 2h + \lambda_2/S_2 \cdot L_s}. \quad (11)$$

If  $h \neq 0$ , the inequality  $1/2 < \frac{dp_2}{d\xi_2} < 1$  is satisfied. The first order condition for (9) is computed as

$$\frac{dC(t=2)}{d\xi_2} = (dp_2/d\xi_2 - 1) \frac{\lambda_2}{B_2 h} e^{(p_2 - \xi_2) \frac{\lambda_2}{B_2 h}} (L_b - p_2) + dp_2/d\xi_2 (1 - e^{(p_2 - \xi_2) \frac{\lambda_2}{B_2 h}}) = 0, \quad (12)$$

where  $dp_2/d\xi_2$  is given by (11).

The equations (10) and (12) form a group of functions with two unknowns  $p_2$  and  $\xi_2$ . Solving them together,  $\xi_2$  is canceled out to generate a function only about  $p_2$ :

$$\left( -\frac{S_2 h}{-S_2 h + \lambda_2 L_s - \lambda_2 p_2} \right)^{\frac{1}{B_2}} = \left( \frac{B_2 (-S_2 h + \lambda_2 L_s - \lambda_2 p_2)}{\lambda_2 p_2 S_2 - \lambda_2 p_2 B_2 - B_2 h S_2 + \lambda_2 L_s B_2 - \lambda_2 L_b S_2} \right)^{\frac{1}{S_2}}. \quad (13)$$

Generally speaking this function cannot be solved analytically. We consider two cases:

**Analytical solutions when  $S_2 = B_2 = 1$ .** By assuming  $S_2 = B_2 = 1$ , the function is reduced to a quadratic equation. We compute  $p_2$  as

$$p_2 = L_s - h \pm \sqrt{hL_b - hL_s + h^2}.$$

Only the solution between  $L_s$  and  $L_b$  makes sense intuitively. Therefore,

$$p_2 = L_s - h + \sqrt{hL_b - hL_s + h^2}.$$

Plug the solution of  $p_2$  into (10) to get the value of  $\xi_2$ :

$$\xi_2 = L_s - h + \sqrt{hL_b - hL_s + h^2} - \ln \left( -\frac{h}{-L_s - \sqrt{hL_b - hL_s + h^2} + L_s} \right) h.$$

Further calculations show that the second order conditions for (8) and (9) are satisfied too. By plugging the solutions of  $p_2$  and  $\xi_2$  into (5), we get the expression for vacancy rate in the second period:

$$v_2 = \frac{h}{\sqrt{h(L_b - L_s + h)}}.$$

From the solutions for  $p_2$ ,  $\xi_2$  and  $v_2$ , we find the following comparative statics:

$$\begin{aligned} dp_2/dh &> 0; & d\xi_2/dh &> 0; & dv_2/dh &> 0; \\ dp_2/dL_b &> 0; & d\xi_2/dL_b &> 0; & dv_2/dL_b &< 0; \\ dp_2/dL_s &> 0; & d\xi_2/dL_s &> 0; & dv_2/dL_s &> 0. \end{aligned} \tag{14}$$

These results are consistent with what we expect and also with some previous static state studies of vacancy rates. As the search becomes more difficult, the asking price, reservation price, and the vacancy rates increase. If the buyers' immediate cost  $L_b$  is higher, they are more willing to tolerate a higher price, resulting in a higher asking price, reservation price, and lower vacancy rate. In the other direction, if the sellers' payoff when keeping the unit vacant is low, they set a lower price to increase the probability of finding a matching tenant. In this situation the vacancy rate is lower.

**In a general situation when  $S_2 \neq B_2 \neq 1$ .** (13) cannot be solved analytically. Similar comparative statics can be obtained numerically for plausible values of the

parameters. The above comparative statics still hold. Moreover, we get comparative statics numerically for  $S_2$  and  $B_2$ :

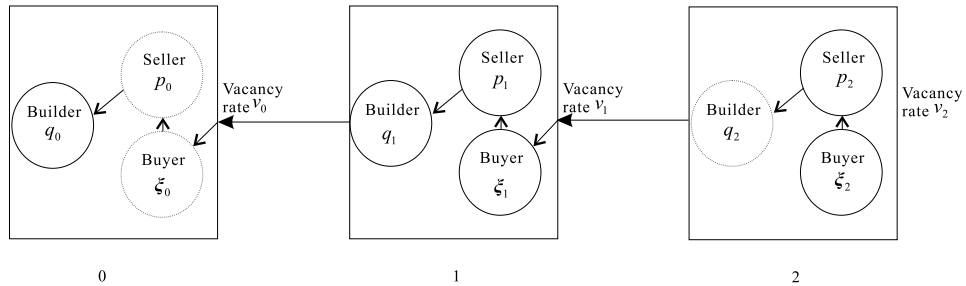
$$\begin{aligned} dp_2/dS_2 &< 0; & d\xi_2/dS_2 &< 0; & dv_2/dS_2 &> 0; \\ dp_2/dB_2 &> 0; & d\xi_2/dB_2 &> 0; & dv_2/dB_2 &< 0; \end{aligned} \tag{15}$$

The above numerical comparative statics show that higher availability of units benefits the buyers. The asking price and reservation price are both lower, and the vacancy rate is higher. In the other direction, sellers benefit when more potential tenants are looking for rental space. The asking price and reservation prices are both higher, and the vacancy rate is lower. The details of the numerical simulations applied in this step are discussed in the next section.

After solving for the decisions of landlords and potential tenants at the final stage, we step back one period and study the decisions of buyers ( $\xi_1$ ), decisions of sellers ( $p_1$ ) and decisions of builders ( $q_1$ ). Equation  $S_2 = S_1 v_1 + q_1$  depicts the transition of states between  $t = 1$  and  $t = 2$ . The builders make decisions based on the knowledge about the distribution of  $\theta_2$ . Their decisions also depend on assumptions about the land market. Given the builders' decisions at  $t = 1$ , we solve for the decisions of landlords and potential tenants at  $t = 1$ , and then the builders' decisions at  $t = 0$ .

Our recursive process of solving the optimal solutions of the system is illustrated in Figure 4.

Figure 4: Recursive Solution Sequence



Generally this dynamic system cannot be solved analytically. We use numerical simulations to derive the inferences from the model.

### 3.3 Numerical Simulations

To solve our model numerically, we need to assign plausible values to parameters. We chose the values  $\alpha = 0.5$  and  $\beta = 0.5$ . Such values of  $\alpha$  and  $\beta$  represent that the increase in the number of available units and the number of potential buyers have the same effect on the rate of meeting. We assumed  $\rho = 0.9$ ,  $CC = 103$ ,  $L_s = 100$ ,  $L_b = 120$ . The values were chosen to satisfy  $L_b > L_s$ , so that the buyers and sellers have incentive to trade; and  $CC > L_s$ , so that the builders to make the development decision cautiously. (If  $CC \leq L_s$ , then the builders always choose to build, because the construction cost can be covered even if the unit cannot be sold.) We set  $l_s = 10$ , and  $l_b = 12$  for similar reasons. To make things simple, we assumed that  $\theta_1$  has a one-point distribution:

$$Pr(\theta_1 = 1) = 1.$$

$\theta_2$  has a two-point distribution as:

$$Pr(\theta_2) = \begin{cases} p_h, & \text{when } \theta_2 = \theta_h, \\ p_l, & \text{when } \theta_2 = \theta_l. \end{cases}$$

We varied the value of  $h$  and  $\theta_2$  to see how the changes of search behaviors and the change in demand for space affect vacancy formation. We tried a variety of values for  $\alpha$ ,  $\beta$ ,  $CC$ ,  $S_0$  and  $B_0$ . The implications from the model will not change as long as we vary the parameter values within a reasonable range. Therefore, we only list the results with the above parameter settings to illustrate our main conclusions.

We illustrated the relationship between vacancy rate and  $h$ , the market heterogeneity, in Figure 5. As  $h$  takes a higher value, the search is more difficult, then the vacancy rate is higher. When comparing the solid line (the first period vacancy rate) with the dashed line (the second period vacancy rate), we find that the vacancy rate in the first period is much higher than the rate in the second period. This can be explained intuitively: In the first period, even if the buyers and sellers cannot find an immediate match, they have the opportunities to find a match in the second period. But because the second is the final period, buyers and sellers do not have the option of delaying their matching decisions any longer. Unmatched buyers and sellers will

be subject to “final punishment”. Therefore, the vacancy rate in the second period is much lower than the first period vacancy rate at any value of market heterogeneity.

Figure 6 is a three-dimension graph to demonstrate the relationship between vacancy rate and the number of potential tenants in the market, at a given number of new developments. The figure shows that as the number of buyers increases, the vacancy rate is lower. At the same time, an increase in the number of available units bring an increase in the vacancy rate. More comparative statics results are listed in (15).

To show the effects of land-supply constraint, we simulate separately two extreme cases of land supply: In the first case, land supply is infinitely elastic. In the second case, we set five as the maximum total number of new units in the two periods. The starting number for buyers and sellers are:  $S_0 = 5$  and  $B_0 = 10$ . We list the simulation results under these two different land-supply assumptions in Table 1 and Table 2, respectively. We set different values of  $\theta_2$  to demonstrate how expected growth and demand uncertainty affect the vacancy rate. In Table 1 and Table 2, four different settings of  $\theta_2$  are presented. In section a, b and c,  $\theta_2$  is deterministic.  $\theta_2 = 1$  represents a case of no growth in demand in the next period;  $\theta_2 = 1.2$ , a case of positive growth in demand; and  $\theta_2 = 0.8$ , a case of negative growth in demand. Section d lists the simulation results when there is uncertainty. If we assume that the distribution (not the true value) of  $\theta_2$  is known as  $Pr(\theta_2 = 0.8) = 0.5$  and  $Pr(\theta_2 = 1.2) = 0.5$  before  $t = 2$ , then market participants know  $E(\theta_2) = 1$ . Future demand has the equal probabilities of going up and down, with the expectation being equal to 1. Figure 7 and Figure 8 compare the vacancy rates under these four different settings of  $\theta_2$ , when land supply is totally elastic and when land supply is fixed.

From Tables 1, 2 and Figures 7 and 8, we find that land supply affects vacancy rates significantly: if land supply is unconstrained, vacancy rates are higher when the next period has positive growth in demand for space. But if the land supply is highly restricted, there is not much space for new developers to use to response to the growth in demand. That is why in Figure 8 the vacancy rates when  $\theta_2 = 1$  and  $\theta_2 = 1.2$  are similar and lower than the vacancy rates when the land supply is infinite. In Figure 7,

the difference between the solid line (the vacancy rate when there is no uncertainty) and the dashed line (the vacancy rate when there is uncertainty) becomes smaller at the larger the value of  $h$ . That means when the land supply is not constrained, the vacancy rates are higher when uncertainty is present. But the effects of uncertainty become smaller at higher values of market heterogeneity. From this figure we can tell that as the market is more heterogeneous, the effect of uncertainty on the vacancy rate is moderated. In Figure 8 the general levels of vacancy rates vary less dramatically over the different values of market heterogeneities than the vacancy rates in Figure 7. That is, when the land supply is very restricted, the effects of market heterogeneity is lower on the vacancy rates. At the same time, comparing Figure 7 and Figure 8, we find that the vacancy rates when uncertainty presents are similar in these two figures, while the vacancy rates without uncertainty in these two figures are very different. We can infer from comparing the two figures that the uncertainty can moderate the effects of supply-side constraints. Moreover in these two figures the level of vacancy rates are higher at higher values of  $h$ , which is consistent with the relationship shown in Figure 5.

Combining the implications from Table 1 and 2, and Figure 5, 6, 7 and 8 , and from more simulation results when we varied the values of parameters (which are not listed in this paper), we get following implications of our model:

1. When the market is more heterogenous ( $h$  is higher), the search and matching is more difficult. Higher equilibrium vacancy rates are the consequences.
2. When the increase in the future demand for space is expected to be high ( $\theta_2 > 1$ ), the vacancy rate in the current period  $v_1$  is higher when the land supply is not highly constrained.
3. When land supply is highly constrained, the general vacancy rate will be lower than when the land supply is very elastic. The reason is, the supply-side of the market is less adjustable with restricted land-supply. Therefore under good demand-side conditions, the vacancy rates tend to be lower to fulfill the extra demands.

4. Search and uncertainty interact with each other: As the market becomes more homogeneous, i.e. the search cost is low, the effect of uncertainty on the vacancy rate is more significant. Search diminishes the effect of uncertainty on vacancy rates.
5. Search and supply-side constraints also have a negatively interactive effect on the vacancy rates. When the supply is inelastic, the potential searchers on the market will have less incentive to wait for longer time, because the future available alternatives are limited. Therefore, the supply-side constraints moderate the effect of market heterogeneity.
6. When land supply is restricted, new development cannot fully respond to an increase in the demand for space (because the potential developments are restricted), but it can respond to a decline in the demand for space. When the demand is uncertain, the effects of land constraints are moderated. The rationale for such effect is: even though being aware of the constraints in the supply side, when uncertainty is high, market players (potential tenants and landlords) will behave conservatively and wait longer before making decisions, and therefore result in higher vacancy rates.

We can compare the implications from our model with previous research results. The effect of search on the vacancy rate in a one-period problem is consistent with studies on steady state vacancy rates such as those by Arnott (1989), Igarashi (1991) and Wheaton (1990). Our model predicts that both new development and matching in the current market are more cautious when facing uncertainty, which is consistent with previous studies on the effects of uncertainty on development and search. But our model suggests further implications. For example, it predicts that when both new development and search are more cautious, the net effect of uncertainty on the vacancy rate is positive; i.e. uncertainty induces a higher vacancy rate. This result goes beyond Grenadier's conclusion about the persistence of the vacancy rate. The interactive effect of search and uncertainty is consistent with the conclusion in Williams (1995) about the effects of search on option value. But our study differs

from Williams in its exploration of the interactive effect on vacancy rates. Our study also shows the significant effect of supply-side constraints, such as land supply, on vacancy rates. Other types of supply-side constraints, such as government regulations, can be similarly studied. The supply-side effects have not been fully explored in previous theoretical models of vacancy formation. Our model also shows the joint negative effect of potential search and supply-side constraints on the vacancy rates, which has not been explored yet.

Our main results are easily explained intuitively. Frictional search and demand uncertainty both have negative effects on the match-making process, and therefore induce a higher vacancy rate. But a frictional search diminish the potential gains from delaying the matching decisions. For example, even though future demand may increase, a landlord also may not be able to find a match later because of the randomness of search and matching. The interactions of land constraints and demand uncertainty can be explained, too. When the land supply is limited, the responses at the supply-side to possible future boom of the demand for space are limited and therefore the resulting vacancy rates are lower. But the effects of demand uncertainty will moderate the effects of supply-side constraints. Therefore, uncertainties combined with different land supply situation have different effects on vacancy rates.

## 4 Empirical Tests

### 4.1 Hypotheses

In the previous section, we developed a theoretical model of dynamic vacancy formation. The model implies that many factors contribute to the differences of vacancy rates across areas and over time. In the following we list the model's implications of what factors affect vacancy rates and how these factors interact. Our goal with the empirical test is to decompose the vacancy rate into its composing factors and their interactions.

1. Demand-side factors such as higher employments induce lower vacancy rates.



2. Factors increasing the search and matching costs, such as dispersion in rent and higher turnover rates produce higher vacancy rates.
3. Expected growth in demand induces higher vacancy rates if land supply is not highly restricted.
4. Uncertainty in demand for office space affects the value of options embedded in the lease contracts and in the vacant land to be developed. Therefore, higher uncertainty provokes higher vacancy rates from the last sections' simulation result.
5. Supply side factors such as constraints on land supply affect vacancy rates. When the supply side is more elastic, the vacancy rate is higher.
6. Some factors interact with each other: Search frictions moderates the effects of uncertainty in the demand for space. In addition, the demand uncertainties moderate the effects of supply-side constraints on the vacancy rates.

Previous empirical studies have found that some of the factors listed above affect the vacancy rates of rental housing markets. In the previous empirical studies, people are interested in finding long-term equilibrium vacancy rates, or called the “natural vacancy rates”. The difference between the natural vacancy rates and the observed vacancy rates are used to explain the rental adjustment. The classic study of Rosen and Smith (1983) estimated the “natural vacancy rates” for different local markets assuming such rates are constant over time. Some of the determinants of the natural vacancy rates have been documented. For example, Rosen and Smith (1983) and Gabriel and Nothaft (1988) found that for the rental housing market, the higher the mobility rate, the heterogeneity of households and the dispersion in rents, the higher the equilibrium vacancy rates. For the office market, Shilling, Sirmans, and Corgel (1987) found that the expected growth in demand induces higher normal (equilibrium) vacancy rate. In the influential study of Voith and Crone (1988) the “natural vacancy rate” is composed by a market specific term and a time-varying term which is common to all markets. In their study they pointed out some factors that

may affect the time-varying factors of the natural vacancy rate, such as the risk-free interest rate. But it is hard to believe that the time-varying factors of the “natural vacancy rates” should be the same to all markets. It is true that there are some common time-varying factors across markets, such as the interest rate, but many of the time-varying factors, such as the expected growth in the demand for a market, or the uncertainty in the demand for a market, vary not only over time but also across MSAs. Therefore even the Voith and Crone (1988) method is a great improvement over the previous studies, it is not flawless. Grenadier (1995b) improved the method by using a polynomial rather than a dummy variable to estimate the time-varying component of the natural vacancy rates. But it still assumes that such components are constant across all local markets.

In our study, we did variety of estimations. Firstly we followed the Voith and Crone (1988) to estimate “natural” vacancy rates. Then we estimated short-term equilibrium vacancy rates which is also based on the model in Voith and Crone (1988). Then we found proxies for the determinants documented in the theoretical model part to explain the “natural” vacancy rate, short term equilibrium vacancy rates, and also the observed vacancy rates. In our estimations we included far more explanatory variables than previous studies to test the factors that determine vacancy rates in rental office market.

## 4.2 Methodology

Because of the simultaneity of market-observed vacancy rates and rent, previous literature, starting with Rosen and Smith (1983) estimates equilibrium equilibrium vacancy rates and studies the determinant of equilibrium vacancy rates. We follow the empirical model of Voith and Crone (1988) to estimate the equilibrium vacancy rates for each local market at different times. The basic idea of Voith and Crone is to decompose the variance in office vacancy rates to market-specific, time-specific and random components. The framework is described by the following equations:

$$V_{it} = \alpha_i + \epsilon_{it}, \tag{16}$$

$$\epsilon_{it} = \rho\epsilon_{i,t-1} + \beta_t + \mu_{it}. \quad (17)$$

Combining equations (16) and (17) we obtain

$$V_{it} = \alpha_i(1 - \rho_i) + \beta_t + \rho_i V_{i,t-1} + \mu_{it}, \quad (18)$$

where  $V_{it}$  is the observed vacancy rate for market  $i$  in period  $t$ ;  $\alpha_i$  is a market-specific level of vacancy rate at  $t = 1$ ;  $\epsilon_{it}$  is the deviation from the baseline vacancy rate;  $\rho_i$  is the persistence of the deviation for market  $i$ ;  $\beta_t$  is the time-specific component of the vacancy rate common to all local markets;  $\mu_{it}$  is a shock with zero mean.

$\alpha_i + \beta_t$  is the estimated “natural” vacancy rate for market  $i$  at time  $t$ ; while  $\alpha_i(1 - \rho_i) + \beta_t + \rho_i V_{i,t-1}$  is short-run equilibrium vacancy rate, which is obtained by eliminating the zero mean random shocks,  $\mu_{it}$  from the observed vacancy rate. The parameters in (18) can be estimated from the regression equation

$$V_{it} = \sum_{i=1}^N \alpha_i(1 - \rho_i)D_i + \sum_{t=2}^T \beta_t M_t + \sum_{i=1}^N \rho_i D_i V_{i,t-1}. \quad (19)$$

After estimating the equilibrium vacancy rates, we ran linear regression to test the factors that may significantly affect the vacancy rate. In the model part, we disentangled the vacancy rate into its contributing factors: demand-side factors, supply-side factors and builder-related factors. Because our theoretical model is constructed to explain the vacancy formation at market level, rather than at property level, we need to find appropriate proxies at market level for all factors that we are going to test.

The number of potential tenants,  $B_t$  in our model, can be approximated by number of employments, or measured by the unemployment rate. The number of available units,  $S_t$  in our model, is measured by the total rentable space in the market. New completions in a market are a natural choice for measuring  $q_t$ , the number of new starts of projects in each period.  $h$ , denoting how difficult the search and matching is for a specific market, can be approximated by rental dispersion, or turnover rate. We choose the growth rate of employments as the proxy for  $\theta_t$ , the growth in demand for space. The uncertainty in the demand for space is approximated by the uncertainty of employment over time. Our model also considers supply-side restrictions, which are often in the form of natural barrels to further development, or the government

regulations such as zoning. Reservation price  $\xi_t$  is unobservable, but asking price  $p_t$  can be measured by asking rent level. Now we are ready to write out the estimation equation as (20),

$$\hat{V}^n = f(RDISP, UNCTY, GROWTH, TO, G, UNEM_{-1}, INVT_{-1}, COMP_{-1}, R_{-1}, CONSTR), \quad (20)$$

where  $\hat{V}^n$  is the estimated equilibrium vacancy rate; *RDISP* is the dispersion of rent within a market; *UNCTY* is the uncertainty of future employments; *GROWTH* is the expected growth of employments; *TO* is the average turnover rate of a local market; *G* is the topographical constraints of a local market; *UNEM* is unemployment rate; *INVT* is the inventory level; *COMP*<sub>-1</sub> is the new completions in the last period; *R*<sub>-1</sub> is the rent level in the last period; and *CONSTR* is the constraints of supply side.

Our model implies an interactive relationship between some factors. When doing multiple linear regression (MLR), an interaction term is included when the magnitude of the effect of one independent variable on the dependent variable varies as a function of a second independent variable. Therefore to test the interactive effects of uncertainty and costly search, and the interactive effects of uncertainty and land supply on vacancy rates, we include interaction terms. From the implications of our theoretical model, we include the interactive terms of uncertainty (*UNCTY*) with rental dispersion (*RDISP*), and the interactive terms of *UNCTY* with constraints on the supply side (*CONSTR*).

### 4.3 Data and Variable Constructions

Our data set is constructed from several resources, including Grubb & Ellis local market data aggregated from the building-by-building information; the Bureau of Labor Statistics; the Bureau of Economic Analysis; and a Metropolitan Statistical Area level data set constructed by Malpezzi, Green and Chun.

Information about rental office markets was provided by Grubb & Ellis. The data are comprised of quarterly vacancy rates, completions, absorptions, inventory

and Class-A-asking-price for 49 U.S. cities (metropolitan areas), over the period from the first quarter of 1995 through the first quarter of 2005 (many cities have some missing observations). Because we restricted our analysis to markets with at least 12 observations of vacancy rate data, we use only 27 local markets in our tests.

Grubb & Ellis provides nominal Class-A-asking rent levels. By quarterly Gross Domestic Product data from Bureau of Economic Analysis, we obtain GDP deflated rent level. Inventory level and lag-one period completions were obtained directly from the Grubb & Ellis rental data set. All these variables are measured at quarterly frequency.

Besides these quarterly data, supplementary information from Grubb & Ellis allowed us to construct some measures of rental market characteristics. We used average lease term (yearly data, from 1998 to 2005) to measure the turnover rate ( $TO$ ). The longer the average lease terms, the lower the turnover rate. The range of rent within each market (the difference between the highest and the lowest rent level, over the average rent level) is computed as the proxy for rental dispersion ( $RDISP$ ). This measure is only available cross sectionally, not over time. Therefore this measure is set to be equal over time for any local markets.

We collected monthly employments and unemployment rates ( $UNEM$ ) for these 27 local markets from the Bureau of Labor Statistics. Quarterly data of unemployment rates and number of employments can be extracted from the monthly data. The expected growth rate of employment,  $GROWTH$ , is computed by the three-year average yearly percentage change of employment preceding the sample quarter.

It is quite surprising that even though Grenadier documented the effects of demand uncertainty on the vacancy rate ten years ago, so far there is no empirical study on the vacancy rate testing the effects of uncertainty in demand for space. To test the effect of demand uncertainty, we need an appropriate measure for it. The method we applied is borrowed from a classic way to measure the uncertainty of firms' return in finance empirical literature. The basic idea is to estimate the historical volatility of the demand for space, and use the historical volatility as the expected uncertainty in the next period. For example, in Aggarwal and Samwick (1999), the estimates

of risks in return are based on the variances of monthly total returns over the 60 months preceding the sample year. In our context, we use the unemployment rates as the proxy for the demand for space. Then the uncertainty is computed from the rolling standard deviation of the unemployment rates by the unemployment rates 36 months preceding the sample quarter. For example, to compute the uncertainty in unemployment rate of Dallas in first quarter of 2005, we need to compute the standard deviation of the unemployment rate based the rates from the first quarter of 2001 to the forth quarter of 2004. We also tried different window sizes and found no essential change of our results.<sup>7</sup> Because 36 months is one of the widely used windows in finance literature, we use it in this paper.

We took two measures of topographical features of local land market from Malpezzi (1996) to measure the physical constraints of land supply: “Adjacent to water” is a dummy variable for a metropolitan area located on a major coastline (ocean or Great Lake). “Adjacent to park” is a dummy variable for a metropolitan area located adjacent to a large national park, military reservation, or other major constraints on expansion. These two measures have cross-sectional but not over time variations.

Besides natural constraints, government regulations may also restrict the supply of rental office space. It is ideal if we can find a proper measure for regulations of local office market, but such measures are scarce. Malpezzi (1996) and Malpezzi, Chun, and Green (1998) construct a regulation index for local housing markets. The difficulty is that we cannot measure how much the regulatory environment for the housing market are correlated to the regulatory environment for a rental office market. Currently we assume that the local governments tend to behave consistently in housing and rental office markets. Even though we know it is not a strong argument, in the absence of anything better, we still include the regulatory index for housing markets as an explanatory variable in our estimation.

---

<sup>7</sup>If the window is shorter, the correlation between this volatility measure and the level of unemployment rates will be more correlated.

## 4.4 Estimation Results

Before doing any estimation let's take a look at our raw data. In Table 3 and Table 4, we present the mean value of the market observed vacancy rates for each year and each local market included in our tests. From these two tables we find that vacancy rates vary a lot across local markets and also over time. The whole sample mean is at about 14.1%. The local market with the highest average vacancy rate is Oklahoma, with a high rate of 23.3%; the market with the lowest average vacancy rate is Seattle, with a low rate of 9.1%. Austin and Dallas, two main cities in Texas also have high vacancy rate of over 20%. From the overtime average vacancy rate we find that over the period that we observed, ( from 1995 to the first quarter of 2005) 2000 has the lowest vacancy rate at 9.1%, while 2003 and 2004 have an equally high rate of 18.4%. These numbers fit well into our prior knowledge about the market conditions.

The first step of our estimation was to estimate the natural (long-term) vacancy rates, and the short-term equilibrium vacancy rates. Then using the estimated long-term or short-term equilibrium vacancy rates, or the observed vacancy rates as the left-hand-side variable, we are ready to estimate the equation (20). The purpose of trying different dependent variables are: as we stated earlier, the existent method of estimating the natural vacancy rates has shortcomings of unable to count into the factors that both vary across markets and over time. By using vacancy rates under different meanings, we tested whether the factors proposed in our model contribute to explain the natural vacancy rates or the observed vacancy rates. Our trials of regressions shows that the key variables in our model, including the search frictions, the demand uncertainty, and the supply-side constraints, have consistent signs when using different meanings of vacancy rates as explanatory variables. The right-hand-side variables have the highest explanatory power for the real observed vacancy rates. Therefore we only presented the regression results by using the observed vacancy rates as the dependent variable. Table 5 presents the regression result when the interaction terms are not included. Table 6 is similar to Table 5, but the result is from the regression with the interaction terms: the interaction of uncertainty and

search frictions, and the interactions of uncertainty and supply-side constraints. From the results from these two regressions we found that most of the estimation results confirm what our model suggests:

The vacancy rate is affected by search-related factors. The rate is higher in areas which allow a shorter lease term, i.e. higher turnover rate; and in areas where the rents are more dispersed, therefore the search costs are higher. In table 5 the rental dispersion has the expected positive sign, but not significant. When including the interaction of uncertainty and rental dispersion, the effect of rental dispersion is significant at 10% level, and the interaction term has a negative sign (although only not very significant), as we predicted from the theoretical model.

The vacancy rate is higher in areas that have higher demand uncertainty. We also find that the absorptions are slower when the uncertainty is higher.<sup>8</sup> In a volatile environment, market players behave conservatively and the option values in vacant units are higher. Therefore more vacant units are held. In our regression results, the expected growth of demand is positively correlated with the vacancy rate, which supports our model prediction, and also consistent with the results of Shilling, Sirmans, and Corgel (1987). This result also makes intuitive sense: when the market is expected to grow fast, there will be stronger demand for space in the future. Therefore, property owners have incentive to hold more vacant units to satisfy the growing future demand. An analogy between inventory level and vacant units can help us understand this positive coefficient.

In the supply side, topographic factors and regulatory environment affect the vacancy rate significantly. The tighter the constraints on the land supply and new property development, the lower the vacancy rate. Lag-one period completions have positive effects on the vacancy rate, consistent with Rosen and Smith (1983).

Our results imply that the rent level of the last period is negatively related to the vacancy rate. It can be explained as follows: When the rent level is high, the opportunity costs of holding idle resources, i.e. the vacant units, are high. Therefore

---

<sup>8</sup>We regress absorptions on the uncertainty measure, and the coefficient is significantly negative. The results are not presented but can be obtained from the author upon request.



less units are kept vacant.

The mortgage rates have very significant negative effect on the vacancy rate. As the mortgage rate is higher, the opportunity cost of keeping one unit vacant is also higher. In such situation, landlords would like to hold less vacancies. From the tenants' perspective, when the mortgage rate is higher, the general economic condition is also better. In such situation the potential tenants would more like to set up a matching with landlords.

The effects of interaction terms are also significant. We construct the interactions terms of the demand uncertainty and some other local market characteristics: the interaction between uncertainty and land constraints, the interaction between uncertainty and regulation; the interaction between uncertainty and rental dispersion; the interaction between uncertainty and average lease term; the interaction between rental dispersion and the land supply conditions. Because of the variables, especially the interaction terms are highly correlated, we only eliminate some interaction terms from our regression. From the regression results presented in Table 6, We find that interaction terms slightly improve our estimation. Now the adjusted R-square is about 0.72, slightly better than 0.71 in the estimation without interaction terms. Most explanatory variables have the same signs as the regression results in Table 5. After identifying the interaction term between uncertainty and rental dispersion, the effect of rental dispersion itself becomes significant. The effect of uncertainty is still significant, as the case without interaction terms. Moreover, as predicted by our theoretical model, the joint effect of uncertainty and rental dispersion on the vacancy rate is negative. The interaction of uncertainty and the dummy variable of "adjacent to water" is significant. When a market is on the coast area or the Great Lakes, the joint effect of uncertainty and land supply has a positive effect on the vacancy rates: the effects of land-supply constraints is moderated by the uncertainty. Different from our expectation, the effect of regulation on the vacancy rate is positive, which is also moderated by the uncertainty. Such results may be a contradiction to our model implications. It may imply that our regulation index is not suitable: the regulation on the housing market and housing market are not highly positively correlated. The

interaction of frictional search and supply-side constraints has negative to vacancy rate, as predicted by the theoretical model.

## 5 Conclusions

In this paper we set up a dynamic model to study the interaction of search and matching in the current rental markets and potential future development. Our model predicts that potential development significantly affects the vacancy rate. The vacancy rate will be much higher when the potential development is not highly constrained. The uncertainty in demand for space affects both the decisions of developers and the players in the current rental markets, and results a higher vacancy rate if the supply side is not tightly constrained. The frictional search and demand uncertainty, these two types of “market frictions” that generate higher vacancy rates, tend to moderate each other’s effect. The supply-side constraints also moderate the effect of frictional search on the vacancy rate. Our empirical tests provide strong support for most of the predictions of our model.

Our study documented both theoretically and empirically the interactive effects of frictional search, demand uncertainty, and supply-side constraints, and their interactive effects on the vacancy formation. We empirically test the importance of uncertainty on the formation of vacancy rates. Our study has important practical implications: when facing uncertainties, different local markets may react differently depending on some other fundamental conditions of the specific market. For example, how difficult (frictional) the search is on the market, and how restricted the supply side is. When the supply side in a market is very restricted, the vacancy rate on the market tends to be lower. But the presentation of the uncertainty with supply constraints will jointly contribute to a higher vacancy rate.

For the examples we raised in the beginning of the paper: the vacancy rates in Houston and San Francisco. We found that the different changes in the vacancy rates in these two cities can be better explained by the changes in the uncertainties, the difference in the market heterogeneities (1.57 in Houston and 0.38 in San Francisco),

and the difference in the supply-side constraints (more naturally restricted and also regulated land market for San Francisco).

The next step of the current research is to study the relationship between the vacancy rate adjustment and the rental adjustment. Our current study focuses on the short-term equilibrium vacancy rate. It has been well documented that the vacancy rate has tendency to adjusted to a long run equilibrium level. In fact our model in a special setting up deals with the case of long-term equilibrium vacancy rate.<sup>9</sup> An interesting question is: how fast the market vacancy rate adjusts to the long-term equilibrium level? Our conjecture is: in a market that search is more difficult, the adjustment process will be more difficult. This interesting thinking will be explored in our future research.

It is possible to extend our model in several directions. For example, real estate investment, especially on the commercial side, is very lumpy. Such lumpiness will affect development decisions and may therefore affect the vacancy rate. Intuitively, lumpiness in supply produces some range of inactions to shocks. Intuitively such lumpiness may induce higher volatility in the vacancy rate. Will such lumpiness also affect the level of the vacancy rate? Another relevant but different question is the demand-side lumpiness also different across markets. Such demand-side lumpiness affects the search and matching behavior, and therefore may also affect the vacancy rates. Our basic model may also be extended to analyze the effects of lumpiness to the vacancy rate.

We can set the developers to be a monopoly rather than many competing players to study the effects of monopoly power on the vacancy rates. We can also explore the vacancies or space inventories held by demand side in the market (that is often in the form of phantom vacancies). If we assume that the tenants can change his/her status according to the current economic conditions, the tenants may hold some units

---

<sup>9</sup>If we assume that there is no uncertainty and the supply side is stationary over time, then we can get a long run equilibrium vacancy rate. This rate is only affected by the market heterogeneity, or the search friction. This result is consistent with previous study on stationary vacancy rate, therefore we do not list it in the current paper.

as inventory. As the economic conditions change, tenants would like to adjust their inventory level according to their expectation about the future market condition. This type of adjustment is often in the form of sublease space. Such adjustments of the demand side could be considered in our model.

## References

- Aggarwal, R. K., and A. A. Samwick, 1999, "The Other Side of the Trade-off: The Impact of Risk on Executive Compensation," Journal of Political Economy, vol. 107, 65–105.
- Arnott, R., 1989, "Housing Vacancies, Thin Markets, and Idiosyncratic Tastes," Journal of Real Estate Finance and Economics, 2, 5–30.
- Blank, D. M., and L. Winnick, 1953, "The structure of the Housing Market," The Quarterly Journal of Economics, 67, 181–208.
- Buttimer, R., and S. H. Ott, 2004, "Commercial Real Estate Valuation and Development with Leasing Uncertainty," working paper.
- Childs, P. D., S. H. Ott, and T. J. Riddiough, 2002a, "Optimal Valuation of Claims on Noisy Real Assets: Theory and Application," Real Estate Economics, 30, 415–443.
- Childs, P. D., S. H. Ott, and T. J. Riddiough, 2002b, "Optimal Valuation of Noisy Real Assets," Real Estate Economics, 30, 385–414.
- Gabriel, S. A., and F. Nothaft, 1988, "Rental Housing Markets and Natural Vacancy Rate," AREUEA Journal, 16, 419–429.
- Glascok, J., S. Jahanian, and C. F. Sirmans, 1990, "An Analysis of Office Market Rents: Some Empirical Evidence," AREUEA Journal, 18, 105–119.
- Grenadier, S., 1995a, "The Persistence of Real Estate Cycles," Journal of Real Estate Finance and Economics, 10, 95–119.
- Grenadier, S., 1996, "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets," Journal of Finance, 51, 1653–1679.
- Grenadier, S., 1999, "Information Revelation Through Option Exercise," Review of

- Financial Studies, 12, 95–130.
- Grenadier, S. R., 1995b, “Local and National Determinants of Office Vacancies,” Journal of Urban Economics, 37, 57–71.
- Hendershott, P. H., B. D. MacGregor, and R. Y. C. Tse, 2002, “Estimation of the Rental Adjustment Process,” Real Estate Economics, 30, 165–183.
- Igarashi, M., 1991, “The Rental Vacancy Relationship in the Rental Housing Market,” Journal of Housing Economics, 1, 251–270.
- Malpezzi, S., 1996, “Housing Prices, Externalities, and Regulation in U.S. Metropolitan Areas,” Journal of Housing Research, 7, 209–241.
- Malpezzi, S., G. H. Chun, and R. K. Green, 1998, “New Place-to-Place Housing Price Indexes for U.S. Metropolitan Areas, and Their Determinants,” Real Estate Economics, 26, 235–274.
- Read, C., 1988, “Advertising and Natural Vacancies in Rental Housing Markets,” AREUEA Journal, 16, 354.
- Read, C., 1991, “Maintenance, Housing Quality, and Vacancies under Imperfect Information,” AREUEA Journal, 19, 138–153.
- Rosen, K. T., and L. B. Smith, 1983, “The Price-Adjustment Process for Rental Housing and the Natural Vacancy Rate,” The American Economic Review, 73, 779–786.
- Shilling, J. D., C. Sirmans, and J. B. Corgel, 1987, “Price Adjustment Process for Rental Office Space,” Journal of Urban Economics, 22, 90–100.
- Stokey, N. L., R. E. J. Lucas, and E. C. Prescott, 1989, Recursive Methods in Economic Dynamics, Harvard University Press.
- Titman, S., 1985, “Urban Land Prices Under Uncertainty,” American Economic Review, 75, 505–514.
- Vandell, K. D., 2003, “Tax Structure and Natural Vacancy Rates in the Commercial Real Estate Market,” Real Estate Economics, 31, 245–267.
- Voith, R., and T. Crone, 1988, “National Vacancy Rates and the Persistence of Shocks in U.S. Office Markets,” AREUEA Journal, 16, 437–458.
- Wang, K., and Y. Zhou, 2000, “Overbuilding: A Game-Theoretic Approach,” Real

- Estate Economics, 28, 493–522.
- Wheaton, W. C., 1987, “The Cyclic Behavior of the National Office Market,” AREUEA Journal, 15, 281–299.
- Wheaton, W. C., 1990, “Vacancy, Search, and Prices in a Housing Market Matching Model,” The Journal of Political Economy, 98, 1270–1292.
- Williams, J. T., 1995, “Pricing Real Assets with Costly Search,” Review of Financial Studies, 8, 55–90.

# Appendix

Figure 5: First and Second Period Vacancy Rates and Market Heterogeneity

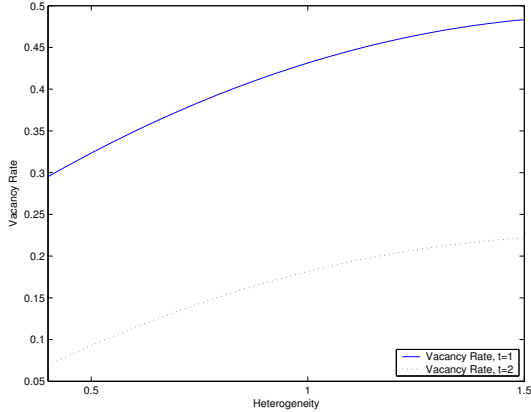


Figure 6: Variation of the Vacancy Rate with Number of Buyers and Sellers

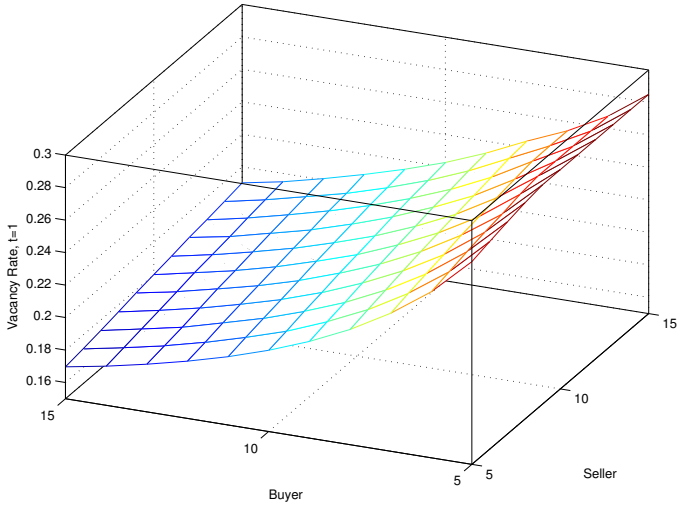


Figure 7: Vacancy Rate, Heterogeneity, and Uncertainty, at Unconstrained Land Supply

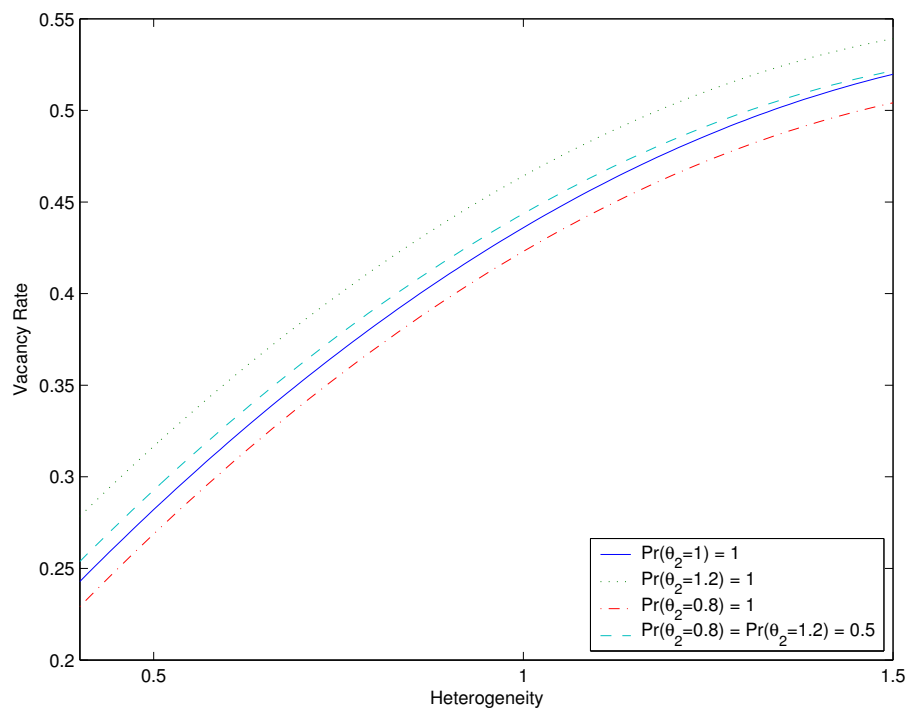




Figure 8: Vacancy Rate, Heterogeneity, and Uncertainty, at Fixed Land Supply

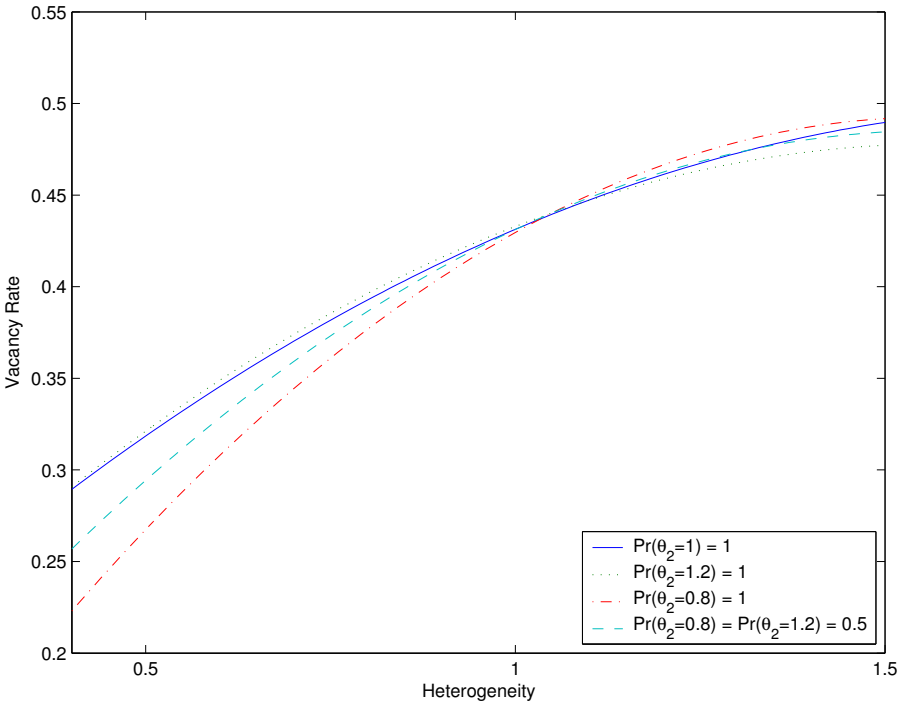


Table 1: Simulation results: when land supply is infinitely elastic.  $h$ : rent dispersion, an index for search costs;  $v_1$ : vacancy rates in the first period;  $v_2$ : vacancy rates in the second period;  $q_0$ : new completions in the first period;  $\xi_1$ : reservation prices of buyers in the first period;  $q_1$ : new completions in the second period;  $p_1$ : asking prices in the first period;  $C_1$ : expected costs of buyers in the first period;  $C_2$ : expected costs of buyers in the second period;  $\Pi_1$ : expected profits of sellers in the first period;  $\Pi_2$ : expected profit of sellers in the second period.

	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	average vacancy rate
Section a: a deterministic case: $\Pr(\theta_2 = 1) = 1$													
$q_0$	1	1	2	3	3	3	5	4	5	6	6	7	
$q_1$	2	3	3	3	3	4	3	4	4	3	4	3	
$C_1$	106.67	106.50	106.36	106.22	106.49	106.39	106.19	106.46	106.32	106.43	106.36	106.45	
$\Pi_1$	103.42	103.33	103.21	103.14	103.24	103.23	103.11	103.23	103.16	103.19	103.16	103.19	
$v_1$	0.24	0.27	0.33	0.35	0.40	0.41	0.43	0.45	0.48	0.49	0.51	0.53	0.41
$p_1$	103.59	103.53	103.42	103.34	103.48	103.52	103.36	103.57	103.51	103.52	103.55	103.56	
$\xi_1$	104.04	104.04	103.97	103.95	104.14	104.25	104.14	104.40	104.40	104.46	104.55	104.62	
$C_2$	109.89	108.85	108.61	108.40	108.65	108.24	108.32	108.34	108.23	108.58	108.32	108.64	
$\Pi_2$	103.19	103.11	103.12	103.11	103.19	103.11	103.13	103.13	103.09	103.18	103.09	103.16	
$v_2$	0.06	0.09	0.11	0.13	0.14	0.17	0.19	0.20	0.22	0.23	0.25	0.25	0.17
Section b: a deterministic case: $\Pr(\theta_2 = 1.2) = 1$													
$q_0$	3	3	4	4	5	6	6	7	8	8	9	9	
$q_1$	2	3	3	4	4	3	4	4	4	4	4	4	
$C_1$	106.06	106.03	105.97	105.96	105.89	106.07	106.05	105.97	105.89	106.07	105.99	106.15	
$\Pi_1$	103.27	103.24	103.18	103.19	103.14	103.20	103.20	103.15	103.10	103.16	103.11	103.17	
$v_1$	0.28	0.31	0.36	0.38	0.42	0.45	0.46	0.48	0.51	0.51	0.53	0.54	0.44
$p_1$	103.43	103.43	103.38	103.44	103.40	103.45	103.51	103.47	103.44	103.53	103.50	103.59	
$\xi_1$	103.89	103.96	103.96	104.08	104.10	104.21	104.33	104.35	104.37	104.52	104.54	104.69	
$C_2$	109.76	108.93	108.72	108.31	108.21	108.66	108.37	108.29	108.21	108.40	108.33	108.51	
$\Pi_2$	103.17	103.12	103.14	103.09	103.09	103.21	103.14	103.12	103.09	103.13	103.09	103.12	
$v_2$	0.07	0.09	0.11	0.13	0.15	0.16	0.18	0.20	0.22	0.23	0.25	0.26	0.17

h	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	average vacancy rate
Section c: a deterministic case: $\Pr(\theta_2 = 0.8) = 1$													
$q_0$	0	0	0	0	1	1	2	2	3	3	4	4	
$q_1$	1	2	2	3	3	3	3	3	3	3	3	3	
$C_1$	107.65	107.11	107.45	107.09	106.82	107.07	106.81	107.02	106.78	106.96	106.73	106.90	
$\Pi_1$	103.12	103.28	103.38	103.34	103.19	103.29	103.18	103.26	103.16	103.24	103.14	103.20	
$v_1$	0.22	0.27	0.31	0.34	0.38	0.39	0.43	0.44	0.47	0.48	0.50	0.50	0.41
$p_1$	103.20	103.44	103.56	103.58	103.42	103.55	103.44	103.56	103.45	103.57	103.47	103.58	
$\xi_1$	103.01	103.88	104.06	104.14	104.03	104.21	104.15	104.32	104.27	104.44	104.40	104.56	
$C_2$	109.42	109.27	109.50	108.64	108.37	108.62	108.39	108.61	108.40	108.61	108.42	108.61	
$\Pi_2$	103.12	103.18	103.29	103.16	103.13	103.20	103.15	103.20	103.14	103.19	103.12	103.15	
$v_2$	0.07	0.09	0.10	0.13	0.15	0.16	0.18	0.19	0.22	0.23	0.25	0.26	0.17
Section d: an uncertain case: $E(\theta_2)=1$ , with $\Pr(\theta_2=1.2)=0.5$ , $\Pr(\theta_2=0.8)=0.5$													
$q_0$	1.5	1.5	2	2	3	3.5	4	4.5	5.5	5.5	6.5	6.5	
$q_1$	1.5	2.5	2.5	3.5	3.5	3	3.5	3.5	3.5	3.5	3.5	3.5	
$C_1$	106.86	106.57	106.71	106.52	106.36	106.57	106.43	106.49	106.33	106.51	106.36	106.52	
$\Pi_1$	103.27	103.26	103.28	103.27	103.17	103.25	103.19	103.21	103.13	103.20	103.13	103.19	
$v_1$	0.25	0.29	0.34	0.36	0.40	0.42	0.44	0.46	0.49	0.50	0.52	0.52	0.42
$p_1$	103.34	103.44	103.47	103.51	103.41	103.50	103.48	103.52	103.45	103.55	103.49	103.59	
$\xi_1$	103.45	103.92	104.01	104.11	104.07	104.21	104.24	104.34	104.32	104.48	104.47	104.62	
$C_2$	109.59	109.10	109.11	108.47	108.29	108.64	108.38	108.45	108.30	108.50	108.37	108.56	
$\Pi_2$	103.14	103.15	103.21	103.13	103.11	103.20	103.15	103.16	103.11	103.16	103.10	103.14	
$v_2$	0.07	0.09	0.10	0.13	0.15	0.16	0.18	0.20	0.22	0.23	0.25	0.26	0.17

Table 2: Simulation results, when land supply is constrained,  $N=5$  is the maximum number of new developments.  $h$ : rent dispersion, an index for search costs;  $v_1$ : vacancy rates in the first period;  $v_2$ : vacancy rates in the second period;  $q_0$ : new completions in the first period;  $\xi_1$ : reservation prices of buyers in the first period;  $q_1$ : new completions in the second period;  $p_1$ : asking prices in the first period;  $C_1$ : expected costs of buyers in the first period;  $C_2$ : expected costs of buyers in the second period;  $\Pi_1$ : expected profits of sellers in the first period;  $\Pi_2$ : expected profits of sellers in the second period

	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	average vacancy rate
Section a: a deterministic case: $\Pr(\theta_2 = 1) = 1$													
$q_0$	2	2	3	3	3	3	3	3	4	4	4	4	
$q_1$	2	3	2	2	2	2	2	2	1	1	1	1	
$C_1$	106.12	106.02	106.35	106.64	106.90	107.15	107.37	107.57	107.71	107.89	108.05	108.21	
$\Pi_1$	103.08	103.08	103.13	103.26	103.37	103.47	103.55	103.63	103.61	103.70	103.76	103.82	
$v_1$	0.29	0.31	0.37	0.38	0.40	0.41	0.42	0.43	0.47	0.47	0.48	0.48	0.41
$p_1$	103.22	103.25	103.29	103.45	103.59	103.74	103.84	103.95	103.91	104.03	104.12	104.23	
$\xi_1$	103.63	103.73	103.82	104.04	104.25	104.46	104.62	104.78	104.77	104.96	105.10	105.26	
$C_2$	109.09	108.30	108.97	109.22	109.46	109.69	109.88	110.07	110.49	110.67	110.82	110.98	
$\Pi_2$	103.07	103.00	103.19	103.28	103.36	103.44	103.50	103.55	103.65	103.70	103.73	103.77	
$v_2$	0.07	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.17	0.18	0.19	0.14
Section b: a deterministic case: $\Pr(\theta_2 = 1.2) = 1$													
$q_0$	3	4	4	4	4	4	5	5	5	5	5	5	
$q_1$	2	1	1	1	1	1	0	0	0	0	0	0	
$C_1$	106.06	106.46	106.79	107.08	107.34	107.57	107.75	107.94	108.12	108.29	108.45	108.60	
$\Pi_1$	103.27	103.34	103.47	103.58	103.67	103.77	103.79	103.86	103.94	103.99	104.06	104.11	
$v_1$	0.28	0.34	0.36	0.38	0.39	0.40	0.44	0.45	0.46	0.47	0.47	0.48	0.41
$p_1$	103.43	103.50	103.66	103.80	103.93	104.07	104.06	104.17	104.29	104.38	104.48	104.57	
$\xi_1$	103.89	104.01	104.24	104.45	104.65	104.84	104.89	105.05	105.23	105.37	105.53	105.67	
$C_2$	109.76	110.47	110.64	110.80	110.95	111.12	111.59	111.71	111.85	111.96	112.09	112.20	
$\Pi_2$	103.17	103.37	103.47	103.56	103.63	103.70	103.81	103.86	103.91	103.94	103.98	104.01	
$v_2$	0.07	0.07	0.09	0.10	0.11	0.12	0.12	0.13	0.14	0.15	0.16	0.16	0.12

h	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	average vacancy rate
Section c: a deterministic case: $\Pr(\theta_2 = 0.8) = 1$													
$q_0$	0	0	0	1	1	2	2	2	3	3	3	3	3
$q_1$	2	2	3	3	3	3	3	3	2	2	2	2	2
$C_1$	106.72	107.11	106.78	106.55	106.82	106.58	106.81	107.02	107.16	107.34	107.51	107.68	107.68
$\Pi_1$	103.17	103.28	103.24	103.10	103.19	103.08	103.18	103.26	103.26	103.33	103.40	103.45	103.45
$v_1$	0.25	0.28	0.30	0.36	0.38	0.42	0.43	0.44	0.47	0.48	0.49	0.49	0.40
$p_1$	103.31	103.44	103.45	103.30	103.42	103.30	103.44	103.56	103.51	103.62	103.72	103.80	103.80
$\xi_1$	103.70	103.88	103.96	103.85	104.03	103.96	104.15	104.32	104.31	104.47	104.63	104.76	104.76
$C_2$	109.03	109.27	108.36	108.10	108.37	108.13	108.39	108.61	109.04	109.24	109.43	109.59	109.59
$\Pi_2$	103.06	103.18	103.07	103.04	103.13	103.08	103.15	103.20	103.31	103.36	103.40	103.43	103.43
$v_2$	0.07	0.09	0.12	0.14	0.15	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.16
Section d: an uncertain case: $E(\theta_2)=1$ , with $\Pr(\theta_2=1.2)=0.5$ , $\Pr(\theta_2=0.8)=0.5$													
$q_0$	1.5	2.0	2.0	2.5	2.5	3.0	3.5	3.5	4.0	4.0	4.0	4.0	4.0
$q_1$	2.0	1.5	2.0	2.0	2.0	2.0	1.5	1.5	1.0	1.0	1.0	1.0	1.0
$C_1$	106.39	106.79	106.78	106.81	107.08	107.08	107.28	107.48	107.64	107.82	107.98	108.14	108.14
$\Pi_1$	103.22	103.31	103.35	103.34	103.43	103.42	103.48	103.56	103.60	103.66	103.73	103.78	103.78
$v_1$	0.26	0.31	0.33	0.37	0.38	0.41	0.43	0.44	0.47	0.47	0.48	0.49	0.40
$p_1$	103.37	103.47	103.55	103.55	103.68	103.69	103.75	103.86	103.90	104.00	104.10	104.19	104.19
$\xi_1$	103.80	103.95	104.10	104.15	104.34	104.40	104.52	104.68	104.77	104.92	105.08	105.21	105.21
$C_2$	109.39	109.87	109.50	109.45	109.66	109.63	109.99	110.16	110.45	110.60	110.76	110.90	110.90
$\Pi_2$	103.11	103.28	103.27	103.30	103.38	103.39	103.48	103.53	103.61	103.65	103.69	103.72	103.72
$v_2$	0.07	0.08	0.10	0.12	0.13	0.15	0.15	0.16	0.17	0.18	0.19	0.20	0.14

Table 3: Summary statistics: The observed vacancy rates for different years across MSA.

year	number of obs.	mean	min	max	std. dev.
Observed Vacancy Rate					
1995	43	0.134	0.077	0.193	0.035
1996	60	0.120	0.003	0.196	0.046
1997	62	0.106	0.036	0.180	0.038
1998	68	0.100	0.025	0.166	0.036
1999	75	0.105	0.029	0.200	0.034
2000	76	0.092	0.016	0.173	0.041
2001	82	0.130	0.043	0.223	0.035
2002	102	0.174	0.095	0.254	0.031
2003	112	0.184	0.110	0.256	0.032
2004	112	0.184	0.103	0.256	0.034
2005	28	0.173	0.103	0.242	0.037

Table 4: Summary statistics: the observed and equilibrium vacancy rates for different local markets over time.

market	number of obs.	mean	min	max	std. dev.
		Observed Vacancy Rate			
Albuquerque	11	0.155	0.134	0.170	0.010
Atlanta	38	0.144	0.087	0.231	0.052
Austin	13	0.210	0.180	0.232	0.018
Boston	40	0.106	0.031	0.198	0.059
Chicago	40	0.135	0.079	0.199	0.042
Cleveland	40	0.163	0.112	0.217	0.033
Dallas/Fort Worth	38	0.203	0.158	0.256	0.036
Denver	40	0.136	0.082	0.224	0.054
Detroit	39	0.134	0.069	0.211	0.046
Houston	40	0.151	0.101	0.201	0.032
Kansas City	32	0.183	0.146	0.201	0.018
Las Vegas	11	0.139	0.120	0.154	0.013
Los Angeles	40	0.156	0.115	0.190	0.020
Miami/Dade County	34	0.140	0.085	0.171	0.023
Nashville	40	0.109	0.057	0.164	0.031
New York City	40	0.107	0.054	0.157	0.027
Oakland/East Bay	34	0.100	0.030	0.159	0.037
Oklahoma City	11	0.232	0.216	0.245	0.009
Omaha	11	0.181	0.165	0.195	0.010
Philadelphia	25	0.142	0.084	0.189	0.035
Phoenix	40	0.144	0.088	0.218	0.046
Pittsburgh	28	0.173	0.126	0.214	0.028
Richmond	11	0.142	0.120	0.163	0.014
St. Louis	34	0.124	0.087	0.187	0.030
San Antonio	13	0.202	0.186	0.210	0.007
San Francisco	41	0.105	0.017	0.241	0.087
San Jose/Silicon Valley	37	0.104	0.003	0.217	0.071
Seattle	40	0.091	0.023	0.182	0.058

Table 5: Determinants of the Observed Vacancy Rate

Effect	Estimates	Standard Errors	$t$	Pr > $ t $
Intercept	0.14	0.044	3.17	0.0016
Demand uncertainty	.015	.003	4.34	<.0001
Rent dispersion	.00002	.00012	0.17	.8648
MSA located adjacent to coast, etc.	-.025	.004	-7.12	<.0001
MSA located on park, etc.	-.006	.003	-1.96	.051
Regulation index	.0004	.0007	0.56	.5791
Average lease term	-.0004	.00014	-3.15	.0017
Lag one period unemployment rate	.007	.0018	4.01	<.0001
Expected growth rate of employ- ments	0.42	0.11	3.68	.0003
Lag one period completions	6.9E-9	2.4E-9	2.93	.0035
Lag one period rent level	-.00165	.00025	-6.54	<.0001
Log of lag one period inventory	.013	.002	6.09	<.0001
Lag one period mortgage rate	-0.033	.003	-10.89	<.0001
Adjusted R <sup>2</sup>	0.703			
N	528			

The dependent variable is the observed vacancy rates, estimated by regression equation (19). Demand uncertainty is computed by the rolling standard deviation of unemployment rates of 36 months preceding the sample quarter. Rental dispersion is computed from the difference of the highest and the lowest rents, scaled by the average rent level. MSA located adjacent to park, etc. and MSA located on coast, etc. are two dummy variables to measure the topographic characteristics of a local market from Malpezzi (1996). Regulation Index is a variable measuring how strict the governmental regulations are, constructed in Malpezzi (1996) and MGC (1998). Expected growth rate of employments is computed as the three-year average yearly percentage changes of employments preceding the sample quarter. Log of lag one period inventory is the logarithm of the lag one period inventory level.



Table 6: Determinants of the Observed Vacancy Rate: Effects of Interactions

Effect	Estimates	Standard Errors	$t$	Pr > $ t $
Intercept	.053	.054	0.97	0.3311
Demand uncertainty	.057	.018	3.07	.0023
Rent dispersion	.0006	.0003	2.24	0.0255
MSA located on coast, etc.	-.047	.008	-5.92	<.0001
MSA located adjacent to park, etc.	.0007	.005	0.16	0.87
Regulation index	.0034	.0015	2.26	.024
Average lease term	-.0006	.0001	-4.13	<.0001
Lag one period unemployment rate	.008	.002	4.51	<.0001
Lag one period rent level	-.002	.0003	-6.51	<.0001
Expected growth in employments	.501	.114	4.41	<.0001
Lag one period completions	6.22E-9	2.34E-9	2.65	.008
Log of lag one period inventory	.014	.002	6.52	<.0001
Lag one period mortgage rate	-.030	.003	-9.67	<.0001
Interaction of uncertainty and rental dispersion	-.0004	.0002	-1.84	0.067
Interaction of uncertainty and located on coast	0.014	.006	2.53	.0117
Interaction of market heterogeneity and adjacent to park	-.0018	.001	-1.84	.0658
Adjusted R <sup>2</sup>	0.715			
N	528			

The dependent variable is the observed vacancy rates, estimated by regression equation (19). Demand uncertainty is computed by the rolling standard deviation of unemployment rates of 36 months preceding the sample quarter. Rental dispersion is computed from the difference of the highest and the lowest rents, scaled by the average rent level. In this regression we classified the rental dispersion into two groups: high and low. MSA located adjacent to park, etc. and MSA located on coast, etc. are two dummy variables to measure the topographic characteristics of a local market from Malpezzi (1996). Regulation Index is a variable measuring how strict the governmental regulations are, constructed in Malpezzi (1996) and MGC (1998). Expected growth rate of employments is computed as the three-year average yearly percentage change of employments preceding the sample quarter. Log of one period inventory is the logarithm of the lag one period inventory level; All variables named with “interaction” are the interaction terms of the correspondent variables.