ARE INTERNATIONAL REAL ESTATE MARKETS INTEGRATED: EVIDENCE FROM CHAOTIC DYNAMICS

RESEARCH REPORT
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EXECUTIVE SUMMARY

Chaos, also referred to as nonlinear dynamic analysis, is a nonlinear deterministic process that appears random. A system may be nonlinear, and yet can be either simple or chaotic. If a price series has some degree of nonlinear dependency on their past movements, it may generate complex behavior and be described by chaotic theory. In the case of specific "chaotic" nonlinearities, such as the logistic map, the dependency can evolve from passive and predictable states to violent and unpredictable states. Real estate markets may be characterized by non-linear "chaotic" behavior resulting from the presence of market frictions and significant transaction costs, the interaction of informed and irrational market players as well as economy wide fluctuations. Consequently, the dynamic behavior of returns may give rise to asymmetric dynamics for returns of different size.

In the context of market integration, the usual notion is that national real estate markets are segmented because of differing degrees of market imperfections, such as illiquidity and transaction costs, with each market exhibiting unique microstructure characteristics that further distinguish one market from other markets. The disparity in characteristics in turn might induce non-linearities in real estate market returns and help explain differences in the level and behavior of returns across markets. Our study is believed to be the first comprehensive work to combine the non-linearities and chaos dynamics of national real estate market returns with market integration in international real estate. Six major national real estate markets are included in this study; those of the USA, UK, Australia, Japan, Hong Kong and Singapore, as well as the World real estate market over the period January 1990 through June 2006. For each market, we examine four data series (EPRA/NAREIT weekly original series, EPRA/NAREIT weekly hedged series; EPRA/NAREIT daily original series and EPRA/NAREIT daily hedged series) to form an "on the balance of evidence" picture.

The BDS statistics applied to the standardized residuals of the AR and GARCH model reject the null hypothesis that the real estate return data are independently and identically distributed, suggesting that the linear dependencies and conditional heteroskedasticity are not the causes of the nonlinear structure in the data. On the other hand, tests of chaos, based on the K-map components of a logistic model indicate that real estate market returns exhibit chaotic behavior in some instances. Of the 28 real estate return series, there are six weekly original series, two weekly hedged series, five daily original series and two daily hedged series that have at least one K-map term that is statistically significant. This K-map evidence thus implies an exponential relationship by which current real estate market returns affect future real estate market returns. Further tests on a full logistic map which is made up of a non-linear component (K-map) and a behavioral component (Z-map) reveal that many of the Z-map coefficients are statistically significant and positive, a finding which supports the notion of market integration among the six national real estate markets and with the world stock market. The Z-map in this study serves either as an indicator of the degree of market integration among the six national real estate markets or proxy for the degree of integration of the national real estate markets with the world stock market (represented by the MSCI world equity index).

Table 1 provides a summary that allows for the interaction of the Z-map term (represented by the MSCI equity index) with the K-map terms (chaos) for 12 real estate series. For the 9 original series, evidence of varying levels of market integration is documented. In addition, the lagged same country's market returns (K-map) combine with the MSCI returns (Z-map) help determine the future evolution of the same country's market returns; the effects of chaos are however weaker. On the contrary, there appears to be some dominance of K-map influence in the three hedged real estate series with two of the three Z-map coefficients being statistically insignificant. If the hedged real estate is considered a better proxy for the direct real estate market, the results imply that national direct real estate markets are still segmented with the world stock market, and that past non-linear returns might affect the level of future returns to a greater degree (K-map).

Table 1
Non-linear TAR-GARCH (1, 1)-M regressions
of market returnsand MSCI index: the K-map and Z-map

Types of index	Market	Significant K-map terms	Z-map coefficient	Adj R^2 (due to K-map)	Adj R^2 (due to Z-map)	Total Adj R^2 (K+Z maps)
Weekly-original	US	R^2	0.403***	2.6%	19.7%	22.3%
	UK	R^3	0.539***	2.1%	16.3%	18.4%
	AUS	R^3	0.344***	2.5%	11.6%	14.1%
	JP	R^3, R^4	1.217***	3.2%	16.4%	19.6%
	WRE	R^2, R^4	0.697***	1.7%	45.3%	47.0%
Weekly-hedged	HK	R^2, R^4	0.051***	2.1%	2.5%	4.6%
	SG	R^2	-0.032	4.4%	-0.6%	3.8%
Daily-original	UK	R^3	0.318***	3.3%	7.5%	10.8%
	JP	R^2	0.963***	1.6%	12.3%	13.9%
	HK	R^3, R^4	0.549***	3.5%	8.6%	12.1%
	WRE	R^3	0.495***	4.9%	34.5%	39.4%
Daily – hedged	HK	R^3, R^4	0.007	1.9%	-0.3%	1.6%

Notes: (1) For each index, statistically significant nonlinear lagged returns (K-map terms) is analyzed with $R_t = a_0 + b_1(R_{t-1}) + b_2(R^2{}_{t-1}) + b_3(R^3{}_{t-1}) + b_4(R^4{}_{t-1}) + \varepsilon_t$, (2) A non-linear TAR-GARCH (1, 1) – M regression is run to estimate the joint impact of the K-map and Z-map (proxied by the MSCI world equity index). The non-linear TAR-GARCH (1, 1) - M model has the following specification:

$$R_t = a_0 + a_1 R_{t-1} + a_2 R_{t-2} + a_3 R_{t-3} + a_4 R_{t-4} + b\sigma_t{}^2 + c(Dummy) + dMSCI_t + \varepsilon_t$$

$$\sigma_t{}^2 = e_0 + e_1\varepsilon_{t-1}{}^2 + e_2\sigma_{t-1}{}^2 + e_3\sigma_{t-1}{}^2 I_{t-1} + e_4(Dummy)$$

$a_{2-}a_4$ are the coefficients for the K-map terms; d is the coefficient for the Z-map (MSCI) term
*** indicates two-tailed significance at the 1percent level

Finally, if the non-linear expressions are more descriptive than the linear relationship, are they also better predictors? In general, the comparisons of the forecasting accuracy show that incorporating the K-map components in the model appears to improve the forecasting performance over the conventional linear model. However, this finding is at best inconclusive due to the mixed results obtained for some individual series.

Overall, the important contributions of this study include significant evidence of nonlinearity and low deterministic chaotic behavior in some real estate market returns. These results are important because knowing that real estate market returns exhibit chaotic behavior can help us understand the evolution of real estate market returns better and find ways of predicting them. In this respect the results also have practical implications, because they suggest that international integration models for real estate market returns should include some nonlinear terms to be correct. In the international context, if real estate markets are in fact chaotic, the co-movements among them may also take non-linear form. Accordingly, global investors should be cautious in formulating their diversification strategies since gains from diversification may not occur in the long-run.

Abstract

A logistic model of chaos is employed to test real estate returns in several countries for non-linear dependencies. These nonlinearities could arise as a nonlinear function of past observations. Furthermore, evidence of varying evels of market integration among the major real estate markets, and with the MSCI world stock market is documented for these series. This study also accounts for non-linear chaotic behavior in forecasting models in order to obtain potentially more accurate real estate forecasts.

1. INTRODUCTION

The purpose of this study is to study the behavior of real estate market returns over time with regard to their non-linear structures and chaos and explore the modeling implications for real estate and stock market integration. "Chaos" is a technical term that refers to irregular, complex behavior that seems random, but actually has some hidden order. This behavior is typical of deterministic non-linear systems, such as the logistic map (Larrain, 1991).[1] The presence of chaos, or low-dimensional non-linearities, in financial market price movements is indicative of a deterministic link wherein past price observations predict the outcome of future observations. Furthermore, while the linear links between the past prices and future prices, in general, tend to be more stable, the non-linear or exponential links tend to be associated with wider price fluctuations. In the case of specific "chaotic" non-linearities, such as the logistic maps, the link can evolve from passive and predictable states to violent and unpredictable states. Finally, as Hsieh (1991) explains, one important reason to test for the presence of chaos is that it can help investors and policy makers to understand and explain fluctuations in the economy and financial markets which appear to be random.[2]

There are several possible reasons why non-linearities may be observed in real estate market returns. At the onset, the strategic interactions among market participants, the process

[1] Chaos has been identified in hydrodynamic turbulence, lasers, electrical circuits, chemical reactions, biological reactions and climatic change. Hsieh (1991) considers several examples about the chaotic process. They are the univariate chaotic system (tent maps and logistic maps), bivariate chaotic system (Henon maps), trivariate chaotic system (Lorenz maps), Mackey-Glass equation and general chaotic maps. The logistic maps generate "low dimensional" chaos, in that the linear structure can be easily detected. They generate the sequence of x_r according to: $x_t = Ax_{t-1}(1-x_{t-1})$, where A is between 0 and 4 and x_0 is between 0 and 1. For small values of A, the system is stable and well behaved. However as the value of A approaches 4, the system becomes chaotic.

[2] In the chaotic growth model, the economy follows nonlinear dynamics, which are self-generating and never die down.

by which information is incorporated into asset prices, and the dynamics of economy-wide fluctuations are all inherently non-linear. Specifically, non-linearities could arise because of the presence of real estate market imperfections, such as illiquidity and significant transaction costs. Consequently, this may affect the speed with which profitable real estate transactions can be implemented and lead to the clustering of price changes. Another possible reason is that not all the market participants are rational. For example, while some investors react to information instantaneously, others instead delay their response until other investors respond first. Also, some other investors overreact to bad news and under-react to good news. Hence, the market's response to information might not necessarily give an aggregate linear feedback all the time. Third, the characteristics of the real estate and financial market microstructure, such as restrictions on short-sales and other trading rules, information disclosure requirements and rules and/or control on foreign ownership might lead to delays in executing arbitrage transactions. This in turn might induce non-linearities in the real estate returns. Finally, a "chaotic" real estate market may cause sudden bursts of volatility and occasional large price movements. For example, the July 1997 Asian economic crisis was associated. with large movements in real estate market returns and volatility in some Asian economies.

The finding of a non-linear return structure would open the possibility for non-linear pricing models that could better explain certain aspects of the real estate return-generating process.[3] Additionally, the accuracy of forecasting property returns may be reduced unless non-linear structures are incorporated into property forecasting models. Another implication is that it would raise questions about market efficiency and about the linear form of asset pricing models used in tests of market integration. The question whether real estate markets are segmented or integrated is always debatable among practicing professionals and academics alike. The usual notion is that national real estate markets are still segmented because of various market imperfections, with each market exhibiting unique characteristics

[3] The random walk properties of stock returns is an outcome of the Efficient Markey Hypothesis (EMH) which states that stock prices exhibit unpredictable behavior given available information. The EMH has guided theoretical and empirical research on financial markets.

that distinguish one market from other markets. The disparity in characteristics is thought by some to explain differences in the level and behavior of returns across markets. In this study, the term "market integration" is defined as the relationships (i.e. co-movements) of the different national real estate returns, as well as the relationships between each of the national real estate returns and the world stock market returns. If the various national real estate markets are integrated, it implies that there is at least a "common factor" in international real estate returns (Liow and Webb, 2006).

A number of studies have found evidence of non-linearities in the short-term movements of stock market and REIT returns (Hsieh, 1991; Chu, 2003 and Ambrose et al, 1992). However, very little is understood about this issue for the major international real estate markets. With recent studies highlighting the portfolio diversification benefits of including publicly listed international real estate in a portfolio (Conover et al, 2002; Worzala and Sirmans, 2003), considerable attention has been given to examining various aspects of securitized real estate performance in Asia and internationally. It is therefore timely and important to examine whether there is chaos or non-linear dependence in international real estate returns and the resulting implications on real estate and stock market integration. This effort is important and offers an alternative perspective to institutional investors who mainly rely on the random walk hypothesis for common stock and real estate asset pricing.

Six major real estate markets are included in this study; those of the USA, UK, Australia, Japan, Hong Kong, and Singapore; and the World real estate market over the period 1990 through June, 2006. Since inadequate time series data for direct real estate returns in all countries renders the examination of the non-linear dynamics impossible, this study uses listed property as a proxy for private property. Both weekly and daily real estate security series are examined. A total-hedged real estate security return series (both weekly and daily) is also examined. On a scientific level, this study thus requires the adoption of conclusions from presenting the tests on four datasets to obtain "balanced" inferences. This approach is specially needed as the subject (non-linearities and chaotic dynamics) is relatively underresearched in the international real estate literature. Following Larrain (1991), this study uses a logistic map that is capable of nesting both K-map and Z-map models to examine whether non-linearities exist in the real estate markets, the presence of which can potentially impact real estate market integration as well as with the world stock market. Additionally, return for each market is specified as a non-linear Threshold Autoregressive GARCH – in mean [TAR – GARCH (1, 1) - M] process to ensure that the non-linearities in returns are not due to volatility changes. Finally, the forecasting performance of the non-linear and linear TAR-GARCH models is compared to ascertain if the former can outperform the latter in long range forecasting.

This is believed to be the first comprehensive study to examine the non-linearities and chaos dynamics of international real estate markets and market integration from the combined K- and Z-map perspective. However, Ambrose et al, (1995), Newell et al, (1996), Newell and Matysiak (1997), and Liow (2006) have investigated the individual non-linear behavior of US REITs, the listed UK property company shares and international real estate securities from the "long-memory" and "chaotic" perspectives without considering the market integration/segmentation and forecasting performance issues.

2 RELATED LITERATURE

In contrast to the widely-accepted linear perspective adopted in the study of financial asset returns, several stock market studies have questioned this assumption in favor of non-linearities in mature markets. In addition, other research into the non-linear behavior of stock markets was motivated by suggestions that low-dimensional non-linear deterministic systems (i.e. "chaotic") may generate price behavior as apparently random series (Hsieh, 1991; Scheinkman & Lebaron, 1989). Savit (1988) suggests that asset returns may not follow a stochastic process and instead might be generated by deterministic chaos such that the forecasting errors grow exponentially and the process appears to be stochastic. Scheinkman and LeBarin (1989) find that stock returns may follow a non-linear dynamic system in the

US. Hsieh (1991) also finds evidence of non-linearities in the daily returns of US stocks. Other stock market studies include Wiley (1992), Kohers et al, (1997), Antoniou et al, (1997), Opong et al, (1999), Chu (2002), and Appiah-Kusi and Menyah (2003) whereby the linear stochastic model or random walk has been shown to be inappropriate in explaining the complex dynamics of the stock markets. For example, using a Hurst statistic and the BDS test, Opong et al. (1999) examine the main indices in the UK market (All shares, FTSE100, FTSE250 and FTSE350) and are able to reject the hypothesis that the series are random, independent and identically distributed. They also find that some cycles or patterns show up more frequently than would be expected in a truly random series.

Chaos is a subset of non-linear analysis. According to Larrain (1991), financial price series that appear to be random may actually have some hidden order or auto-dependency. If a series has a certain degree of non-linear auto-dependency, it will generate complex behavior which can be described by fractal and chaotic tests. These tests reveal to what degree the series is random (unexplainable), or to what degree it is dependent on its past performance. Non-linearity may be explained in terms of a non-linear feedback mechanism in price movements. For example, the univariate quadratic logistic map (equation 1 below) is a function that maps the value at time t-1 into the value at time t. The second term in the equation is a negative non-linear feedback term which competes with the linear term in stabilizing the series. This term (feedback mechanism) is non-linear if the corrective measure taken by the market is not proportional to the original deviation.

$$P_t = aP_{t-1}(1 - P_{t-1}) = aP_{t-1} - aP^2_{t-1} \qquad (1)^4$$

In applying the non-linear techniques to the US Treasury bills, Larrain (1991) points out that the important question to address is how and when past interest rates (technical factors) combine with fundamental factors that will help determine the evolution of future

[4] Equation (1) is an example of a logistic map. Hsieh (1991) provides a non-technical explanation of Equation (1). Select P_0 between 0 and 1, and generate the sequence of P_t according to equation (1), where a is between 0 and 4. For small values of a, the system is stable and well behaved. But as the value of a approaches 4, the system become chaotic. Hence, for some values of the parameter (a), the dynamics may be simple; while for other values, the dynamics may be chaotic.

interest rates. Larrain (1991) develops a model (equation 2) made up of a non-linear component, which is called the K-map, and a behavioral component, which is called the Z-map:

$$R_{t+1} = f(R_{t-i}{}^{n}) + g(Z_{t-i}) \quad (2)$$

Here, the structural Z-map model, the second term on the right-hand side of equation (2), assumes that adaptive adjustments take place slowly with substantial lags. Equation (2) is expanded into equation (3):

$$R_{t+1} = \{K - map : a + b(R^{n})_{t} - c(R^{n+1})_{t}\} + \{Z - map : d(Y)_{t} + e(P)_{t} + f(M)_{t} + g\sum(Y - C)_{t} \quad (3)$$

Here, the exponent n depicts the degree of non-linearity of interest rates, R. The terms Y, P, M and (Y-C) represent the real GNP, consumer price index, nominal money supply and savings, respectively, where C is real personal consumption.

Following Larrain (1991), Sewell et al. (1996) examine the issue of stock market and foreign exchange rate integration between the US and five Asian countries. Their analyses reveal that some of the time series examined display non-linear dependence and there exists varying levels of market integration among the time series. On the contrary, there has been only limited examination of the potential presence of chaotic behavior and non-linear dependence in real estate markets to date. Ambrose et al, (1992) test real estate investment trusts (REITs) returns for evidence of the presence of a fractal structure, a subset of non-linear dynamics. By determining the presence of non-linear dependence versus random behavior, they hope to conclude whether there was difference in the structure between the real estate and stock markets and, as a consequence, whether there was evidence of market segmentation. Using rescaled range analysis and Lo's test, no evidence of chaotic (long memory) behavior was found by them. Liow (2006) broadens the analysis into an international environment. He investigates whether persistence in international real estate market return and volatility takes the form of long memory using both original (unhedged) and hedged real estate security returns series. The range of econometric tests applied in his study is also more extensive than the basic tests by Ambrose et al, (1992) and provide the potential to differentiate between non-linear (long memory) and linear stochastic structure. Newell et al, (1996) examine Australian property trusts returns over the 1980-1994 period. Using an extensive range of tests, which include the BDS test, Lo's test, correlation dimension, Lyapunov exponents and three moments test, Newell et at, (1996) find little evidence of chaos, with stochastic non-linear models being more appropriate to model the Australian property trust returns. Applying the same methods, Newell and Matysiak (1997) find that the underlying structure present in the various UK property series is not chaotic, and that non-linear stochastic models seem the most appropriate models to capture the underlying dynamic process.

3. DATA

Since direct property performance series do not have sufficient data points for the analyses conducted in this study, the real estate data used are weekly and daily FTSE EPRA/NAREIT total return indexes maintained by the European Real Estate Association (EPRA) for the six most significant real estate markets in the respective regions. The markets analyzed are the USA, the UK, Australia, Japan, Hong Kong, Singapore and a world real estate index. This data approach is consistent with that of Ambrose et al, (1982) in utilizing the US REIT returns over 1962-90, since the US direct real estate performance series (e.g. NCREIF) do not have sufficient data points to effectively determine the presence of chaos. The starting date for the EPRA data indexes from Datastream is 29 December 1989. Therefore this study period starts from January 1990 and covers through June 2006, the longest weekly (861 observations) and daily (4304 observations) time series that are available from Datastream. Relying on weekly return series restricts the analysis to shorter time series with less than 1000 points, and nonstationarity can become an issue as the time period increases. On the contrary, the use of a daily return series, although it provides sufficient data points, might exhibit artificial dependence in the non-linearity tests (Gilmore, 1993). Recognizing that there is no ideal solution, this study includes both weekly and daily tests to form an "on the balance of evidence" picture. Figure 1 plots the weekly real estate indices.

(Figure 1)

The country level and world equity indices are compiled by the Morgan Stanley Capital Index (MSCI) and were obtained from the Datastream on-line information system. The MSCI stock market indices are widely used by international fund managers for asset allocation decisions and performance measurement, as well by researchers for academic studies. The returns (R) are obtained by taking the natural logarithmic difference of the index times 100. Taking the first difference may not only ensure that the time series are stationary, but is a common practice in standard econometric work to "whiten" a time series. All the return series are expressed in US dollars and are therefore adjusted for foreign exchange fluctuations and facilitates cross-country comparisons. As such, the perspective of the US investors is taken in this study.

Although the EPRA database is constructed to be representative of the universe of public real estate companies, it is likely that returns on these securities might not accurately reflect returns in private real estate markets. In order to address this issue, this study also examines a total-hedged real estate security return series following the procedures developed by Liang et al. (1996). To the extent that real estate security performance is affected by general market conditions, the former may contain information about the latter. In contrast, the total-hedged real estate security data remove the stock market influence from the raw real estate security index and the resulting index generates a return series that should track real estate factors more closely.

Following Liang et al. (1996), equation (4) below is first run to estimate the total-hedge ratio (h):

$$RESI_t = c + h * MKT_t + \varepsilon_t \quad (4)$$

Where: $RESI_t$ = weekly (daily) real estate security index's total return and MKT_t = weekly (daily) total return on the market index.

Successive values of h are derived via 52-week or 260-day rolling windows. These values are used to hedge the 53[th] week's (261[th] day's) stock market movements. The resulting total-hedged RESI is calculated as (equation 5):

$$RESI_{hedged} = RESI - h(MKT - tbill) + \varepsilon \qquad (5)$$

Where $RESI_{hedged}$ = return for the total-hedged RESI index and tbill = return on the 3-month US T-bill index.

Because of the loss of observations due to the rolling regression procedures, the formal analysis with the hedged series commences from January 1991 through June 2006 with 809 weekly and 4044 daily observations for all data series.

4. METHODOLOGY

The empirical framework follows the modeling approach of Larrain (1991) to investigate whether or not the six major real estate markets are integrated among themselves and with the world capital market in the presence of return non-linearities. Essentially, the modeling involves the use of a logistic map that is capable of nesting both linear and non-linear specifications. As indicated in equation 1, a logistic map is a type of non-linear function that maps the price of an asset at time t-1 to the price at time t. The analysis in this study covers five steps which are briefly described below

(a) The non-parametric BDS is employed (Brock et al, 1986) to distinguish between stochastic systems and non-linear systems that may involve chaos. The BDS test has been used to test for nonlinear behavior in a wide range of financial data including national stock indices and exchange rates. It detects departures from the independent and identical distribution (IID) hypothesis by measuring the statistical significance of the correlation integral showing the correlation dimension of the time series. The non-IID alternatives include non-stationarity and linear and non-linear dependence. A white noise process is completely disorderly and has infinite dimension; a purely random process also has infinite dimension; a chaotic or non-linear system has a finite dimension. Methodologically, the correlation integral calculates the probability that any two points that are part of two trajectories in phase space are ε units apart. As ε increases, the probability scales according to increases with the fractal dimension of the phase space.

All the real estate return series with an autoregressive (AR-p) model are filtered to remove linear dependences, where the lag length p is selected based on the Minimum Akaike Information Criterion (MAICE). The residuals are tested with the BDS procedure. A Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is also employed to remove conditional heteroskedascity. The residuals from the GARCH models are again tested with the BDS procedure. The null hypothesis tested here is that each of the real estate series is IDD. Under the null hypothesis that the index series under study is random, the null hypothesis can be rejected with 95% confidence when the BDS statistic exceeds 2.0. If the BDS statistics reject the null of IDD, we then proceed with step 2 to investigate whether deterministic chaos could exist for the real estate series.

(b) The effects of the non-linear components in the logistic map are referred to as the K-map. In step 2, whether non-linearities exist in the index series is analyzed with equation (6):

$$R_t = a_0 + b_1(R_{t-1}) + b_2(R^2{}_{t-1}) + b_3(R^3{}_{t-1}) + b_4(R^4{}_{t-1}) + \varepsilon_t \qquad (6)$$

Where R_t is the return and ε_t is an error term at time t. Equation (6) is also labeled as a K-map logistic equation. As pointed out by Larrain (1991), equation (4) is an alternative form of logistic map that generates chaotic behavior. The main purpose of using this approach is not to determine the precise nature of any non-linearity, but rather to ascertain whether any non-linearity exists. Furthermore, the chaotic behavior of non-linear difference equations is rather insensitive to the details of the non-linear term, so long as it is non-linear. Hence, by extending equation 6 to include lagged values raised to the fourth power, the non-linear process should be approximated. If none of the non-linear terms is found to be significant, a linear model is implied; otherwise, evidence of non-linearity would indicate changes in the risk-return relationship that might induce non-linear returns in accordance with the implications of the logistic map.

(c) Once the statistically significant lagged values have been determined, in step 3 a full logistic map is formed which is made up of a non-linear component (K-map) and a behavioral component, which is called the Z-map. Specifically the Z-map in this study can be defined in two ways. First, it serves as an indicator of the degree of market integration among the six national real estate markets. As an example, suppose that all of the K-map variables for Japan are significant when equation (6) was estimated, the Z-map model for Japan would be:

$$JPR_t = a_0 + b_1(JPR_{t-1}) + b_2(JPR^2{}_{t-1}) + b_3(JPR^3{}_{t-1}) + b_4(JPR^4{}_{t-1}) + b_5(USR_t) + b_6(UKR_t) + b_7(AUR_t) + b_8(SGR_t) + b_9(HKR_t) \ldots\ldots\ldots\ldots\ldots (7)$$

The coefficients b5 – b9 in equation (7) would capture the Z-map effect and allow an examination of the degree of market integration among the national real estate markets.

Alternatively, the MSCI world equity index is used to proxy for the Z-map effects on the degree of integration of the national real estate markets with the world capital market (Equation 8):

$$JPR_t = a_0 + b_1(JPR_{t-1}) + b_2(JPR^2{}_{t-1}) + b_3(JPR^3{}_{t-1}) + b_4(JPR^4{}_{t-1}) + c(MSCI_t) \qquad (8)$$

The coefficient *c* would capture the Z-map effect. The interaction between the K-map and Z-map is another issue of interest from equation (6). Specifically, if the K-map dominates the Z-map it indicates that the country specific real estate factors dominate world-wide capital market factors, and hence is an indicator of market segmentation. On the contrary, Z-map dominance indicates a greater degree of market integration between the real estate and the world stock market.

(d) The relationship between the conditional real estate market returns and conditional volatility is investigated for those markets where at least one of the K-map coefficients is statistically significant. Equation 8 is expanded and estimated by the Threshold Autoregressive GARCH – in mean specification [TAR – GARCH (1, 1) - M] as follows:

$$R_t = a_0 + a_1 R_{t-1} + a_2 R_{t-2} + a_3 R_{t-3} + a_4 R_{t-4} + b\sigma_t^{\ 2} + c(Dummy) + dMSCI_t + \varepsilon_t \qquad (9)$$

$$\sigma_t^{\ 2} = e_0 + e_1 \varepsilon_{t-1}^{\ 2} + e_2 \sigma_{t-1}^{\ 2} + e_3 \sigma_{t-1}^{\ 2} I_{t-1} + e_4 (Dummy) \qquad (10)$$

In the mean equation 9, the market return R $_t$ is specified as a first-order autoregressive process with the non-linear variables (K-map), world capital market (MSCI) (Z-map), its own conditional variance and a Asian financial crisis dummy (DUMMY), which takes a value of 1 for the period July 1997 – August 1998; and zero otherwise. The main objective in including this crisis dummy is to control for regime swifts in the three Asian real estate markets (i.e. Japan, Hong Kong, and Singapore) following the Asian financial crisis in July 1997. A research study by Kallburg et al (2002) finds that there was a reduction in the real estate market returns and increases in the volatility and correlations following the crisis. The parameter b for the conditional variance tests the linkage between the return and conditional volatility. A significant value for b, which can be either positive or negative, means that the conditional volatility contributes to the risk premium.

The conditional variance in equation (10) is assumed to be predicted by the previous variance $\sigma_{t-1}^{\ 2}$, the square of the previous shock $\varepsilon_{t-1}^{\ 2}$ and the crisis dummy. Additionally, the positive and negative shocks are differentiated by the indicator variable I_{t-1} that takes a value of 1 when the previous shock is negative and zero otherwise. The asymmetric effect is thus captured by the hypothesis that e_3>0. A positive e_3 implies that a negative innovation increases conditional volatility.

(e) The forecasting accuracy of the non-linear TARCH (1, 1) - M and linear TARCH (1, 1) – M models is evaluated in term of static forecast (one-step ahead) and two years–step dynamic forecast. The main objective is to answer the following question: if the non-linear expressions are more descriptive than the linear relationship, are they also better predictors? Four criteria are employed: root mean square errors (RMSE), mean absolute errors (MAE), mean absolute percentage errors (MAPE), and Theil's inequality coefficient.

5. RESULTS

5.1 Descriptive statistics

Table 1 provides details of the mean, standard deviation, skewness, kurtosis and Jarque-Bara statistic of both the original and hedged real estate return series. It can be seen that for all six national markets and World real estate the use of the weekly and daily hedged indices results in a reduction in the standard deviation of the series. In some cases this reduction is quite significant. For example, in the case of the Hong Kong market, the weekly standard deviation of the hedged series is 1.69%, in comparison to an original figure of 4.38%. In addition, with the exception of Japan, each of the seven markets sees a slight reduction in the mean weekly returns. It is also noticeable that in three cases (UK, US, and WRE) the mean daily returns for the hedged series are similar to those of the original series. With minor exceptions, all series are significantly skewed and leptokurtic relative to the normal distribution. Finally, Figure 2 and Figure 3 plot the weekly and daily hedged ratios for the real estate series. The average weekly hedge ratios are 0.498 (US), 0.624 (AUS), 0.668 (UK), 0.837 (WRE), 1.157(HK), 1.233 (JP) and 1.300 (SG). The respective average daily hedge ratios are however smaller: 0.359 (US), 0.519 (UK), 0.589 (AUS), 0.717 (WRE), 1.083 (HK), 1.092 (JP) and 1.160 (SG).

(Table 1, Figure 1 and Figure 2 here)

5.2 Non-linearity and Chaos – BDS results

The BDS statistics are needed to determine whether each of the real estate series is stochastic or is a non-linear system. The values for the BDS statistic are shown in Table 2 (weekly), Table 3 (hedged weekly indices), Table 4 (daily) and Table 5 (hedged daily indices). The analyses are conducted on the return residuals after each return series passed through its corresponding autoregressive and GARCH filters. All residual series are examined for embedding dimensions between 2 and 5 (weekly series) and between 2 and 10 (daily series) respectively.[5] The value of ε is 0.5, 1.0, 1.5, and 2.0 times the standard deviation of the series.

(Tables 2 to 5 here)

With most of the results exceeding 2, the BDS results indicate that the original and hedged real estate series are not independently and identically distributed. The main exception is with the UK hedged weekly series where the null cannot be rejected in nine of sixteen cases. This indicates that the series is probably stochastic[6]. These significant values of the BDS test for the real estate series points to a possible non-linear structure in the series. The preliminary conclusion is that for most of the real estate series, some non-linear structure is probably present; but provides no evidence as to whether this non-linear structure is deterministic (i.e. chaotic) or stochastic.

5.3 K-map results

Based on the BDS statistics reported above, it appears that the UK hedged weekly real estate series is stochastic. Further analysis is therefore not justified.

The K-map results conducted on equation 6 are reported in Table 6. There is some evidence that the non-linear (K-map) terms influence the real estate index returns. Of the 28 real estate series, there are six weekly indices (US, UK, AUS, JP, SG, and World Real Estate), two hedged weekly indices (HK and SG), five daily indices (UK, JP, HK, SG, and World Real Estate) and two hedged daily indices (HK and AUS), respectively, that has at least one K-map term that is statistically significant. The Adj R^2 values for these 15 series

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[5] The number of n-histories (embedding dimensions) constructed from each series is limited to (no of obs)/200 or 10 to allow a comparison across a wide range of dimensions.

[6] Some scattered insignificant BDS values are observed for the UK weekly series, Australia and Hong Kong weekly hedged series and the UK daily series. With some reservation, the study will proceed with the K-map and Z-map analyses and report the results together with other series whose null of IID is consistently rejected by the BDS test.

range between 0.7 and 4.9 percent. Comparatively, only four hedged series have a significant K-map term. One possible interpretation is that since stock prices are consistent with a non-linear data generating process (Scheinkman and LeBaron, 1989), we would probably expect the hedged real estate return series (with the stock market influence removed) to have no or weaker evidence of non-linearities than the original real estate series, and this was the case.

(Table 6)

5.4 Non-linearities and market integration

Table 7 reports the full logistic map results based on equation 7 for the 13 non-linear real estate series (excluding the two world real estate series). The model contains both lagged exponential same country market returns (K-map) and other real estate markets' returns (Z-map). The regression results in Table 7 thus give an indication of the relative strength of same country lagged exponential returns versus other markets' returns.[7] There is some evidence that the K-map terms influence the markets. Of the thirteen real estate return series, there is at least one significant K-map coefficients in five cases; two K-map coefficients in seven cases; in one case (HK hedged daily series) the three non-linear coefficients are highly significant and appear to indicate K-map dominance. Furthermore, there are five, nine, and eight, respectively, significant coefficients for the R^2, R^3 and R^4 terms. For n = 2 (i.e. R^2), the nature of the dependency of returns on past returns is either negative (3 cases) or positive (2 cases). These two coefficients represent, respectively, non-linear growth and declining rates for past returns. For n = 3 (R^3), the nature of the dependency of returns on past returns is mainly negative (6 cases, 66.7%); whereas the nature of the dependency of returns on past returns is mainly positive (5 cases, 62.5%) for n = 4 (R^4).

Table 7 also shows that in the cases of the weekly and daily real estate series, most of the Z-map coefficients are statistically significant and positive, a finding which supports the notion of market integration among the major national real estate markets. The Adj R^2 values

[7] A non-linear specification could also be appropriate for the Z-map components (i.e. other real estate markets' returns). Since the focus is on the non–linear dynamics in self-returns, non-linearity in the Z-map components was not included.

range between 10.1 and 38.5 percent. The implication is that the lagged same country's market returns (K-map) combine with other markets' returns (Z-map) help determine the future evolution of the same country's market return; and that the effects of chaos are weaker. On the contrary, there appears to have some dominance of K-map influence in the four hedged real estate series; the K-map coefficients are large and significant and there are relatively more insignificant z-map coefficients. One possible explanation is that if the hedged real estate security series are considered a better proxy for the direct real estate market, the findings imply the national direct real estate markets are still largely segmented, and that past non-linear returns affect the level of future returns to a greater degree (K-map).[8]

In order to further test whether these results are valid over varying historical time periods, recursive regressions on equation 8 are conducted to test for constancy of the significance of the K-map components. Figure 4 plots the recursive t-values for the 22 significant K-map coefficients over the full study period. Focusing on the 95% significant bands where (t >. 1.96) and (t< - 1.96), the evidence indicates that although several K-map coefficients are statistically insignificant over some estimation periods, all of them are statistically significant (positive or negative) more than 50 percent of the time, with intermittent fluctuation in the level of significance observed for some K-map coefficients.

(Figure 4 here)

The estimates of the mean equation (9)'s parameters are reported Table 8. As observed, the tests for evidence of non-linearities in the return-generating process are confirmed by the statistically significant coefficient estimates for at least one a_n (where n = 2, 3 and 4) estimates for 12 of the 15 markets. The three exceptions are Singapore (weekly returns), Singapore (daily returns), and Australia (hedged daily returns) where none of the non-linear coefficients is statistically significant. Compared with other countries, it appears

[8] However, Liang et al. (1996) also point out that although there is a connection between the hedged-equity REIT return index and the Russell-NCREIF index, the notion of the hedged REIT returns being more representative for real estate returns than the raw REIT data is not universally accepted. Accordingly, caution should be exercised in interpreting the results associated with the hedged real estate index.

that the Hong Kong real estate market has the strongest K-map components. Its weekly hedged, daily, and daily hedged real estate series have two significant non-linearity coefficients each. Similarly the Japanese and world real estate series (weekly) also have two significant K-map terms each. The estimates for the Z-map component (MSCI) also confirm that all the original series (weekly and daily) are strongly correlated with the MSCI (coefficient *d*) thereby indicating close integration of the public real estate markets with the world stock market. Hence, there is some evidence both the K-map and Z-map terms influence the real estate markets. On the contrary, the Z-map term is statistically insignificant in 2 of the 3 hedged real estate series that are influenced by the K-map terms. Hence, if the hedged real estate series is regarded as a better proxy for the unsecuritized real estate market, the results indicate that the real estate markets are more sensitive to the country-specific factors (as indicated by the K-map terms) and less (/not) influenced by the global factors (proxied by the Z-map term), a finding that is in broad agreement with Ling and Naranjo (2002).[9] Consistent with expectations, a significant and negative Asian financial crisis coefficient (c) for the Japan, Singapore, Hong Kong, and the World real estate market is also found. Except for the UK (weekly), Hong Kong (weekly hedged) and world real estate (daily) markets, no significant statistical relationship between the expected return and conditional volatility (b) for the remaining real estate return series is found.

(Table 8 here)

With respect to the estimates for the TGARCH conditional variance equation (10), the GARCH parameter estimates (e2) are all statistically significant and are all much larger than the ARCH coefficient estimates (e1), implying that the prediction of the volatility is dominated by the AR component. The volatility persistence, measured by (e1 + e2) is high, but always less than 1 for all series. Except for five series (weekly: US, AUS, and World; daily: UK; and daily hedged: Hong Kong), the hypothesis of no asymmetric effect (e3 = 0) is statistically rejected for the remaining 9 series. Moreover a positive e3 (13 cases) implies that

[9] Ling and Naranjo (2002) find evidence of a strong world-wide factor in international securitized real estate returns. Moreover an orthogonalized country-specific factor is also highly significant.

a negative shock increases the conditional volatility in these markets. This finding is in agreement with Liow (2006), who finds asymmetry in the 12 country conditional volatilities international real estate securities markets.

Finally, Table 9 summarizes the results of the forecasts for the 12 non-linear real estate series on a post sample from 2004:6 to 2006:6. For three cases (Japan and worth weekly series and Singapore hedged weekly index), the non-linear model outperforms the linear model in every criterion in both the static and dynamic forecasts. On the contrary, the linear model appears to be a better predictor than its non-linear counterpart for the AUS weekly index and HK hedged weekly real estate index. Additionally, there is mixed evidence on the forecasting performance for the remaining seven non-linear series. For static forecast, the average RMSE, MAE, MAPE, and Thail's coefficients are 0.0142, 0.0111, 201.03 and 0.6766 for the non-linear model, as opposed to 0.0148, 0.0115, 207.51, and 06906 for the linear model. These represent a 3.63, 3.75, 3.22, and 2.06 percent reduction in the four forecast errors with the non-linear model. For the dynamic forecast, the average values show that there is also a 2.68 percent (RMSE), 2.52 percent (MAE), and 2.00 percent (Thail's coefficient) reduction in the forecast errors with the non-linear model. However, there is a 6.67 percent increase in the MAPE forecast error with the non-linear model. In all, the comparisons of the two models' forecasting accuracy show that incorporating the K-map components in the TGARCH model appear to improve the forecasting performance over the conventional linear TGARCH model. However, this finding is at best inconclusive due to the mixed results obtained for some individual series.

(Table 9 here)

6. ADDITIONAL INVESTMENT IMPLICATIONS

Our results have additional implications for global investors. The popular Johansen cointegration test is built on the basis of linear autoregressive model and implicitly assumes that underlying dynamics are in linear form or can be made linear by a simple transformation.[10] In addressing the issue of long-run relationships among real estate markets, the presence of non-linear "chaotic" processes in some real estate markets' returns means that the standard Johansen- linear cointegration test could be misleading since the true nature of the adjustment process is non-linear and the speed of adjustment varies with the magnitude of the disequilibrium (Bierens, 1997). The results of Li (2006) indicate that it is possible that much more evidence of market integration could emerge from nonlinear, rather than linear cointegration analysis, suggesting that comovements among national stock markets may well take nonlinear forms. Hence, when nonlinear cointegration among national real estate markets is also allowed for, real estate assets available to global investors would become even less than when only linear cointegration is allowed. Thus, global investors should be cautious in formulating their diversification strategies if real estate markets are, in fact, chaotic, gains from diversification may not occur in the long-run.[11]

7 CONCLUSIONS

In this study a logistic model of chaos is used to analyze the temporal variation of real estate market returns on six real estate series and a hedged index. The tests were undertaken in an examination of market integration that included how common factors (Z-maps) and country specific factors (K-maps) influence international real estate returns. There is evidence in some of the real estate series of non-linear dependencies. These non-linearities could arise from a representation of the time series as a non-linear function of past observations. Furthermore, evidence of varying levels of market integration is detected for these series. The implication is that the past same country market's returns (both linear and non-linear) combined with other markets' returns or with the MSCI world stock market returns help determine the future evolution of the same country real estate market's return.

[10] To this end, the Johansen cointegration analysis tests empirically the EMH in an international context.
[11] It is beyond the scope of this study to examine non-linear cointegration among the national real estate markets. Future research can employ the non-parametric cointegration test proposed by Bierens (1997) and Breitung (2002).

The analyses also suggest a need to account for non-linear chaotic behavior in forecasting models in order to obtain potentially more accurate real estate return forecasts.

Overall, important contributions of this study are the findings of statistically significant evidence of nonlinearity and low deterministic chaotic behavior in some real estate market returns. These results are important because knowing that real estate market returns exhibit chaotic behavior can help understand the evolution of real estate market returns better and find ways of predicting them. In this respect the results also have practical implications, because they suggest that international integration models for real estate market returns should include some nonlinear terms.

Of course, whether or not the direct real estate market is chaotic cannot be fully resolved without rigorous empirical evidence. One obvious reservation is that inferences based on indirect property (or even the hedged indices) as a proxy for direct property are definitely subject to qualification, and criticism. Specifically, whether the non-linear chaotic structures in real estate security performance for some countries are inherited by the direct property market cannot be answered at this stage. Second, we have not addressed the methodological issue regarding the role played by outliers (extreme observations) in testing for chaos; i.e. are they the results of large exogenous shocks or are they inherently related to the dynamic behavior of the model. Another question is whether the findings are useful from a practicing professional's standpoint. For example, if real estate markets are chaotic, can this be used to implement a more effective trading strategy or risk measurement/management approach?

Nevertheless, this study indicates the possible existence of chaos in major securitized real estate markets and suggests that a technical, non-linear lagged return structure capable of irregular and violent behavior is capable of explaining and forecasting the specific return-generating processes. Furthermore, the possible presence of chaotic processes in some real estate markets indicates that different portfolio management practices may be appropriate for global investors.

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Table 1 Descriptive statistics of real estate market returns

	AUS	HK	JP	SG	UK	US	WRE	MSCIW
Panel A: Unhedged weekly real estate security returns: Jan90 - Jun05								
Mean	0.0026	0.0022	0.0000	0.0008	0.0017	0.0030	0.0017	0.0010
Std. Dev.	0.0205	0.0438	0.0491	0.0510	0.0243	0.0183	0.0193	0.0191
Skewness	-0.2734	-0.4932	0.4325	-0.8689	0.0090	-0.4067	-0.4514	-0.2369
Kurtosis	3.4463	7.4182	4.5373	22.1310	4.5658	6.1242	4.9458	4.9996
Jarque-Bera	17.87	735.19	111.62	13238.39	87.97	373.90	165.07	151.49
Panel B: Hedged weekly real estate security returns: Jan91 - Jun05								
Mean	0.0021	0.0009	0.0015	0.0007	0.0016	0.0028	0.0016	0.0013
Std. Dev.	0.0137	0.0169	0.0296	0.0282	0.0200	0.0160	0.0143	0.0186
Skewness	-0.2597	0.3245	0.2125	-0.1261	0.1060	-0.1543	-0.2725	-0.2839
Kurtosis	3.6066	7.2418	4.1394	6.3680	4.7040	5.6525	5.1156	5.2234
Jarque-Bera	21.49	620.70	49.85	384.52	99.39	240.37	160.89	177.51
Panel C: Unhedged daily real estate security returns: Jan90 - Jun05								
Mean	0.0005	0.0004	0.0000	0.0002	0.0003	0.0006	0.0003	0.0002
Std. Dev.	0.0096	0.0184	0.0217	0.0204	0.0101	0.0073	0.0076	0.0080
Skewness	-0.0459	0.1709	0.5618	0.6383	0.1140	-0.4128	-0.2905	-0.1128
Kurtosis	4.5482	12.0760	7.5228	14.9425	7.7276	8.4640	7.1374	6.3049
Jarque-Bera	431.38	14793.14	3894.79	25869.24	4017.40	5476.23	3130.43	1967.89
Panel D: Hedged daily real estate security returns: Jan91 - Jun05								
Mean	0.0004	0.0002	0.0002	0.0001	0.0003	0.0006	0.0003	0.0003
Std. Dev.	0.0069	0.0071	0.0148	0.0137	0.0089	0.0063	0.0062	0.0079
Skewness	-0.1103	0.5035	0.2633	0.1256	0.3385	-0.3522	-0.3929	-0.1360
Kurtosis	4.9793	8.2795	7.6286	6.7347	7.8634	8.6251	5.8174	6.4851
Jarque-Bera	668.30	4867.54	3656.70	2360.87	4062.70	5415.21	1441.58	2059.09

Table 2

BDS statistics on the null of independent and identical distribution (i i d) using holding period returns filtered with AR (p) and GARCH (1, 1) models for the weekly real estate return series: January 1990 – June 2006

	m	AR-Filtered							GARCH - Filtered						
		US	UK	AUS	JP	HK	SG	WD	US	UK	AUS	JP	HK	SG	WD
0.5	2	3.747	2.253	3.181	4.022	4.755	5.898	3.749	4.341	1.701*	3.411	4.329	4.454	6.565	3.936
0.5	3	4.365	3.031	4.619	4.122	5.579	7.958	4.413	4.915	2.583	4.915	4.442	6.378	8.492	4.842
0.5	4	5.762	3.576	5.051	5.193	7.574	8.862	4.398	6.367	3.516	5.448	5.792	8.327	9.382	4.758
0.5	5	6.876	3.963	5.547	6.063	9.311	10.966	4.516	7.819	4.117	5.928	7.039	9.936	11.303	4.302
1	2	4.199	1.862*	3.054	4.571	4.919	7.181	3.757	4.573	1.879*	3.374	4.867	4.786	7.493	3.885
1	3	4.438	2.955	4.719	4.984	5.939	9.008	4.324	4.861	2.904	4.913	5.283	6.293	9.119	4.547
1	4	5.109	3.296	5.168	6.047	6.897	9.963	4.704	5.527	3.354	5.289	6.337	7.245	9.986	4.968
1	5	6.272	3.043	5.935	6.721	7.923	11.389	4.835	6.697	3.087	6.103	6.992	8.129	11.025	5.077
1.5	2	5.046	1.366*	3.346	5.138	5.012	7.987	3.914	5.366	1.486*	3.855	5.509	5.084	8.682	3.941
1.5	3	5.244	2.611	4.797	5.456	5.905	9.537	4.706	5.469	2.519	5.088	5.932	6.535	10.096	4.912
1.5	4	5.452	3.005	5.207	6.293	6.438	10.387	5.111	5.688	2.991	5.366	6.779	7.122	10.672	5.467
1.5	5	6.146	2.687	5.827	6.609	7.117	11.161	5.238	6.412	2.653	6.001	7.055	7.672	11.106	5.598
2	2	5.824	1.203*	3.399	5.649	5.286	8.631	4.028	5.955	1.079*	4.109	5.919	5.715	9.099	4.021
2	3	6.045	2.535	4.593	5.959	6.078	9.905	5.211	6.073	2.251	5.109	6.351	6.921	10.623	5.316
2	4	5.893	3.119	4.932	6.661	6.209	10.641	5.558	5.898	2.964	5.205	7.057	7.068	11.236	5.812
2	5	6.069	2.746	5.281	6.798	6.656	11.081	5.703	6.111	2.611	5.588	7.162	7.373	11.451	6.001

Notes:

1. The lag length *p* is selected based on the Minimum Akaike Information Criterion (MAICE). The results are 1(AUS), 1 (JP), 1(UK), 2 (HK), 2 (US), 2(WD) and 5 (SG)
2. Embedding dimension (m) limited to n/200 or 5, where n =861
3. Pairs can be no further apart than this distance, which is 0.5, 1.0, 1.5 or 2 times the standard deviation of the series
4. *, - indicates that the BDS statistic is not significant at the 5 percent level, hence the null hypothesis of i.i.d cannot be rejected

Table 3

BDS statistics on the null of independent and identical distribution (i i d) using holding period returns filtered with AR (p) and GARCH (1, 1) models for the weekly hedged real estate return series: January 1991 – June 2006

ε/σ		AR-Filtered							GARCH - Filtered						
	m	US	UK	AUS	JP	HK	SG	WD	US	UK	AUS	JP	HK	SG	WD
0.5	2	4.823	0.408*	1.703*	7.305	1.701*	3.983	1.765*	4.823	0.408*	2.444	7.237	2.075	5.377	1.263*
0.5	3	4.869	0.861*	2.746	9.910	2.924	6.024	2.152	4.870	0.860*	2.973	9.869	3.848	7.245	2.065
0.5	4	6.296	1.949*	3.251	11.562	3.836	7.207	2.029	6.296	1.950*	3.970	11.604	5.259	8.394	2.502
0.5	5	7.411	1.736*	3.835	13.333	4.870	7.913	2.225	7.411	1.736*	4.592	13.510	6.618	8.552	3.456
1	2	4.628	0.730*	1.961*	6.992	1.614*	4.423	2.549	4.628	0.730*	2.752	6.829	1.973*	5.905	2.171
1	3	5.201	1.352*	2.775	9.389	2.684	6.269	2.821	5.201	1.352*	3.219	9.493	3.272	7.673	2.802
1	4	6.175	2.095	3.492	11.174	3.566	7.160	2.945	6.175	2.095	3.946	11.354	4.335	8.621	3.134
1	5	7.709	2.109	4.147	12.944	4.352	7.895	3.005	7.079	2.109	4.676	12.963	5.135	9.305	3.353
1.5	2	5.099	1.227*	2.111	6.358	2.576	4.167	2.617	5.099	1.227*	2.848	6.316	2.847	5.342	2.328
1.5	3	5.488	1.728*	2.715	8.054	3.154	5.618	2.673	5.490	1.728*	3.137	8.193	3.676	6.773	2.685
1.5	4	5.903	2.406	3.339	9.369	3.891	6.409	2.908	5.903	2.405	3.670	9.573	4.519	7.505	3.061
1.5	5	6.197	2.434	3.759	10.430	4.395	7.322	2.937	6.197	2.434	4.117	10.594	5.041	8.229	3.175
2	2	5.019	1.724*	1.674*	5.949	3.719	3.432	2.522	5.020	1.724*	2.443	5.548	3.861	4.650	2.242
2	3	5.414	2.324	2.169	7.224	3.784	4.272	2.465	5.414	2.325	2.652	6.972	4.248	5.456	2.328
2	4	5.632	2.887	2.707	8.183	4.735	4.766	2.609	5.632	2.887	3.020	8.059	5.277	5.832	2.525
2	5	5.609	2.859	2.759	8.735	5.339	5.481	2.571	5.609	2.859	3.095	8.639	5.858	6.278	2.549

Notes:

1. The lag length *p* is selected based on the Minimum Akaike Information Criterion (MAICE). The results are 0 (US), 0 (UK), 1 (AUS), 1(JP), 1 (SG), 2 (WD) and 2(HK)
2. Embedding dimension (m) limited to n/200 or 5, where n =811
3. Pairs can be no further apart than this distance, which is 0.5, 1.0, 1.5 or 2 times the standard deviation of the series
4. *, - indicates that the BDS statistic is not significant at the 5 percent level, hence the null hypothesis of iid cannot be rejected

Table 4

BDS statistics on the null of independent and identical distribution (i i d) using holding period returns filtered with AR (p) and GARCH (1, 1) models for the daily real estate return series: January 1990 – June 2006

ε/σ	m	AR-Filtered							GARCH - Filtered						
		US	UK	AUS	JP	HK	SG	WD	US	UK	AUS	JP	HK	SG	WD
0.5	2	4.804	2.253	3.645	4.188	4.681	6.078	11.235	17.794	7.247	6.151	10.579	11.379	16.212	12.899
0.5	3	5.097	3.031	4.817	4.392	5.646	8.336	13.197	22.498	9.711	6.536	14.281	13.603	21.136	14.822
0.5	4	6.246	3.576	5.316	5.708	7.527	9.562	15.016	27.468	11.540	7.558	16.994	16.176	25.469	16.595
0.5	5	7.586	3.963	5.794	7.197	9.035	11.772	16.309	33.530	13.760	8.963	20.347	18.957	30.310	17.992
0.5	6	8.676	4.110	5.705	8.196	12.147	13.990	17.694	42.463	16.131	10.669	24.383	22.094	36.305	19.376
0.5	7	10.521	3.544	4.893	10.219	16.023	16.842	19.628	53.475	18.984	12.466	28.935	25.407	44.206	21.794
0.5	8	10.766	1.129*	2.843	12.819	21.577	20.323	21.187	69.322	22.989	13.602	34.963	29.636	55.064	23.802
0.5	9	10.116	0.441*	5.774	15.777	23.741	25.898	22.196	91.727	28.022	14.335	40.738	35.420	68.564	26.271
0.5	10	9.299	-0.491*	5.167	23.580	30.687	32.867	22.361	124.283	35.447	16.781	48.558	42.396	88.442	27.767
1	2	4.901	1.862*	3.389	4.660	5.037	7.009	12.865	18.912	8.269	6.459	11.163	13.660	17.516	13.868
1	3	4.938	2.955	4.661	5.111	6.015	9.163	15.359	23.174	10.503	7.105	14.702	16.106	21.954	16.366
1	4	5.529	3.296	4.891	6.201	6.911	10.282	17.445	26.778	11.758	8.004	17.006	18.858	24.859	18.206
1	5	6.595	3.043	5.721	6.887	7.891	11.702	18.975	30.112	13.340	8.953	19.077	21.487	27.840	19.625
1	6	7.481	3.203	6.302	7.640	9.235	12.949	20.469	34.103	14.836	9.837	21.065	24.167	31.342	21.025
1	7	8.568	3.588	6.906	8.859	10.716	14.372	21.971	38.513	16.627	10.855	23.233	26.831	36.375	22.626
1	8	9.396	4.068	7.295	9.793	12.372	15.773	23.533	43.667	18.460	11.843	25.968	29.706	40.246	24.199
1	9	10.153	4.738	8.136	11.438	14.620	17.632	24.875	49.643	20.693	12.665	29.280	33.055	45.797	25.625
1	10	11.308	4.921	8.423	13.406	17.226	19.540	26.223	56.874	23.216	13.624	33.276	36.969	52.563	27.058
1.5	2	5.543	1.366*	4.090	5.373	5.372	7.923	14.261	18.117	9.328	6.559	11.593	14.770	17.989	14.599
1.5	3	5.460	2.611	5.110	5.730	6.203	10.011	17.187	21.412	11.157	7.227	15.161	18.706	21.816	17.534
1.5	4	5.623	3.005	5.206	6.551	6.696	10.912	18.343	24.014	12.090	8.040	17.293	18.783	23.831	19.491
1.5	5	6.261	2.687	5.762	6.814	7.308	11.632	20.579	25.742	13.135	8.822	18.817	20.689	25.527	20.699
1.5	6	6.741	2.975	6.134	7.088	8.080	12.176	21.792	27.489	14.028	9.500	20.055	22.430	27.235	21.818
1.5	7	7.236	3.094	6.394	7.555	8.850	12.770	22.854	29.357	15.082	10.127	21.274	23.961	28.939	22.874
1.5	8	7.690	3.159	6.418	7.789	9.484	13.316	23.863	31.304	16.163	10.850	22.639	25.522	30.769	23.868
1.5	9	8.005	3.580	6.811	8.487	10.354	14.064	24.691	33.326	17.331	11.474	24.099	27.127	32.627	24.676
1.5	10	8.501	3.728	7.018	9.293	11.326	14.797	25.453	35.515	18.449	12.163	25.711	28.785	34.591	25.446
2	2	5.933	1.203*	4.680	5.807	5.807	8.602	14.891	17.188	9.672	6.692	11.947	15.813	18.256	14.524
2	3	5.918	2.535	5.411	6.204	6.482	10.673	17.979	19.751	11.296	7.246	15.488	17.265	21.799	17.493
2	4	5.725	3.119	5.262	6.909	6.582	11.423	20.101	21.901	12.083	8.075	17.705	18.766	23.399	19.447
2	5	5.862	2.746	5.478	7.010	6.960	11.859	21.068	22.958	12.673	8.748	18.678	19.918	24.473	20.488
2	6	6.082	3.054	5.679	7.058	7.363	12.143	22.072	23.832	13.246	9.231	19.602	20.857	25.387	21.439
2	7	6.217	3.029	5.809	7.237	7.676	12.329	22.821	24.608	13.821	9.623	20.373	21.568	26.139	22.161
2	8	6.405	2.814	5.572	7.210	7.876	12.444	23.439	25.378	14.447	10.160	21.114	22.271	26.883	22.798
2	9	6.385	3.187	5.934	7.476	8.184	12.729	23.881	26.119	15.040	10.576	21.767	22.909	27.592	23.218
2	10	6.587	3.203	6.099	7.902	8.512	12.977	24.285	26.874	15.562	11.017	22.383	23.498	28.206	23.618

Notes:

1. The lag length *p* is selected based on the Minimum Akaike Information Criterion (MAICE). The results are 1 (UK), 1 (WD), 2 (JP), 2 (HK), 3 (SG), 4 (US) and 5 (AUS)
2. Embedding dimension (m) limited to n/200 or 10, where n = 4304
3. Pairs can be no further apart than this distance, which is 0.5, 1.0, 1.5 or 2 times the standard deviation of the series
4. *, - indicates that the BDS statistic is not significant at the 5 percent level, hence the null hypothesis of iid cannot be rejected

Table 5

BDS statistics on the null of independent and identical distribution (i i d) using holding period returns filtered with AR (p) and GARCH (1, 1) models for the daily hedged real estate series: January 1991 – June 2006

ε/σ	m	AR-Filtered							GARCH - Filtered						
		US	UK	AUS	JP	HK	SG	WD	US	UK	AUS	JP	HK	SG	WD
0.5	2	14.615	7.785	7.865	10.840	13.386	12.268	7.459	15.608	7.734	7.995	10.839	12.643	11.859	7.627
0.5	3	18.605	10.250	8.824	15.677	15.236	14.679	9.419	19.467	10.128	8.454	15.808	14.386	14.528	9.629
0.5	4	22.727	11.641	8.268	19.744	16.651	17.406	11.666	23.901	11.534	8.599	19.988	15.749	17.424	11.786
0.5	5	27.594	12.625	9.870	24.746	18.814	19.923	14.476	29.174	12.606	10.022	24.964	17.824	19.933	14.346
0.5	6	33.963	12.591	11.319	30.506	21.511	23.417	17.104	35.894	12.649	11.527	30.582	20.268	23.366	16.765
0.5	7	41.523	12.050	13.322	37.817	25.127	27.732	19.252	43.609	12.136	13.419	37.727	23.225	27.797	18.894
0.5	8	51.665	12.453	15.061	48.803	29.113	32.819	22.218	54.025	12.531	14.871	48.377	26.584	32.641	21.486
0.5	9	66.519	12.913	17.288	64.238	34.678	39.213	24.438	69.078	13.480	16.306	63.673	31.616	39.155	23.621
0.5	10	68.236	11.828	18.850	86.286	40.901	46.541	27.321	91.862	13.214	17.217	85.764	37.352	45.858	25.574
1	2	15.494	8.480	7.729	11.272	13.740	13.494	8.079	17.136	8.379	7.777	11.115	13.582	13.209	8.232
1	3	19.270	10.627	8.202	15.047	15.795	16.078	10.116	20.803	10.480	8.345	15.162	15.525	16.017	10.195
1	4	22.285	11.806	8.246	17.553	17.260	18.748	11.966	23.860	11.654	8.433	17.746	16.950	18.819	12.033
1	5	25.154	12.635	9.197	20.484	18.948	21.281	13.897	26.850	12.486	9.349	20.691	18.592	21.358	13.951
1	6	28.371	13.154	10.086	23.315	20.622	24.183	15.829	30.283	13.002	10.252	23.493	20.257	24.253	15.848
1	7	32.061	13.591	11.094	26.640	22.859	27.436	17.565	34.167	13.446	11.219	26.800	22.386	27.581	17.596
1	8	36.300	14.343	12.099	30.692	25.246	31.049	19.252	38.673	14.244	12.178	30.900	24.709	31.306	19.325
1	9	41.315	15.285	13.313	35.789	27.868	35.328	21.134	44.053	15.211	13.328	36.102	27.252	35.714	21.206
1	10	47.687	16.212	14.417	42.240	31.072	40.297	23.434	50.915	16.172	14.362	42.666	30.331	40.846	23.478
1.5	2	15.890	8.888	7.621	11.598	14.451	14.139	8.669	17.231	8.788	7.796	11.385	14.732	13.961	8.775
1.5	3	19.063	10.936	8.282	14.707	16.713	16.319	10.573	20.296	10.819	8.525	14.811	16.921	16.390	10.643
1.5	4	21.217	12.074	8.383	16.348	17.837	18.531	12.001	22.364	11.958	8.619	16.632	18.001	18.677	12.045
1.5	5	22.920	12.741	9.143	18.161	19.140	20.464	13.209	23.993	12.627	9.328	18.439	19.209	20.573	13.259
1.5	6	24.513	13.232	9.812	19.571	20.300	22.399	14.462	25.579	13.106	10.030	19.809	20.331	22.478	14.517
1.5	7	26.210	13.609	10.526	21.083	21.655	24.294	15.496	27.240	13.477	10.724	21.281	21.619	24.382	15.561
1.5	8	27.904	14.164	11.171	22.801	23.068	26.311	16.423	28.934	14.039	11.361	22.971	22.969	26.421	16.492
1.5	9	29.547	14.793	11.913	24.643	24.381	28.499	17.284	30.601	14.680	12.072	24.810	24.227	26.620	17.343
1.5	10	31.380	15.423	12.589	26.669	25.920	30.942	18.304	32.445	15.319	12.762	26.826	25.683	31.073	18.344
2	2	16.176	9.152	7.940	11.404	14.496	15.417	9.443	16.823	9.043	8.236	11.209	15.033	15.423	9.479
2	3	18.785	11.066	8.695	14.394	16.648	16.926	11.245	19.124	10.979	9.013	14.450	17.122	17.224	11.249
2	4	20.337	12.196	8.823	15.731	17.469	18.582	12.246	20.699	12.161	9.062	15.968	17.885	18.925	12.233
2	5	21.265	12.622	9.442	17.053	18.409	19.748	12.989	21.524	12.608	9.616	17.319	18.722	20.038	12.981
2	6	21.912	12.996	9.961	18.087	19.260	20.920	13.802	22.178	12.974	10.131	18.317	19.495	21.160	13.795
2	7	22.611	13.270	10.444	19.008	20.078	21.912	14.382	22.825	13.241	10.591	19.188	20.232	22.146	14.365
2	8	23.235	13.676	10.880	20.049	20.921	22.997	14.871	23.458	13.638	11.014	20.172	21.004	23.231	14.853
2	9	23.783	14.034	11.311	20.980	21.603	24.149	15.283	24.000	13.987	11.412	21.099	21.670	24.367	15.256
2	10	24.332	14.398	11.677	21.859	22.362	25.336	15.791	24.524	14.343	11.764	21.975	22.381	25.535	15.754

Notes:

1. The lag length *p* is selected based on the Minimum Akaike Information Criterion (MAICE). The results are 1 (AUS), 1 (HK), 1 (US), 2 (JP), 2 (SG), 2 (WD) and 3 (UK)
2. Embedding dimension (m) limited to n/200 or 10, where n = 4044
3. Pairs can be no further apart than this distance, which is 0.5, 1.0, 1.5 or 2 times the standard deviation of the series

Table 6 Summary of OLS regression results: the K-map

Country	a0	a1	a2	a3	a4	Adj R2	F-stat	DW
Panel A: Weekly unhedged real estate security indices (861 observations)								
US	0.003***	0.117**	-5.487***	-5.226	1016.005***	0.022	5.93***	1.984
UK	0.001	0.161***	-0.814	-30.894***	320.243**	0.021	5.58***	2.007
AUS	0.002***	0.145***	-2.814	-128.416***	903.677	0.025	6.55***	1.998
JP	-0.001	-0.021	-0.441	-13.182**	99.937***	0.032	8.13***	1.986
HK	0.001	0.046	0.753	2.113	-8.032	0.004	2.03*	2.013
SG	0.0001	0.081	0.324	-2.429**	-3.438	0.011	3.59***	1.997
WD	0.0028***	0.093*	-5.114**	-7.582	1083.207***	0.017	4.79***	2.008
Panel B: Weekly hedged real estate security indices (811 observations)								
US	0.003***	0.034	-1.646	6.689	127.887	-0.002	0.67	1.969
AUS	0.002**	-0.049	5.128	-81.406	-2774.402	0.008	2.66**	2.004
JP	0.002	-0.058	-0.783	-14.627	254.363	0.007	2.36*	1.997
HK	0.0003	-0.053	4.130**	47.939**	-992.808***	0.021	5.41***	2.001
SG	0.003**	-0.141***	-4.054***	1.128	251.797***	0.044	10.37***	2.003
WD	0.0016**	0.043	1.258	-24.545	-772.502	0.005	2.02*	1.989
Panel C: Daily unhedged real estate security indices (4304 observations)								
US	0.003**	0.186***	2.238	-19.266	727.866	0.033	37.73***	2.008
UK	0.0003*	0.154***	-0.608	-101.253***	731.286	0.019	21.26***	2.018
AUS	0.0007***	0.026	-1.066	-21.998	-1188.789	0.002	2.76**	1.997
JP	-0.0006	0.122***	1.522**	-4.061	-74.188*	0.016	28.23***	1.983
HK	0.0006*	0.215***	-1.246	-33.162***	167.787***	0.035	40.43***	2.006
SG	0.0006	0.089***	-1.055	7.738***	-4.895	0.021	24.63***	1.994
WD	0.0001	0.265***	2.594*	-193.537***	-883.352	0.049	56.49***	2.001
Panel D: Daily hedged real estate security indices (4044 observations)								
US	0.00024**	0.344***	3.901	-29.965	-1035.576	0.032	27.33***	1.993
UK	0.0002	0.237***	-0.141	-112.728	912.315	0.0046	4.79***	1.997
AUS	0.0006***	-0.066***	-3.692*	-69.202	2938.264***	0.007	8.49***	2.002
JP	-0.00003	-0.00089	1.497	12.176	-166.406	0.0016	2.68**	1.995
HK	0.0001	0.079***	1.771	176.345**	-5178.718***	0.019	20.54***	1.999
SG	0.0002	-0.060**	-0.556	28.307	-142.202	0.0022	3.23**	1.997
WD	0.0004***	0.016	-4.631	-93.584	6172.495	0.0047	5.81***	1.999

Notes:

OLS regressions are performed on equation 6 for the six national real estate return series (US, UK, Australia, Japan, Hong Kong and Singapore) and the World real estate return series (WD) in unhedged weekly, hedged weekly, unhedged daily and hedged daily specifications:

$$R_t = a_0 + b_1(R_{t-1}) + b_2(R^2{}_{t-1}) + b_3(R^3{}_{t-1}) + b_4(R^4{}_{t-1}) + \varepsilon_t \;\ldots\ldots\ldots \quad (6)$$

***, **, * - indicates two-tailed significance at the 1, 5 and 10 percent levels respectively

Table 7 Summary of OLS regression results for real estate return series: the K-map and Z-map

Country	c0	c1	c2	c3	c4	c5(US)	c6(UK)	c7(AUS)	c8(JP)	c9(HK)	c10(SG)	Adj R2	DW
Panel A: weekly (unhedged) real estate secuirty indices (861 observations)													
US	0.0035***	0.0840**	**-5.9283*** **	na	**931.459*** **	na	0.1245***	0.1109***	-0.0041	0.0189	0.0597***	0.136	1.991
UK	-0.000009	0.1064	na	**-23.381** **	**149.791** **	0.2063***	na	0.2317***	0.0732***	0.0442**	0.0318	0.179	2.023
AUS	0.0016***	0.0806*	na	**-103.709*** **	na	0.1428***	0.1641***	na	0.0309**	0.0856***	0.0019	0.171	2.061
JP	-0.0024	-0.0402	na	**-11.078* **	**71.423*** **	-0.0391	0.3241***	0.1881*	na	0.0652	0.1525***	0.127	1.975
SG	-0.0017	0.1166**	na	**-1.0928*** **	na	0.2839***	0.1094*	-0.0389	0.1086***	0.6182***	na	0.385	2.004
Panel B: weekly (hedged) real estate security indices (809 observations)													
HK	-0.0001	-0.0691	**4.6447** **	**60.395*** **	**-1135.37*** **	0.0109	0.0243	0.0865***	0.0291	na	0.0798**	0.044	2.016
SG	0.0026**	-0.1403***	**-3.9293*** **	na	**247.92*** **	0.0324	0.0204	-0.0389	0.0704*	0.2132**	na	0.063	2.021
Panel C: Daily (unhedged) real estate secuirty indices (4304 observations)													
UK	0.0001	0.1088***	na	**-67.954*** **	na	0.1452***	na	0.1329***	0.0454***	0.0320***	0.0267***	0.083	2.014
JP	-0.0007**	0.1032***	**1.3362** **	na	**-92.005*** **	0.0117	0.2080***	0.2315***	na	0.1392***	0.1309***	0.101	1.996
HK	0.00006	0.1148***	na	**-19.653*** **	**52.652** **	0.1326***	0.0794***	0.2183***	0.0801***	na	0.3867***	0.294	2.057
SG	-0.0002	0.0498**	na	**2.6418** **	na	0.1022**	0.0951***	0.1289***	0.0920***	0.4887***	na	0.273	2.037
Panel D: Daily (hedged) real estate security indices (4044 observations)													
HK	0.0002*	0.0818***	na	**159.396** **	**-4378.10*** **	-0.0065	0.0053	0.0131	0.0132*	na	0.0428***	0.028	1.998
AUS	0.0005***	-0.0833***	**-3.3451* **	na	2689.62	0.0390**	0.0330**	na	0.0002	0.0099	0.0009	0.011	2.006

Notes: The equation is:

$$JPR_t = a_0 + b_1(JPR_{t-1}) + b_2(JPR^2{}_{t-1}) + b_3(JPR^3{}_{t-1}) + b_4(JPR^4{}_{t-1}) + b_5(USR_t)$$
$$+ b_6(UKR_t) + b_7(AUR_t) + b_8(SGR_t) + b_9(HKR_t).$$

***, **, * - indicates two-tailed significance at the 1, 5 and 10 percent levels respectively

Table 8 Non-linear TAR - GARCH (1, 1)-M estimates for real estate return series: 1990 - 2006

	Weekly returns (unhedged)						Weekly (hedged)		Daily returns (unhedged)					Daily(hedged)	
	US	UK	AUS	SG#	JP	WD	HK	SG	UK	JP	HK	WD	SG#	HK	AUS#
							Mean equation								
a0	0.0029**	0.0618***	0.0034	0.0019	-0.0030	0.0016	-0.0403***	0.0044*	0.00038	-0.0010**	0.00067**	-0.000099	0.000476	0.00011	0.0011***
a1	0.1267***	0.1456***	0.3190***	0.0256	0.0714	0.1523***	-0.1785***	-0.1256***	0.0979***	0.0710***	0.1359***	0.3120***	0.0746***	0.0596***	-0.0714***
a2	-4.6856**	na	na	na	na	-5.2539***	-5.5652**	-4.0625***	na	1.2919**	na	-0.9691	na	na	-2.6324
a3	na	-25.257*	-110.843**	-1.1262	-7.7887*	na	15.4467	na	-64.8420**	na	-28.9525***	-139.445***	1.1416	129.3291*	na
a4	na	35.117	na	na	44.714**	766.202**	-772.278***	259.005	na	-74.026	116.166***	na	na	-3408.061**	2937.538
b	722.035	0.0079***	-5.2309	-0.0762	0.5564	6.5018	194.575***	-0.0529	1.8476	0.1886	-0.9845	14.0419***	-1.3123	0.6984	-11.7561
c	na	na	na	-0.0260***	-0.0138**	-0.0065***	-0.0504***	-0.0073*	na	-0.0030**	-0.0030**	-0.0014***	-0.0046***	-0.00063*	na
d	0.4024***	0.5390***	0.3439***	0.8565***	1.2170***	0.6972***	0.0509***	-0.0316	0.3180***	0.9632***	0.5490***	0.5102***	0.4954***	0.0068	0.0400***
							Variance equation								
e0	0.00002**	0.000015**	0.00002**	0.00002**	0.00002	0.00001*	-4.183***	0.000002***	0.000002**	0.000006**	0.00006***	0.000002***	0.000005**	0.000001***	0.00005***
e1	0.1187*	0.0248*	0.0855**	0.0075	0.0268*	0.0648**	0.1428***	-0.0152	0.0497***	0.0533***	0.0593***	0.0719***	0.0716***	0.0864***	0.0431***
e2	0.0206	0.0646***	-0.0393	0.0329*	0.0527**	-0.0139	0.0441***	0.0114***	0.0079	0.0597***	0.0526***	0.0451**	0.0498***	0.0177	-0.0226**
e3	0.8090*	0.9612***	0.8701***	0.9632***	0.9405***	0.9096***	0.5161***	0.9981***	0.9291***	0.9066***	0.8960***	0.8346***	0.8927***	0.8776***	0.9574***
e4	na	na	na	0.00015**	0.000096	0.000003	0.3754***	0.000046***	na	0.000011*	0.000012**	0.000001*	0.000023**	0.000002*	na
Log-L	2390.40	2090.26	2206.15	1628.44	1514.69	2493.51	2221.82	1848.89	14227.3	11185.60	12077.70	16246.98	11784.69	14722.26	14568.69
Adj R2	0.223	0.184	0.141	0.201	0.196	0.479	0.046	0.038	0.108	0.139	0.121	0.394	0.078	0.016	0.008

Notes:

The non-linear TAR-GARCH (1, 1) - M model comprises:

$$R_t = a_0 + a_1 R_{t-1} + a_2 R_{t-2} + a_3 R_{t-3} + a_4 R_{t-4} + b\sigma_t^2 + c(Dummy) + dMSCI_t + \varepsilon_t ; \sigma_t^2 = e_0 + e_1 \varepsilon_{t-1}^2 + e_2 \sigma_{t-1}^2 + e_3 \sigma_{t-1}^2 I_{t-1} + e_4 (Dummy) ;$$

$a_{2-}a_4$ are the coefficients for the K-map terms; d is the coefficient for the Z-map (MSCI) term

None of the K-map coefficients is statistically significant

***, **, * - indicates two-tailed significance at the 1, 5 and 10 percent levels respectively

Table 9
Results of post-sample forecasts (non-linear and linear models)

		Static		Dynamic	
		Non-linear	Linear	Non-linear	Linear
US (weekly)	RMSE	0.01611	0.01627	0.01908	0.01925
	MAE	0.01182	0.01193	0.01565	0.01565
	MAPE	281.092	256.937	283.522	323.642
	Theil's inequality coefficients	0.5939	0.6113	0.6687	0.6748
UK (weekly)	RMSE	0.02175	0.02179	0.02102	0.02119
	MAE	0.01785	0.0178	0.0171	0.01722
	MAPE	140.602	136.645	126.767	128.77
	Theil's inequality coefficients	0.6642	0.6886	0.6541	0.6591
AUS (weekly)	RMSE	0.01824	0.01804	0.01797	0.01775
	MAE	0.01483	0.01481	0.01473	0.01458
	MAPE	190.815	217.093	197.138	179.019
	Theil's inequality coefficients	0.7046	0.6988	0.6903	0.6965
JP (weekly)	RMSE	0.03159	0.03188	0.03144	0.03167
	MAE	0.02376	0.02406	0.02395	0.02421
	MAPE	168.486	169.542	173.18	175.272
	Theil's inequality coefficients	0.5873	0.593	0.5909	0.5945
WD (weekly)	RMSE	0.01044	0.01051	0.01039	0.01054
	MAE	0.00855	0.00861	0.00846	0.00861
	MAPE	101.863	107.539	101.054	108.183
	Theil's inequality coefficients	0.3841	0.3881	0.3806	0.3891
HK weekly (Total-hedged)					
	RMSE	0.01182	0.01091	0.01088	0.01091
	MAE	0.00945	0.00884	0.00876	0.00884
	MAPE	174.628	133.494	160.793	133.493
Theil's inequality coefficients		0.823	0.9042	0.84	0.9042
SG weekly (Total-hedged)					
	RMSE	0.01667	0.02322	0.01662	0.02053
	MAE	0.01375	0.01887	0.01367	0.01659
	MAPE	131.93	246.573	113.896	180.089
Theil's inequality coefficients		0.7586	0.77	0.797	0.814
UK (daily)	RMSE	0.01017	0.01036	0.0101	0.01026
	MAE	0.00707	0.00715	0.00704	0.00711
	MAPE	129.573	124.735	133.516	131.029
	Theil's inequality coefficients	0.7785	0.8025	0.7718	0.8082
JP (daily)	RMSE	0.01447	0.01448	0.01443	0.01441
	MAE	0.01105	0.01104	0.01102	0.011
	MAPE	165.636	165.231	164.71	167.837
	Theil's inequality coefficients	0.6941	0.6952	0.6934	0.6931
HK (daily)	RMSE	0.00991	0.00991	0.0099	0.00991
	MAE	0.00746	0.00746	0.00742	0.00743
	MAPE	261.239	254.426	237.663	241.983
	Theil's inequality coefficients	0.7074	0.7138	0.7186	0.72
WD (daily)	RMSE	0.00476	0.00476	0.00477	0.00476
	MAE	0.00366	0.00366	0.0037	0.00366
	MAPE	278.547	271.425	225.837	230.696
	Theil's inequality coefficients	0.4973	0.4991	0.5043	0.5032
HK daily (Total-hedged)					
	RMSE	0.00484	0.00484	0.00485	0.00487
	MAE	0.00371	0.00372	0.00371	0.00372
	MAPE	387.955	406.42	184.664	125.347
Theil's inequality coefficients		0.9262	0.922	0.9661	0.9847

Note: RMSE denotes root mean square errors, MAE denotes mean absolute errors and MAPE denotes mean absolute percentage errors

Figure 1 Weekly real estate indices and MSCI (USD$)

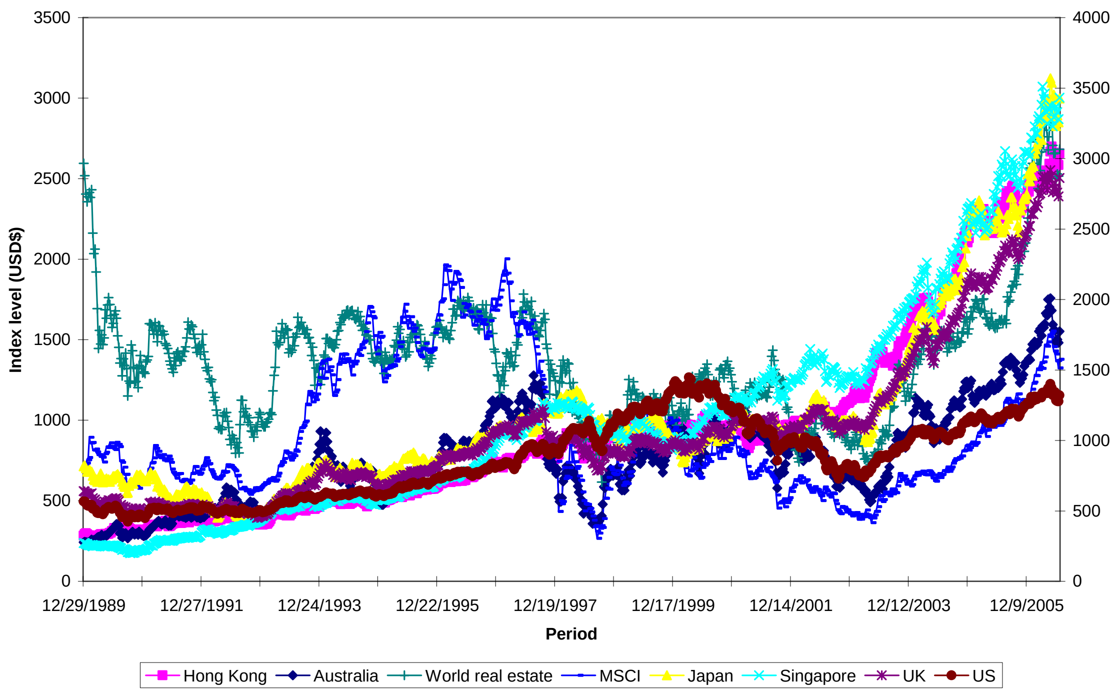


Figure 2: Weeky hedge ratio (1991-2006)

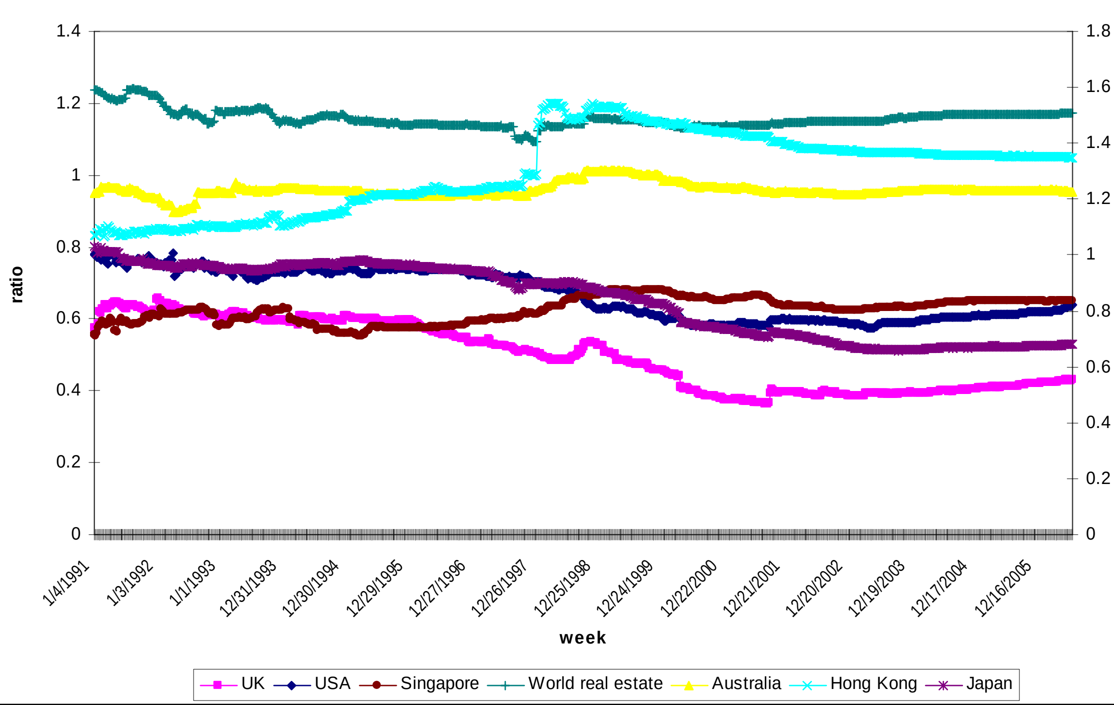


Figure 3: Daily hedge ratio (1991-2006)

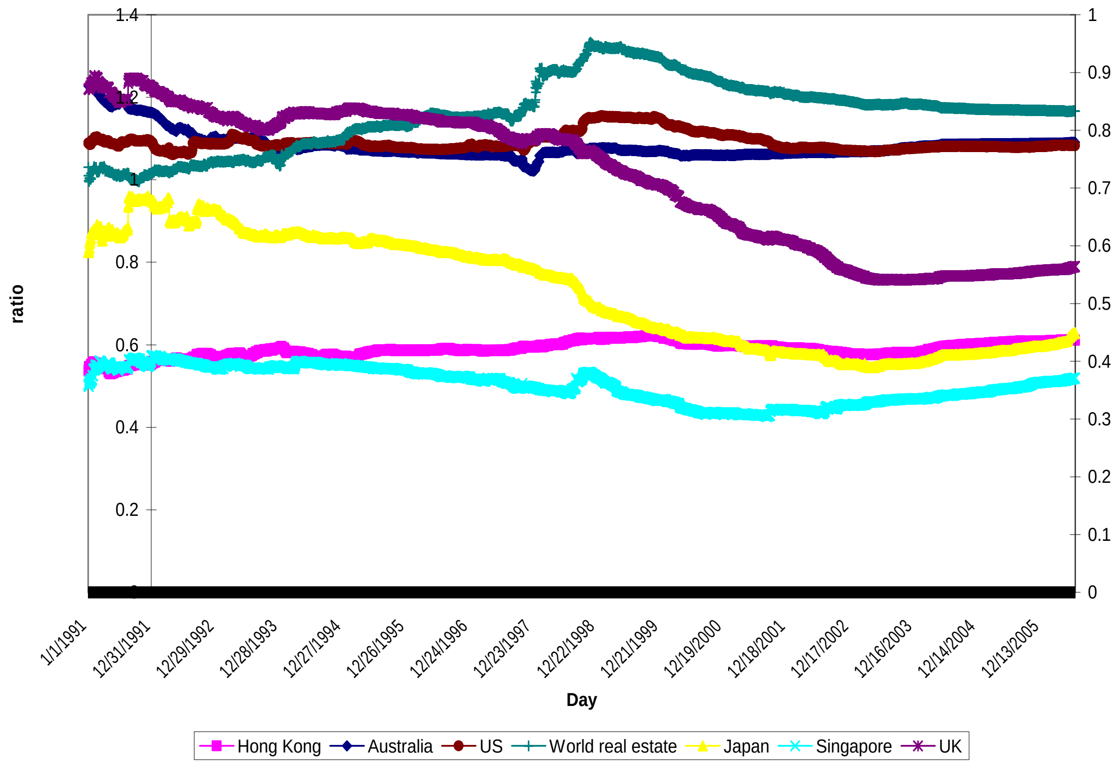
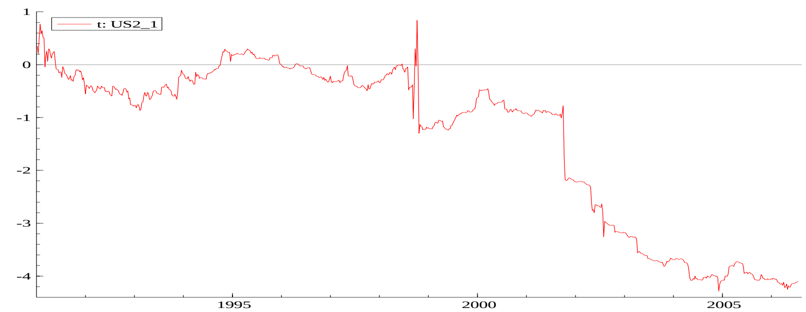


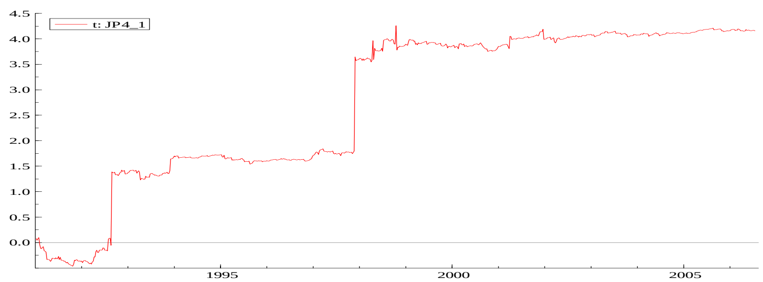
Figure 4 Recursive t-statistics for the K-map variables in the logistic map

AUS – (R^3_{t-1}) – weekly unhedged

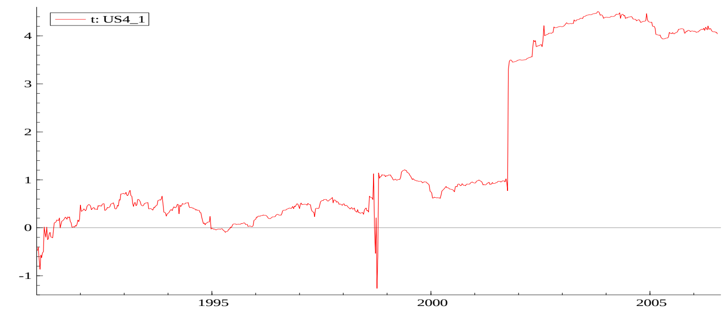
US – (R^2_{t-1}) – weekly unhedged



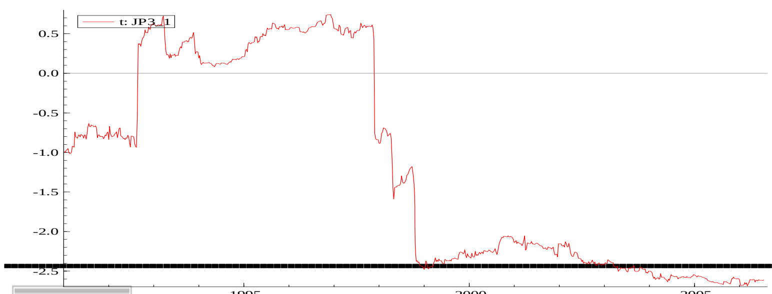
JP – (R^4_{t-1}) – weekly unhedged



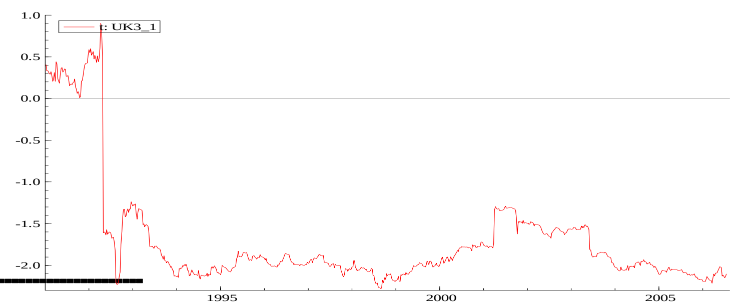
US – (R^4_{t-1}) – weekly unhedged



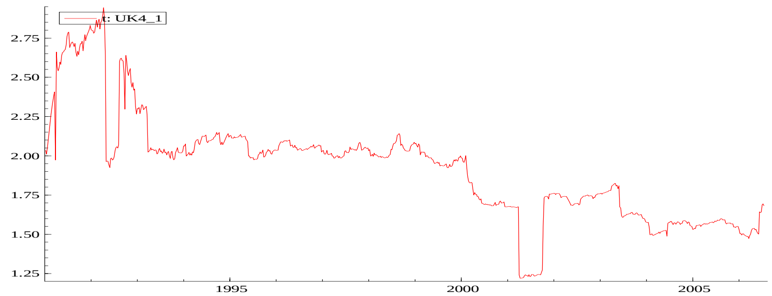
JP - (R^3_{t-1}) – weekly unhedged



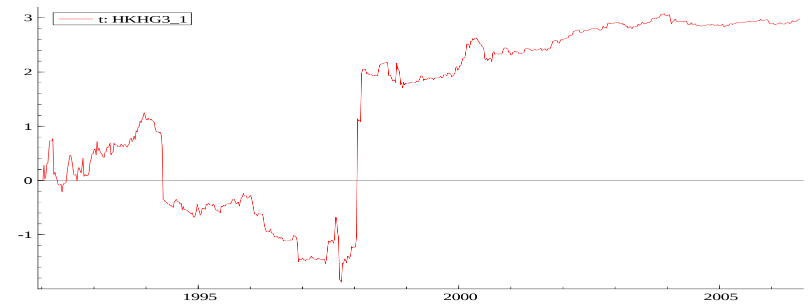
UK – (R^3_{t-1}) – weekly unhedged



UK – (R^4_{t-1}) – weekly unhedged

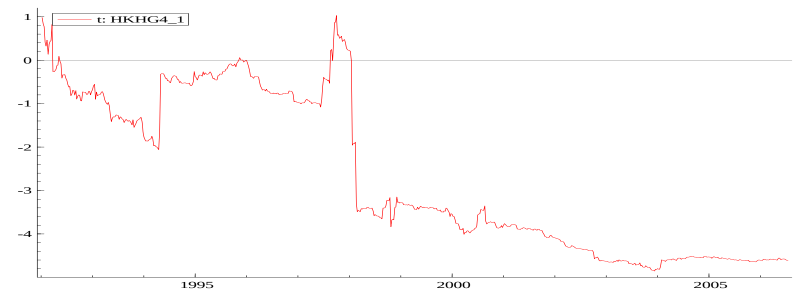


HK – (R^3_{t-1}) – weekly hedged



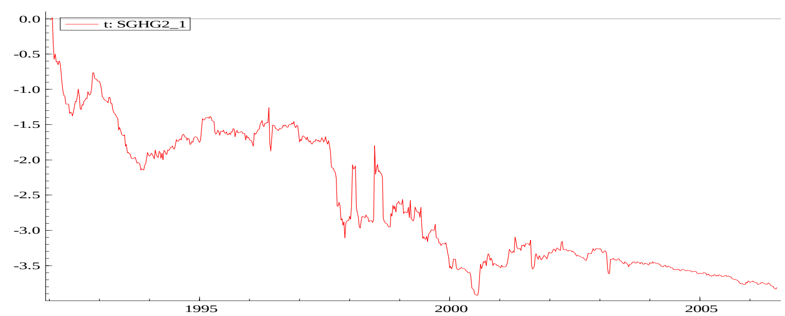
SG - (R^3_{t-1}) – weekly unhedged

HK – (R^4_{t-1}) – weekly hedged

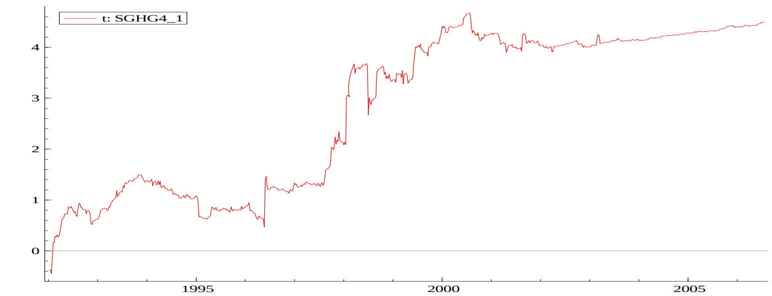


HK – (R^2_{t-1}) – weekly hedged

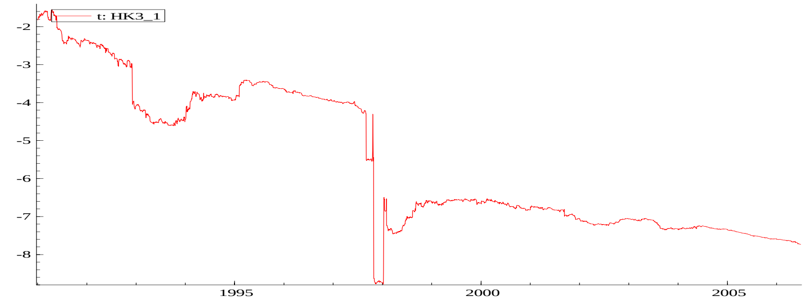
SG – (R^2_{t-1}) – weekly hedged



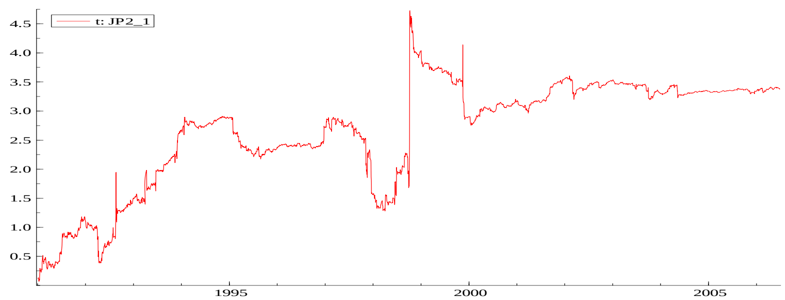
SG – (R^4_{t-1}) – weekly hedged



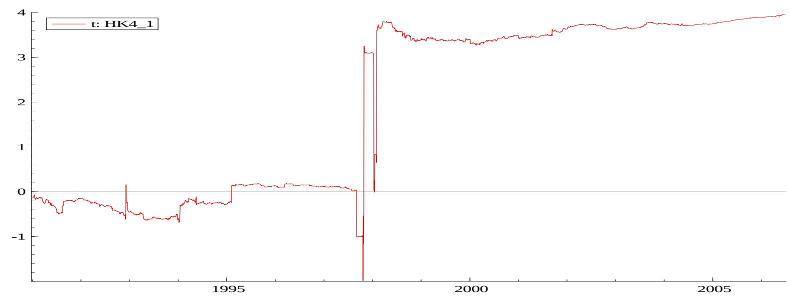
HK – (R^3_{t-1}) – daily unhedged



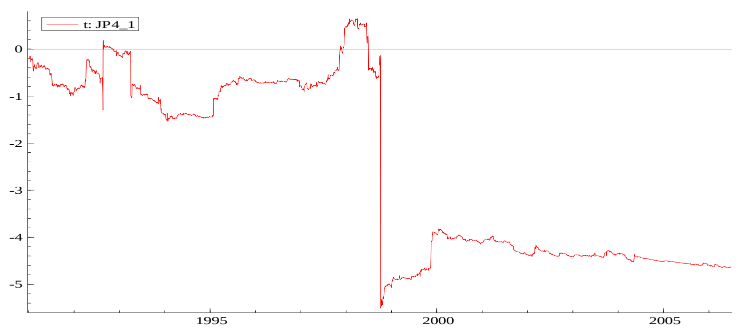
JP – (R^2_{t-1}) – daily unhedged



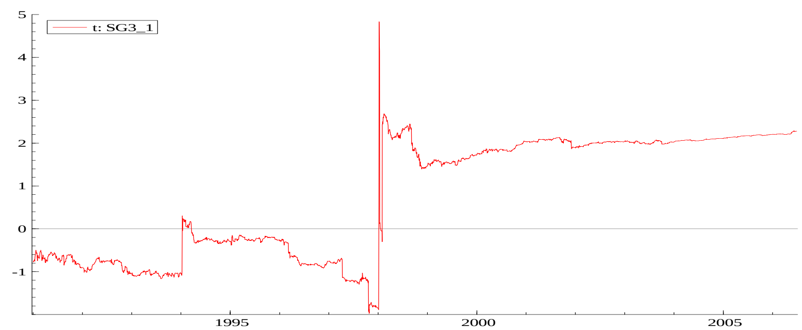
HK – (R^4_{t-1}) – daily unhedged



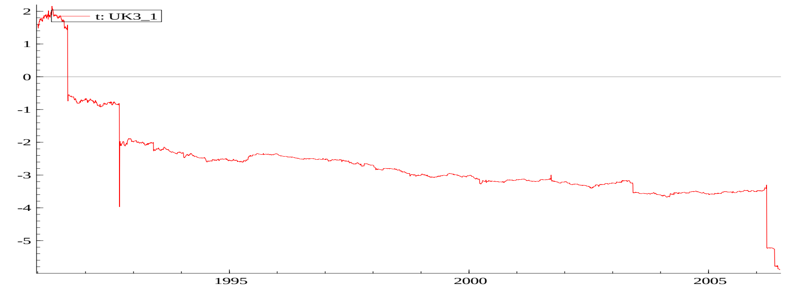
JP – (R^4_{t-1}) – daily unhedged



SG – (R^3_{t-1}) – daily unhedged

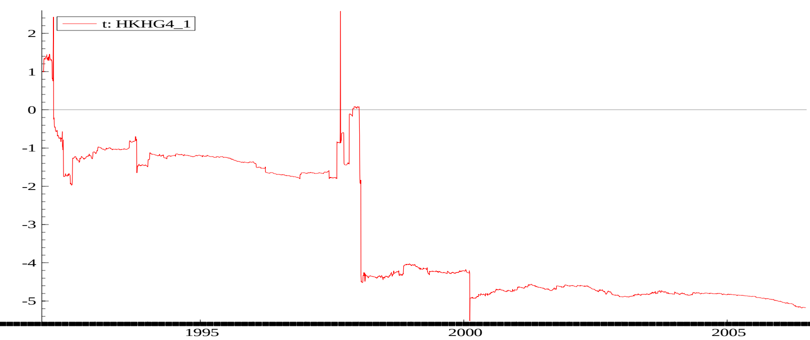


UK – (R^3_{t-1}) – daily unhedged



HK – (R^3_{t-1}) – daily hedged

HK – (R^4_{t-1}) – daily hedged



AUS – (R^2_{t-1}) – daily hedged