The Term Structure of Real Estate Lease Contracts

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Preliminary draft

In financial markets, forward contracts reflect market perception of future price dynamics. Nontransparent markets, like commercial real estate investments, lack such tools. We use a panel of NYC office leases between 2005 and 2016 to estimate a dynamic term structure of forward lease rates (rental revenues), which reflects changing expectations by tenants and landlords about future rental contract conditions. Our imputed term structure is time-varying, generally upward-sloping, and often exhibits an inverted-U shape. We also find that shocks to forward lease rate dynamics are initially most keenly felt in the long-dated lease market and are subsequently transmitted to the short-term lease market. Moreover, consistent with an inefficient informational market, the leasing market takes multiple quarters to fully price the impact of an unanticipated event. Beyond shedding new light on rental market dynamics, our model can be used to quantify risk and reward for real estate strategies. To illustrate this, we examine the financial viability of a nascent "long"-"short" space market strategy commonly used by coworking providers in the last business cycle.
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According to Savills' "Around the World in Dollars and Cents (2016)", the aggregate value of commercial real estate (CRE) that could potentially attract institutional investors is commensurate in size with the bond market. Leases in CRE determine CRE cash flow and are therefore fundamental to the operation and valuation of CRE assets. Importantly for our purposes, the collection of newly executed lease agreement at any given time represents, among other things, the market's assessment of the current and anticipated price of space (per unit time). This is the main object of study in this paper.

We seek to characterize the dynamics of the term structure of "the price of space" over the business cycle, much in the way that one might refer to the term structure of any other commodity, currency, or interest rate. We view a lease contract, after accounting for idiosyncratic characteristics, as essentially a bundle of forward contracts on space (i.e., property square footage). Correspondingly, leases of various maturities originated at roughly the same time can be used to back out a term structure of lease forward rates (e.g., today's contract price that would be paid in five years in exchange for one month's use of one SqFt). We contribute to the literature on property markets by studying leases from an

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approach that, to the best of our knowledge, has yet to be explored.

Using data from CompStak for New York City (NYC) over the period 2005-2016, we employ a Kalman filter to back out spot, 5-year, and 10-year key lease rates. The spot rate should be interpreted as the month-to-month price per square foot for an average tenant. Correspondingly, the 5-year lease rate as determined at date t should be interpreted as the price at which an average tenant could lock-in one square foot of space for one month starting at date $t + 5$. CRE commonly categorizes spatial quality bundles by "Class". We focus in our study on gross leases for Class A (highest quality) office space, and provide a separate analysis for Class B office. We also control for lease stipulations such as tenant improvement allotments, free rent concessions, and rent escalations. Because of insufficient data, we do not control for tenant credit or lease renewals, but we do address this shortcoming in a discussion section.

Our methodology leads to several novel and interesting insights. First, we document that over the observation period the term structure is typically upward sloping. For Class A and B contracts, the slope peaked in 2008 during the Great Financial Crisis (GFC) and bottomed in 2010. For Class A contracts it did not reach the same peak over the sample period, but Class B properties surpassed their 2008 peak in 2015. Second, the curvature of the term structure is typically negative, but flattened out in 2009-2010. Both positive slope and negative curvature of the term structure are consistent across quality classes, but less pronounced for Class B properties. For Class A properties, the general shape of the term structure can be explained by anticipated acceleration of rental growth rates, which eventually are expected to level off and even deteriorate because of depreciation and obsolescence.[1] For class B properties, the interpretation is similar, though relatively muted (i.e., attenuated growth and depreciation expectations).

Another insight from our analysis is that average lease dynamics in NYC across the key rates are well described by a single type of fundamental shock. This contrasts with existing term-structure literature that tends to find different independent shocks corresponding to short-term, persistent, and growth-rate influences (Chiang, Hughen and Sagi, 2015). Moreover, in contrast with common single-factor commodity models (e.g., CIR or Ornstein-Uhlenbeck processes) where the greatest variation is found in the spot rate, our estimates suggest that long-dated space forwards are more volatile than short-dated space forwards. This is consistent with a market in which landlords and tenants are not able to easily adjust their immediate supply or needs for space, so that changes in the rental market are first felt in longer-dated transactions. Indeed, the 10-year forward lease rate leads the others in response to a hypothetical shock in our estimated dynamic model. The spot rate, by contrast is quite sluggish to react (but eventually does). Importantly, across all forward lease rates, the price impact of a shock takes time to be fully incorporated, potentially reflecting informational inefficiencies in the market for leasing space (Hendershott, Lizieri and MacGregor, 2010; Hendershott,

[1] A forward contract locks-in future lease rates in a property that is *now* categorized as Class A.

Lizieri and Matysiak, 1999).

Valuation in CRE typically relies on proforma analyses that ignore variability, subject to assumptions, and to sensitivity tests based on rules of thumb. We illustrate the utility of our approach by applying the estimated model of NYC's lease term structure to a revenue strategy employed by hypothetical co-working company.[2] In particular, we evaluate the risk-reward attributes of a leasing strategy that is "long" a long-dated lease on space and "short" a sequence of short-term leases. Based on our results, long-dated space forward prices are typically higher than spot rents. For our sample, this strategy is generally unprofitable, even assuming full occupancy of the short-term rental. To make the strategy consistently profitable on a risk-return basis for Class A properties in NYC, one or a combination of the following must be true: The long-term lease base rent must be below market (e.g., bottom quartile or tercile), the short-term leased space must be more intensively used (e.g., 20% increased user density), or income must be derived from other sources (e.g., tenant services). In Class B properties, the flatter term structure of lease forwards leaves more scope for potential profitability. This advantage, however, has largely disappeared in the last few years of our sample. Overall, our analysis makes clear that the co-working "long"-"short" space strategy, while potentially lucrative under the right conditions, is by itself not sufficiently financially viable across all market dynamics to justify a co-working business; I.e., additional revenue and risk mitigation is required from other sources. From an underwriting perspective, our work suggests that it may be sensible for co-working spaces to be treated more like hotels than office, but this is currently not the prevailing practice (Chegut and Langen, 2019).

I. Literature

A. *Theoretical contributions*

Earlier lease valuation models can be found in Miller and Upton (1976), McConnell and Schallheim (1983), and Schallheim and McConnell (1985). Miller and Upton (1976) provides a model where the firm decides optimally between leasing or buying and asset, whereas McConnell and Schallheim (1983) and Schallheim and McConnell (1985) analyze the valuation of leases with different embedded options. It is worth nothing that these articles study leases in the broad sense, i.e. not restricted to the real estate setting.

Recent literature on lease valuation is based upon the model presented in Grenadier (1995), in which lease rates result from simultaneous equilibria in the leasing market and the underlying asset market, where competing developers behave optimally. In the model, leases are contingent claims on building values and are determined in the real estate market by conditions such as the num-

[2]Coworking companies are known to employ multiple revenue strategies to support their business model. The "long"-"short" space strategy is just one of them.

ber of competing developers, expectations about future demand for space, and current construction activity. This model is extended to the case of non-perfect competition in the developer market in Grenadier (2005).

Other theoretical studies include Clapham and Gunnelin (2003), who derive a model for the effect of risk aversion and interest rate dynamics;Ambrose, Hendershott and Kłosek (2002), who derive a model for pricing upward-only adjusting leases; and Ambrose and Yildirim (2008), who study the potential impact of the lessee's credit risk on the term structure of lease rates.

B. Empirical contributions

The empirical literature on the term structure of lease rates consists mainly of studies in which the lease term is a regressor on a model where the current lease rate is the dependent variable. Examples of such studies include Brennan, Canaday and Colwell (1984) for the case of office rents in Chicago; Benjamin, Boyle and Sirmans (1990) in which a negative and significant slope is found for US shopping center rents; Wheaton and Torto (1994), who find a positive significant relationship between rents and lease term for more than 50 U.S. cities; and Webb and Fisher (1996), who find a negative but not significant relationship between the lease term and the rental rates for office buildings in the Chicago CBD.

Only a few empirical articles have been concerned explicitly with the term structure of lease rates. Notably, Gunnelin and Söderberg (2003) study office leases in Stockholm, Sweden during the period 1977-1991. In this paper, the term of the lease (in months) is used as an explanatory variable in a regression model for the rental rates, and the slope is given by its estimated coefficient. Term structure dynamics are captured by interactions with year dummy variables, whereas the squared lease term provides an estimate for the curvature of the *yield curve* for leases. Renegotiation and other options are considered via dummy variables as well. They find a statistically significant and positively sloped term structure in 6 years (out of 15), and a negative and significant slope in one of the cases. In a similar study, Englund et al. (2004) estimate a regression mondel in which the term structure in 3 Swedish cities (2,400 properties) during 1998-2002 is determined by interactions among year and lease maturity dummies. Most of their obtained coefficients are not significant, but a test for zero joint effect rejects the null hypothesis, which leads them to claim the existence of a positively sloped term structure.

There exist a number of studies that apply some version of the methodology in Gunnelin and Söderberg (2003) to investigate the term structure in other markets. Bond, Loizou and McAllister (2008) use CBRE data corresponding to 935 office leases in London during the years 1994-2004, and conclude, somewhat imprecisely, that the term structure is positively sloped for every year in their sample. Fang and Ruichang (2009) use this framework and find a negative and significant slope in the Shanghai market during the period 2005-2008. More recently, Hüttel et al. (2016) apply the same framework to agricultural land rental contracts in Germany between 1990 and 2010, although they only observe leases that had not expired before 2006. They find an upward sloping term structure, with a negative and significant quadratic term coefficient for some years.

Stanton and Wallace (2009) develop a contingent-claims model for the valuation of leases with embedded options. The model uses no arbitrage conditions to price any asset whose payoffs depend on the value of the service flow, i.e. the use of space, which in equilibrium is equivalent to the lease rate. The spot lease rate is assumed to follow a geometric Brownian motion (GBM), which restricts the shape of the term structure to be constant over time. The model is calibrated with data from 711 lease contracts originated between 1987 and 1996 that correspond to 47 properties, 11 US states, and a single commercial mortgage underwriter. The data includes information about rent levels, expense pass-through agreements, embedded options, and local market characteristics. After estimation, the authors conclude that the model fits the data poorly.

Agarwal et al. (2011) present a model in which the term structure of leases is determined endogenously by the tenant's capital structure and space market conditions. The value of the service flow follows, as in Stanton and Wallace (2009), a GBM. However, they contrast the cases of a risk-free lessee (in which the rent can be obtained from equating present value of lease payments to service flow), and of a risky lessee, which has a certain probability of default determined endogenously by the firm's capital structure. Comparative statics of the model show how the equilibrium rent is affected by changes in different parameters. In this setup, the shape of the term structure is given by the effect of the lease maturity, which is proved to be positive. Empirical results obtained using data of 2,482 leases in 10 states of the US between January 2001 and March 2002 seem to confirm their results in terms of coefficient signs, but no goodness of fit measure is provided. Based upon the testable hypotheses this model provides, the study offers a better explanation for cross-sectional heterogeneity of lease rates than for the level or shape of the term structure.

More recently, Yoshida, Seko and Sumita (2016) develop a discrete time model for the term structure of leases with a cancellation option. They prove that the lessee's cancellation option induces a positive slope on the term structure in a market with no frictions. However, leasing costs faced by the lessor may generate a U-shaped term structure. To empirically test these claims, they use data from 700 Japanese residential leases signed between 2000 and 2002. The test procedure consists of two steps: First, lease rates are regressed on housing characteristics, year and region fixed effects; afterwards, the error terms are regressed on lease term dummy variables, which yields estimates of average lease rates for different maturities. Model fit is low and the differences among the estimated coefficients for the term structure are not significant. However, the results seem to be consistent with the model: in low-vacancy areas, in which leasing costs are lower for the lessor, the point estimates show a positively sloped term structure, whereas in high-vacancy areas a U-shape is observed.

Our main contribution is in estimating a dynamic model that is flexible enough to capture different components of the term structure, but is also able to capture the joint dynamics of these components. By moving beyond the assumption of simple Brownian motion we are able to model richer dynamics. Instead of relying on a panel regression framework, as in Gunnelin and Söderberg (2003), we employ a Kalman filter approach — much as is done in the term structure literature for interest rates and commodities. The combination of a flexible dynamic model together with an estimation technique more attuned to capturing signals in the data generating process enables us to capture term structure attributes with more confidence.

II. Data

We use proprietary data from CompStak on individual lease contracts corresponding to commercial office properties in New York City.[3] Data for each reported lease contract comes directly from real estate brokers or other entities involved with the transaction. In exchange for the lease contract information, a reporting entity receives other brokers' lease information or "comps". CompStak staff validate each newly entered lease transaction for consistency and plausibility. This data collection process may alleviate, at least partially, concerns related to sample selection bias, data misreporting and measurement error.[4]

Lease transaction data includes details on the characteristics of the lease contract, the property, and tenants. Reported lease contract terms include the transaction date, the commencement date, the term of the lease, the type (e.g., net or gross), the brokers that were involved in the transaction, whether the lease has renewal options, the size of the space, the rent schedule, and any concessions to the tenant. The rent schedule field reports the monthly rent per square foot over the life of the contract (including rent escalations), while the concessions field reports the number of months of free rent and/or tenant improvements (TI) offered to the tenant. The observable characteristics of the property include the space type (e.g., office, retail, etc.), the address, a quality designation ("Class"), the size and age of the building, as well as the number of stories. The data set also includes the name of the landlord and the tenant, as well as the industry of the latter. In our analysis, we use a subset of these features for commercial office spaces in Manhattan.

Table 1 reports the number, by year, of leases that have a full rent schedule. We denote such leases as having full-information (FI). After removing from the leases in the last three columns of the table those that correspond to quarters in which less than 30 contracts were originated, we end up with 3,458 transactions executed between the second quarter of 2005 and the second quarter of 2016.[5]

[3] CompStak is a commercial real estate data company that provides information on comparable lease transactions, or comps, for over 50 United States cities.

[4] CompStak outlines their verification approach on their website.

[5] Quarters with fewer than 30 executed leases are 1998Q1-2005Q1, 2012Q3-Q4 and 2015Q1-Q4.

After further excluding leases with outlier effective rents (outside the 2.5 and 97.5 percentiles) in each of the quality Classes, the resulting sample has 2,465 Class A leases and 749 Class B leases. On average, there are 88 contracts for each quarter in our final sample.

Table 1—: **Office leases by year of contract execution**. The first three columns show the number of full-information office leases executed during each year in our data set, classified by quality class. The last three columns restrict the sample to full-service or gross leases. The number of leases after excluding quarters with less than 30 executed contracts is 3,458.

	All office			Full-service/gross office		
	Class A	Class B	Class C	Class A	Class B	Class C
1998	2	1	0	0	0	0
2000	1	1	0	0	0	0
2001	5	2	0	4	2	0
2002	21	14	1	7	3	0
2003	37	9	2	13	2	1
2004	390	193	17	16	8	0
2005	364	128	8	132	20	1
2006	581	201	24	289	64	13
2007	585	237	29	316	78	3
2008	444	232	31	282	100	8
2009	477	222	21	262	78	6
2010	556	228	19	388	121	13
2011	615	215	24	342	95	9
2012	759	457	48	81	40	6
2013	450	168	21	255	79	5
2014	445	228	19	220	96	4
2015	60	32	5	37	17	2
2016	145	83	12	74	39	8
2017	86	33	7	25	8	1
2018	80	19	4	41	0	0
2019	6	1	1	2	0	0
	6109	2704	293	2786	850	80

We also have information on the type of lease. Gross and full-service leases are essentially equivalent and are all-inclusive (i.e., the tenants are only responsible for the associated quoted rents). Modified gross and net (including "NN" and

Among the 3,458 full-information transactions, 90 Class A and 27 Class B leases are missing the TI field, and 26 Class A and 9 Class B leases are missing the free rent field. In these cases, we assume that that TI and/or number of free rent months are zero.

"NNN") leases place some or all of the burden of operating and maintenance expenses on the tenant. Because we do not have access to information on property expenses, we restrict our analysis to gross and full-service leases, which constitute the largest category of transactions in our data set.

Table 2 reports on key lease characteristics for full-information Class A and Class B leases in our final sample. Lease term is measured in years and is on average a little over 8 years for Class A and B contracts with similar year variation. Lease terms range from two to twenty years and this does not vary much by quality of space (i.e., "Class"). Time to commencement measures the number of months between the execution date and the commencement date, where it takes Class A leases 2.49 months to commence, it takes just 1.97 months for Class B. Time to expiration is the number of years between the execution date and the expiration of the lease. Similar to lease term, it takes just 9 years, on average, for both classes contracts to expire with similar variation.

Table 2—: **Summary statistics.** The table shows summary statistics for the final sample used in our estimations.

	Mean	S.D.	1%	25%	50%	75%	99%
Class A							
Lease term (years)	8.95	3.80	2.00	5.25	10.00	10.50	20.00
Time to commencement (months)	2.49	5.29	0.00	0.00	1.00	3.00	28.00
Time to expiration (years)	9.16	3.87	2.00	5.42	10.00	10.75	20.28
Starting rent (USD)	5.54	1.96	2.58	4.08	5.17	6.62	11.20
Average rent (USD)	5.21	1.95	2.41	3.76	4.77	6.20	10.94
Average rent increase (USD per yr)	0.04	0.05	0.00	0.00	0.04	0.05	0.14
Number of rent bumps	0.93	0.78	0.00	0.00	1.00	1.00	3.00
Average bump duration (months)	54.40	16.95	13.64	46.00	57.00	60.00	120.00
Tenant improvements (USD)	31.15	29.27	0.00	0.00	27.00	55.00	100.00
Free rent (months)	5.04	4.05	0.00	2.00	4.00	7.00	15.00
Class B							
Lease term (years)	8.86	4.02	1.08	5.00	10.00	10.50	20.60
Time to commencement (months)	1.97	3.71	0.00	0.00	1.00	3.00	12.00
Time to expiration (years)	9.02	4.05	1.42	5.33	10.00	10.58	20.72
Starting rent (USD)	3.53	0.91	2.00	2.83	3.33	4.08	6.04
Average rent (USD)	3.31	0.87	1.91	2.62	3.18	3.88	5.59
Average rent increase (USD per yr)	0.02	0.03	0.00	0.00	0.02	0.04	0.10
Number of rent bumps	0.91	0.89	0.00	0.00	1.00	1.00	4.00
Average bump duration (months)	56.99	24.61	11.55	48.00	58.00	60.00	126.24
Tenant improvements (USD)	23.01	23.08	0.00	0.00	17.75	40.00	75.00
Free rent (months)	4.62	3.52	0.00	2.00	4.00	6.00	14.00

Starting and average rent are measured in USD per square foot per month. Average rent is the mean of all monthly payments implied by the lease, taking into account months of free rent and tenant improvements (TI). Starting and average rents are about $2 more per square foot for Class A space, than for Class B space, but Class A space has higher variation.

Average rent increase represents the average yearly rent hike in USD, i.e., the difference between rent due at the end of the lease and starting rent, divided by lease term. The average rent increase for Class A space is twice that of Class B space, \$ 0.04 to \$0.02, respectively. Moreover, the standard deviation is quite high, but this is more representative of skewness in the data.

The number of rent bumps represents the number of times the rent is updated during the leases. For both Class A and B contracts this usually occurs once, but can occur as many as 3 or 4 times. The timing of a bump is on average every four to five years, where the average bump duration denotes the average number of months between rent updates. The average rent increases by just under 1% per year over its term.

Tenant concessions like TI are measured in USD per square foot. The TI for Class A space is about \$8.00 more per square foot. Free rent is expressed in months and the average free rent for a Class A and Class B space is not too dissimilar, 5.04 and 4.62 months, respectively. Importantly, TI are substantial when given, amounting to nearly a full year's rent, while a free rent period is common at a median concession of four months.

Finally, we also have data on the geolocation of the lease contracts. Figure 1 depicts the distribution of Class A and B leases that we use in our estimation. Leases primarily come from Midtown and Lower Manhattan. Class A leases cluster at the southern tip of lower Manahattan (near Wall St.) and the Midtown blocks bounded between 39th St., Central Park, 2nd Avenue and 8th Avenue. Class B leases are somewhat more evenly dispersed.

A. *Relating Lease Contract Specs to a Bundle of Lease Forward Rates*

Our analysis relies on the assumption that the present value of the stream of contract cash flow equals the present value of contract occupancy. Our data set allows us to calculate the monthly cash flow implied by the contract, including concessions, along with the corresponding months of occupancy. Each lease can be viewed as a bundle of forward contracts on space occupancy, with its present value given by the discounted sum of forward prices. Given enough heterogeneity in lease terms one can effectively "unwind" the lease bundles to arrive at the constituent forward contracts. To make the idea concrete, we provide a simple example.

Example. — Consider three recently signed gross leases. The first has a three-period term and commits the tenant to a constant contract rent of 5.0 paid in each of the next three periods. The second lease has a two-period term corresponding to a rent of 4.5 paid in each of the next two periods. The third lease, also with a two-period term, starts in period two and pays a rent of 4.0 and 7.0 in periods two and three, respectively. For the sake of this example, interest rates are constant at 0% per period. Further assuming away counterparty risk, heterogeneity in the quality or value of space, or the presence of valuable renewal options, leads to

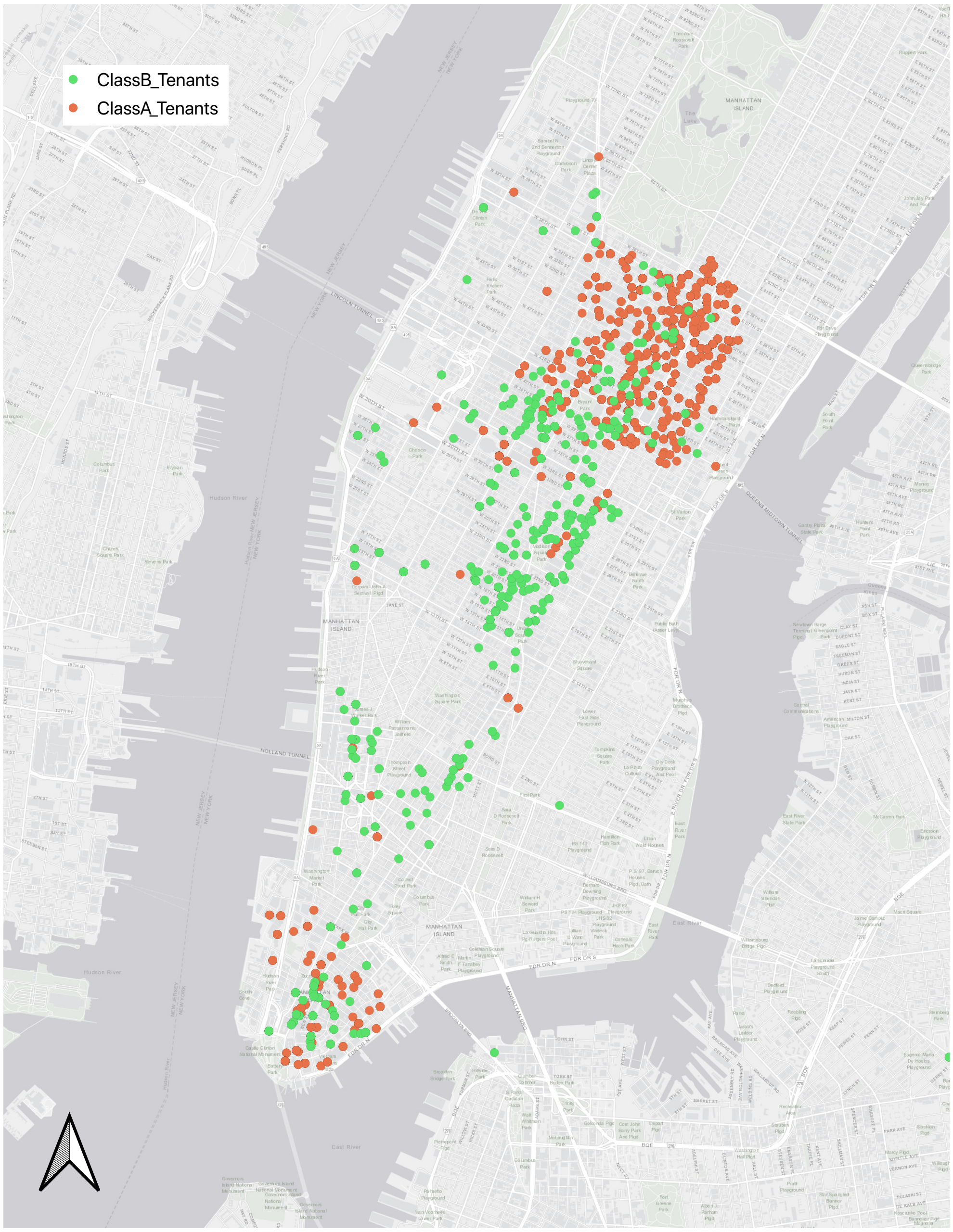


Figure 1. : **Office Lease Spatial Distribution**. The figure depicts the geospatial point distribution of the Class A and Class B Compstak office leases that we use in our estimation across the sample period (2005 to 2016).

three equations each of which sets the present value of the cash flow from a lease to the present value of its forward commitment. Specifically, denoting a current forward commitment to providing space in period i as F_i yields,

$$\begin{aligned} 5+5+5 &= F_1 + F_2 + F_3, \\ 4.5+4.5+0 &= F_1 + F_2, \\ 0+4+7 &= F_2 + F_3. \end{aligned}$$

The (unique) solution to the set of equations above is $F_1 = 4, F_2 = 5$ and $F_3 = 6$. □

To account for non-zero interest rates, one need only discount each cash flow and forward term by its term rate (i.e., multiply each by the price of a corresponding $1 zero-coupon bond). Moreover, in practice, heterogeneity in the quality of space means that the set of linear equations linking the present value of rents and forward space commitments is only approximate, prompting the addition of a noise term. Two complicating factors that we do not address head-on in this paper are the potential presence of valuable (to the tenant) renewal and lease default options. The presence of these options should be reflected by higher imputed forward rates, and the impact may systematically differ across horizons. This means that our imputed forward rates should be interpreted to be gross of average renewal option and systematic credit spreads.[6]

Another challenge to implementing the general idea outlined above is the fact that, as suggested by Table 2, there are very few short-term leases. That is because, leases tend to cluster around certain industry-standard terms (e.g., 5-, 7-, 10-, and 15-year leases). Thus there isn't enough heterogeneity in lease terms to identify all points along the term structure. To deal with this, following the literature on the term structure of interest rates, we assume that all forward lease rates at a given time can be derived from a small set of *key forward rates* via linear interpolation. Specifically, we consider three key forward rates: the spot price of occupancy, corresponding to contracting for the next month of space; the five-year forward rates, corresponding to locking in today one month's occupancy starting in five years; and the ten-year forward rate, similarly defined.[7] Under the linear interpolation assumption, the forward price of one month's occupancy starting in 2.5 years from now is a 50:50 weighting of the spot and the 5-year rates.

[6] In principle, if one were able to control for the credit worthiness of each tenant one would be able to back out a term-dependent credit spread. At this point we do not have such data. Likewise, among the full-information leases we employ in our estimation, only 14 Class A and 5 Class B leases include information about a renewal option (the rest are empty), and in each case the information is too vague to quantitatively interpret.

[7] We restrict attention to three key rates, despite the small amount of clustering around 7- and 15-year leases because the increase in parameters that we estimate in our structural state-space model is quadratic in the number of key rates.

B. Details of the Fundamental Observation Equation

For any lease with full information in our dataset and executed at some calendar month t, we can compute the monthly cash flow sequence commitment from the occupancy commencement to the final month. This includes months of free rent and also accounts for any scheduled contract rent escalations. Correspondingly, TI commitments are subtracted from rent paid in the first month of occupancy. Denote the monthly cash flow sequence for a lease i executed at month t as $(c_{i,t,0}, c_{i,t,1}, \ldots, c_{i,t,T_i})$, where the last contract month starts at $t+T_i$. It is possible for some of the $c_{i,t,\tau}$'s to be zero or positive (e.g., if $t+\tau$ is prior to commencement or coincides with a month of free rent, or a month in which TI is paid).

For each execution month in our sample, we obtain continuously compounded risk-free Zero Coupon Bond (ZCB) rates from OptionMetrics and compute present value (discount) factors.[8] Denoted as $d_{t,\tau}$ the discount factor at month t for a risk-free obligation due at month $t+\tau$. This is also the price of a \$1 zero-coupon bond maturing at $t+\tau$.

Next, we denote as $F_{t,\tau}$ the contract (forward) price, determined at month t, of locking in a commitment to one month of occupancy at month $t+\tau$. As mentioned earlier, we assume that $F_{t,\tau}$ is determined by linear interpolation/extrapolation from $F_{t,0}, F_{t,60}$, and $F_{t,120}$. For instance, the forward price of $F_{t,1} = \frac{59}{60}F_{t,0} + \frac{1}{60}F_{t,60}$. We then denote as v_τ the vector of linear interpolation coefficients applied to the vector of key rates to generate $F_{t,\tau}$. In the example just given, $v_1 = (\frac{59}{60}, \frac{1}{60}, 0)$.

For a given lease i at month t we define $PV_{i,t}$ to be the sum of lease cash flows, $(c_{i,t,0}, c_{i,t,1}, \ldots, c_{i,t,T})$, discounted to the present using the ZCB rates:

$$PV_{t,i} = \sum_{\tau=0}^{T_i} d_{t,\tau} c_{i,t,\tau}, \tag{1}$$

Let $t+\tau_c$ be the occupancy commencement date. The corresponding expression for discounted forward claims on the same space is

$$\sum_{\tau=\tau_{i,c}}^{T_i} d_{t,\tau} F_{t,\tau} = \sum_{\tau=\tau_c}^{T_i} d_{t,\tau} v_\tau \cdot (F_{t,0}, F_{t,60}, F_{t,120})' \tag{2}$$

$$= w_{t,0,i} F_{t,0} + w_{t,60,i} F_{t,60} + w_{t,120,i} F_{t,120}, \tag{3}$$

where $w_{t,0,i} = \sum_{\tau=\tau_c}^{T_i} d_{t,\tau}(v_\tau)_1$, and $(v_{\tau,})_1$ is the first component of v_τ, while $w_{t,60,i}$ and $w_{t,120,i}$ are similarly defined using the second and third components of the key-rate coefficient vector v_τ. Note that the expression in Eq. (2) is linear in the

[8] Rates from OptionMetrics are only available for up to ten years of maturity. For longer horizon payments we use the 10 year rate.

lease key rates.

Our main hypothesis is that there is an underlying prevailing term structure of lease rates in NYC and that it determines the average lease price. In other words, we are asserting that

$$PV_{t,i} = \sum_{a \in \{0,60,120\}} w_{t,a,i} F_{t,a} + \text{mean-zero independent error.}$$

For each lease, the quantity on the left side is observable in our data, as are the $w_{t,a,i}$ coefficients. This is the empirical analogue of the simple example provided earlier. A problem with estimating this equation is that longer leases will have larger $PV_{t,i}$'s and correspondingly larger $w_{t,a,i}$ coefficients (i.e., a short-maturity lease is a smaller bundle of forward obligations than a long-maturity lease). In fact, the $PV_{t,i}$'s will roughly scale with the lease term, suggesting that deviation from average pricing will be more pronounced for longer leases than shorter leases. To control for the expected heteroskedasticity in the error term, we normalize both left and right side of the equation above by the sum of coefficients. To that end, we define the normalized discounted lease cash flow as,

$$nPV_{t,i} = \frac{PV_{t,i}}{\sum_{a \in \{0,60,120\}} w_{t,a,i}}, \tag{4}$$

and restate our observation equation as

$$nPV_{t,i} = \frac{\sum_{a \in \{0,60,120\}} w_{t,a,i} F_{t,a}}{\sum_{a \in \{0,60,120\}} w_{t,a,i}} + u_{t,i}, \tag{5}$$

where the $u_{t,i}$'s are assumed to be independent across leases (i.e., there is no systematic deviation from average pricing for any subset of leases).[9] Equation (5) relates lease cash flow to a weighted average of lease key rates and is central to our estimation methodology. It also has a simple economic interpretation. If the term structure of lease rates were flat, then $F_{t,a}$ would not vary with a and Equation (5) would reduce to $nPV_{t,i} = F_t + u_t$. In other words, the forward rate would simply be the average over calculated $nPV_{t,i}$'s in a given execution period. One can therefore interpret $nPV_{t,i}$ as an effective rent.[10]

Figure 2 plots average effective rent for Class A and B leases in our final sample. Class A effective rents have remained largely constant, if one ignores the run-up to the GFC. By contrast, Class B effective rents appear to have markedly increased

[9] One could model u_t in more detail by making it dependent on available hedonic variables.

[10] It is this calculated effective rent that we use when trimming the Class A and Class B datasets at the 2.5 and 97.5 percentiles.

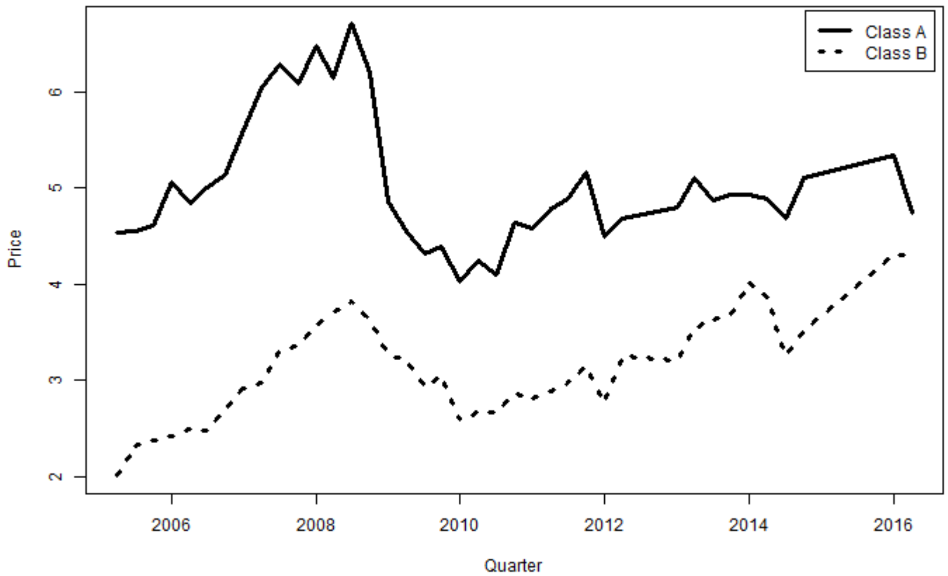


Figure 2. : **Effective rent**. Plotted is the average quarterly effective rental rate calculated in Equation (5). The rate is monthly (per square foot) and separately calculated for Class A and Class B properties in our dataset.

over the sample period.

III. Empirical estimation

Equation (5) is a linear relation linking the key rates to observable quantities. Viewed as a cross-sectional expression for month t, the key rates (i.e. the $F_{t,a}$'s) are coefficients in a linear regression. Correspondingly, we can estimate the key rates each period by OLS. Although this simple analysis is sufficient to reject a flat and constant term structure, the corresponding standard errors are too large to capture the time series behavior and autocorrelation structure of the key rates. To accomplish that, we turn to a more sophisticated Bayesian (Kalman filter) estimation approach.

A. Naïve Estimation of Key Forward Rates

We bucket leases by quarter to estimate key rates via OLS from Equation (5). This is because a monthly estimate lacks sufficient statistical power to adequately resolve the regression coefficients (i.e., the key rates). Thus our estimates of the key rates would correspond to "instantaneous" forward rates averaged over the quarter. Regressions are performed independently for each of the 39 quarters in our sample.

Table 3 reports the resulting distribution of OLS coefficients corresponding to the estimated key rates for Class A properties.[11] The main emerging pattern from this exercise is an upward sloping but concave (inverted U-shape) term structure of lease rates. The distribution of Spot rates ($F_{t,0}$) is typically dominated by that of the 5- and 10-year rates, but the 5-year rate typically dominates both. Treated as independent observations, one can reject the hypothesis that the pooled coefficients are equal. In other words, the term structure does not appear to be flat.

Table 3—: **OLS estimates for Class A properties**. The table shows summary statistics for the OLS point estimates of the spot ($F_{t,0}$), 5-year forward ($F_{t,60}$), and 10-year forward ($F_{t,120}$) rates across all 39 quarters in our final sample. The model for each quarterly regression is given by Equation (5)). The number of class A leases in each quarterly estimation sample ranges from 21 to 114.

	Mean	S.D.	10%	25%	50%	75%	90%
Spot	4.12	1.84	2.21	3.47	4.19	5.32	6.24
5yr	6.04	2.16	3.87	4.87	5.79	7.05	8.87
10yr	4.75	2.11	2.04	3.64	4.79	5.94	6.84

To provide a sense of the time-variation in the coefficients, Figure 3 plots OLS estimates and 95% confidence intervals of the key rates at different points in the real estate cycle. Outside of the period of great market stress in commercial real estate (2008-2011), one observes a small but positive slope to the term structure, with a pronounced inverted-U shape. During the time of stress, the slope flattened (or even became slightly negative) and the curvature largely disappears.

It should be clear from the plotted confidence intervals in Figure 3 that one can reject a flat term structure hypothesis even during specific quarters. Still, this exercise fails to adequately capture time-series dynamics.[12] The time-series autocorrelation for the each of the key rates (estimated via OLS) is close to zero. Such a conclusion, of course, is economically suspect and is likely driven by the large standard errors in each of the cross-sectional estimates. What is missing from the naïve approach taken above is a link between distinct cross-sectional estimates. For instance, if the estimated spot rate is high in the two extreme quarters among three contiguous quarters, then the likelihood should be more

[11] We only report results for Class A properties because there are too few (one third as many) Class B properties to obtain reliable estimates of key rates in most quarters. Later, when employing state-space techniques, we overcome this hurdle.

[12] This issue would also afflict a panel regression approach to estimating the key rates (e.g., Gunnelin and Söderberg, 2003).

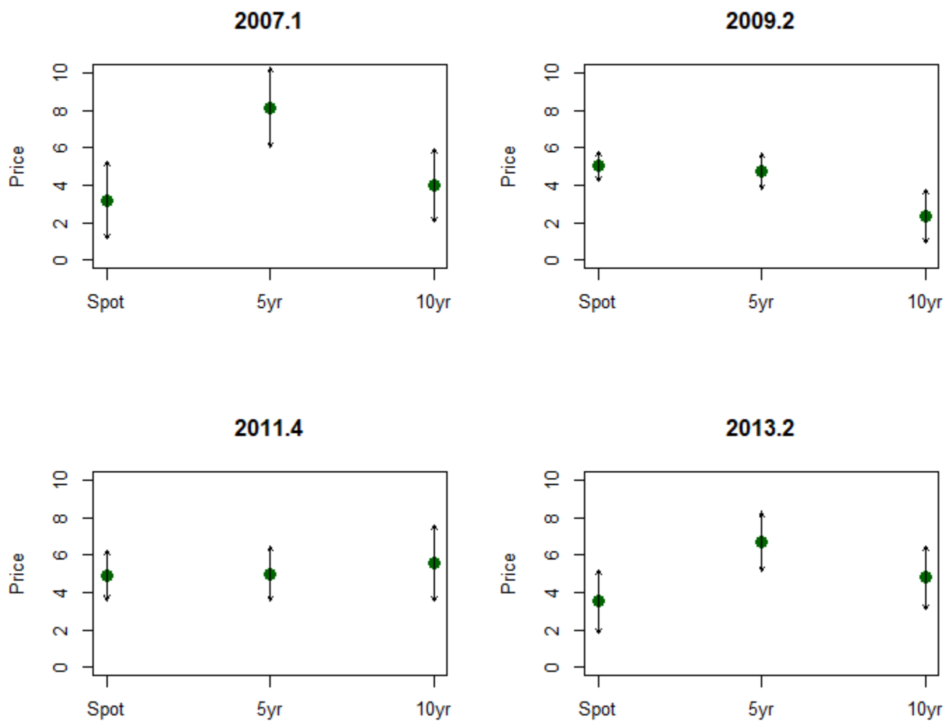


Figure 3. : **OLS estimates for Class A properties** The figure shows OLS estimation results for the spot ($F_{t,0}$), 5-year forward ($F_{t,60}$), and 10-year forward ($F_{t,120}$) in four of the quarters in our sample. Point estimates are depicted by green dots. The black arrows represent $\pm$ 1-standard-error bands for each of the point estimates.

than 50% that the spot rate for the middle quarter is also high. To include this logic in an econometric framework we resort to state-space Bayesian estimation techniques, described next.

B. Structural State-Space Model

We model the (unobserved) key rate dynamics as a VAR(1). This is done both for ease of estimation and also because, in our limited sample period, one cannot reject the hypothesis that effective rents are mean-reverting.[13] Denoting

[13]Mean reversion is unlikely to hold in a sample period spanning several decades where inflation alone will lead to secular growth in rents. we expect that over a longer time period, the dynamics we identify will be complemented by a slow-moving growth factor.

the vector of three key rates as $F_t = (F_{t,0}, F_{t,60}, F_{t,120})'$, the assumed evolution is

$$F_{t+1} = \bar{F} + \rho F_t + \epsilon_{t+1}, \tag{6}$$

where $\epsilon_{t+1} \sim N(0, Q)$ is uncorrelated with F_t and Q is some positive semi-definite matrix, $\bar{F}$ is a constant vector, and ρ is the 3x3 AR(1) matrix with eigenvalues in $[0, 1)$.[14]

As with the OLS estimation, we bucket leases by quarter to estimate average quarterly key rates and restrict attention (at first) to Class A properties. If, in addition to $\bar{F}, \rho$ and Q, the variances of the u_t's in the observation equations are known, then these can be used in conjunction with the Kalman Filter to back out the key rates. To do that, we assume that one quarter prior to the first quarter in our dataset, the key rates are drawn from the prior distribution given by the unconditional first and second moments of the process for F_t.[15]

The above procedure allows us to impute the time series of key rates assuming that the various model parameters are known. To estimate these, we search for the parameter set that maximizes the Gaussian likelihood of observed data. Details are available in most graduate texts on econometric time-series techniques. This approach permits us to simultaneously estimate both the model parameters and a "best guess" at the time series of realized key rates (referred to as the "smoothed" time-series).

In estimating the model, we found that allowing the variance of observation errors (i.e., the $u_{t,i}$'s) depend on the calendar year substantially reduced erratic movements in the imputed key rates.

Because the model features many parameters, our search for the maximum likelihood parameter set is done by first generating a pseudo-uniform (Sobol) grid of 3,000 parameter sets and then used each of these points to initialize a separate search for a local optimum. We then selected the best of all converged searches. Surprisingly, there was a clear cluster around the global optimum, suggesting that we're unlikely to have missed a better fitting parameter set. The corresponding parameters are reported in Tables 4 - 5. Next, we separately estimate the model for Class B leases with the important difference that we assume the AR(1) matrices (the ρ's) for Class A and B leases coincide, and that the variance matrices (the Q's) for the two Classes are proportional. The restrictions substantially reduce the number of Class B parameters we need to estimate. What is surprising is that the observation errors for Class B leases (Table 5) are several times *smaller* than those for Class A. This reduces concern that the parameter restrictions may lead to model misspecification for the Class B leases.

[14]The constraint on the eigenvalues ensures that the process is mean-reverting and non-oscillating.

[15]The unconditional mean of F_t is given by $(I - \rho)^{-1}\bar{F}$. Its associated unconditional variance, V, is given by the solution to the linear matrix-valued equation, $V = Q + \rho V \rho'$. Once again, we caution that the unconditional mean and variance we estimate reflect only the sample period we observe. Over a significantly longer, or inflationary, time period we would expect that the time series of forward rates would not be mean-stationary.

Table 4—: **State equation parameters**. The table shows maximum likelihood estimates for the parameters in Equation (6), obtained via a Kalman Filter. The eigenvalues of ρ and Q were restricted to $[0, 1]$ employing a Schur decomposition of these matrices. The main estimation was performed using our sample of 2,465 class A leases. For the class B estimation, we fixed ρ and Q (except for one eigenvalue) at the Class-A point estimates, and then estimated the rest of the parameters.

	Class A	Class B
$\bar{F}_1$	2.8075	1.8556
$\bar{F}_2$	2.8605	1.9021
$\bar{F}_3$	3.2041	2.1599
ρ_{11}	-0.4504	
ρ_{12}	-0.3205	
ρ_{13}	1.1557	
ρ_{21}	-1.1093	
ρ_{22}	0.7395	
ρ_{23}	0.7510	
ρ_{31}	-1.4622	
ρ_{32}	-0.2787	
ρ_{33}	2.0336	
Q_{11}	0.0000	0.0000
Q_{22}	0.0205	0.0148
Q_{33}	0.0311	0.0224
Q_{12}	0.0006	0.0004
Q_{13}	0.0007	0.0005
Q_{23}	0.0253	0.0182

To more readily compare with the OLS estimation (Table 3), we report the unconditional mean and variance of the key rate vector, F_t, in Table 6. The statistics in the table convey a measure of average key rate statistics over our sample period. For Class A leases, here too one observes the increasing yet concave "average" term structure, reinforcing the conclusions from the less sophisticated analysis and lending the finding of an inverted-U shape an element of robustness. Class B properties, on the other hand, appear to be nearly flat. While this might be true for the sample, on average, the Class B term structure has been consistently sloping up in recent years.

It is interesting to note that the key rate variances in Table 6 (the diagonal terms of the Variance matrices) *increase* with the horizon of the forward contract. This is unusual for commodities exhibiting mean-reverting price. In the standard term structure literature, whether dealing with commodities, currencies, or interest rates, long-horizon forward rates tend to vary less than the spot. This

Table 5—: **Observation equation error variances**. The table displays the maximum likelihood estimates for the elements of the variance-covariance matrix of $u_{t,i}$ in Equation (5), obtained via a Kalman Filter. We assume such matrix to be diagonal and restrict the variance of the observation errors to be the same for all leases signed during a given year. The columns labeled "Count" show the number of leases executed each year in each quality class.

	Class A		Class B	
	Variance	Count	Variance	Count
2005	2.6836	112	0.3293	19
2006	4.0948	278	0.3395	59
2007	4.2758	299	0.4091	78
2008	4.4386	260	0.4669	97
2009	2.4042	254	0.4440	77
2010	1.8325	374	0.3044	110
2011	3.0644	322	0.4046	90
2012	2.3467	60	0.5084	23
2013	3.1751	247	0.8192	79
2014	2.4734	207	0.8957	90
2016	1.9635	52	0.5365	27

is termed the "Samuelson Effect". It appears that lease forward rates exhibit the opposite behavior. This might arise because there is greater scope for adjusting the supply and demand for long-horizon space commitments. For instance, consumers of space may be relatively inflexible concerning their current space needs, while suppliers of space cannot quickly increase their "inventory" of space and may be disinclined to keep short-term space unleased for fear of foregoing rents. In other words, supply and demand for short-term occupancy is relatively fixed. Because more adjustment is possible for future perceived needs and availability, the market for locking-in long-dated space may exhibit greater variability in response to changing economic conditions.

It is also useful to examine the eigenvalue structure of the autocorrelation matrix, ρ, reported in Table 7. What is striking is that the eigenvalues are roughly equal, meaning that shocks to forward rates decay at a roughly constant rate across all horizons. This is consistent with another finding: A decomposition of the variance matrix, Q, (in Table 4) reveals that it has only a single non-zero eigenvalue. What this means is that the estimated dynamical system for the key rates is driven by a *single* source of uncertainty. As discussed above, one interpretation is that changes to the economic conditions are first reflected in changes to the supply/demand equilibrium of long-run lease components, and subsequently transmitted to shorter maturity forward components.[16]

[16]Stated in lay-terms, it may be easier to negotiate over the terminal date of a lease than the start

Table 6—: **Unconditional mean and variances**. The table shows the unconditional mean and variance of the key rate vector, F_t, as implied by the point estimates of the parameters of the state equation (Eq. (6)), displayed in Table 4. The unconditional mean of F_t is given by $(I - \rho)^{-1}\bar{F}$. Its associated unconditional variance, V,is given by the solution to the linear matrix-valued equation $V = Q + \rho V \rho$.

	Class A			Class B		
	Spot	5yr	10yr	Spot	5yr	10yr
Mean	4.4733	5.5577	4.7263	3.1873	3.155	3.2696
	0.2291	0.1756	0.3218	0.1652	0.1266	0.232
Variance	0.1756	0.3167	0.3385	0.1266	0.2284	0.2441
	0.3218	0.3385	0.513	0.232	0.2441	0.3699

Table 7—: **Eigenvalues of ρ**. The table displays the maximum likelihood estimates of the eigenvalues of ρ (Eq. (6)), obtained via a Kalman Filter. These parameters were restricted to $[0, 1]$ using a Schur decomposition of ρ.

Class A		
0.7661	0.7809	0.7757

To examine how an economic shock differentially impacts the key rates and eventually "decays", we undertake an impulse-response exercise. In it, we consider a one standard deviation disturbance to the single fundamental shock underlying the dynamical system for Class A key rates in NYC. We plot the results in Figure 4. At Quarter 0, the key rates are at their unconditional mean and at Quarter 1 we apply the one standard deviation shock. One can see that the impact of the shock is initially most pronounced in the 10-year forward and least seen in the spot. Interestingly, the shock applied in Quarter 1 continues to move prices in the same direction — in other words, prices at Quarter 1 do not fully reflect the impact of the shock. In a market where information is efficiently (i.e., immediately) incorporated in prices, a 10-year forward commitment to space made in Quarter 1 would anticipate predictable increases in forward prices the following quarter: The price impact of the Quarter 1 shock would be fully felt in Quarter 1. By contrast, the estimated dynamical system suggests substantial inefficiency in the space market where it takes around six months for prices to fully reflect the Quarter 1 shock. Also interesting is the fact that, though attenuated, the shock

date of a lease. In turn, the supply of and demand for long-term forward components of leases can more easily adjust to changing economic conditions.

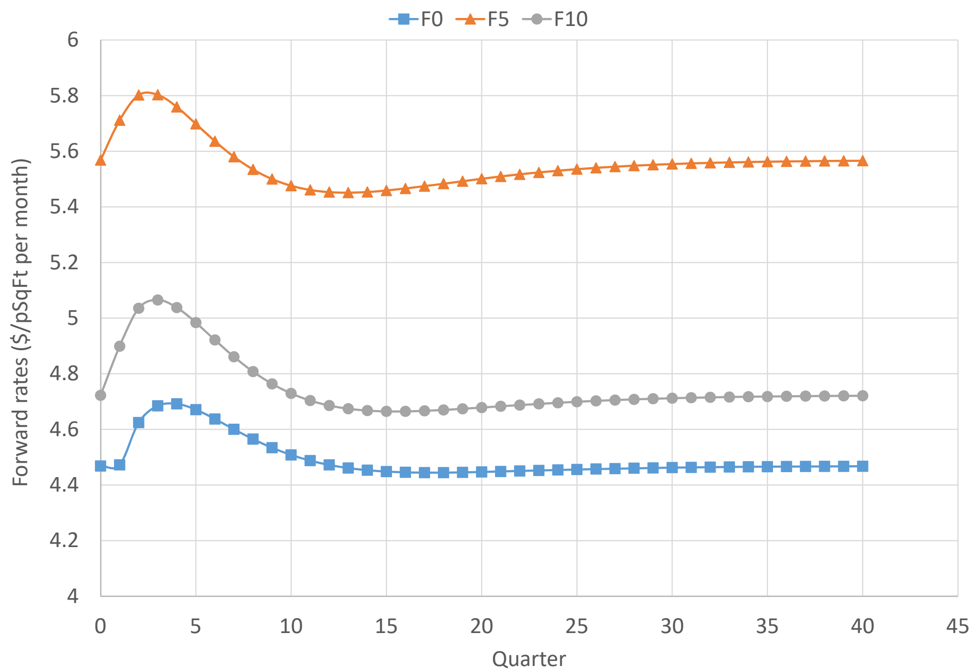


Figure 4. : **Impulse-response plot**. The estimated dynamical system of lease forward key rates in Equation(6) is driven by a single shock that differentially impacts the key rates. Initially, the shock most profoundly impacts the 10-year forward (F10, gray circles) and is subsequently transmitted to the other key rates. Eventually, it finally decays.

does make a non-trivial impact on the spot market but the impact is delayed relative to the influence on long-date forwards. This illustrates that shocks are sluggishly transmitted from long-dated to short-dated forward rates.

From the estimated model parameters, we can use the Kalman filter to produce optimal estimates for the time-series of key rates. There are two ways to estimate these time series. One can produce an estimate of key rates each quarter that depends only on information available up to that quarter. This is termed a "filtered" estimate. Alternatively, each quarter, one can employ all observed information (including observations from subsequent quarters) to impute the key rates. This is termed a "smoothed" estimate. We opt to report the latter because our goal is to provide our best possible estimate for the unobserved key rates.

Figure 5 plots smoothed estimates of the key rates over our sample period, for both Class A and B properties. Several features stand out. Firstly, both the level and the relative rankings of the key rates are time-varying. Second, one can see evidence that the 10-year lease rate leads the shorter maturity rates, as suggested

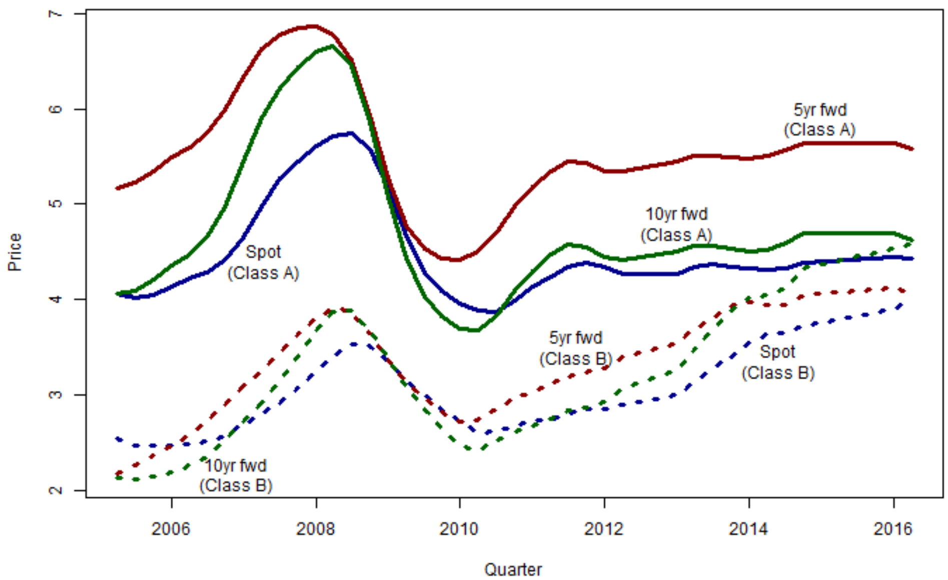


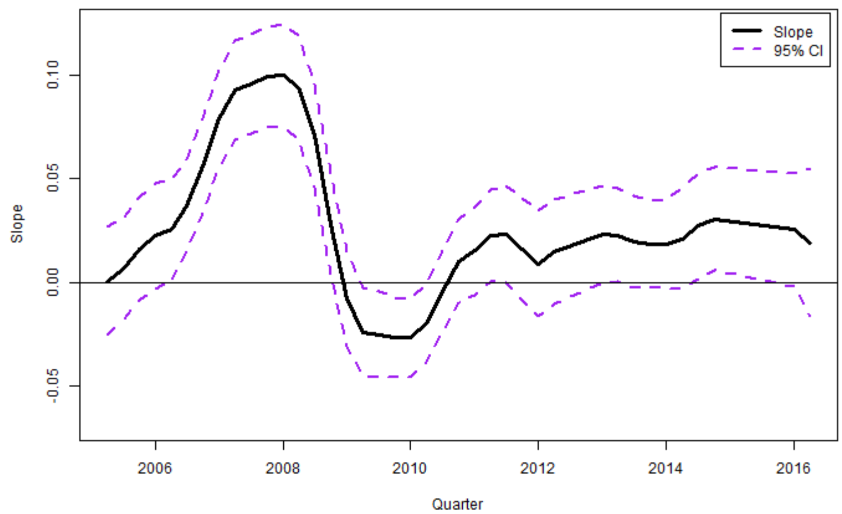
Figure 5. : **Term structure of lease rates**. The figure depicts the estimated key forward and spot rates for class A (solid lines) and class B (dotted lines) properties, as given by the Kalman smoother. Key rates for quarters with less than 30 executed leases were interpolated.

by the impulse-response exercise of Figure 4.

Figures 6 and 7 plot the slope and curvature, respectively, of the term structures implied by the key rates. The slope is calculated as $\frac{1}{10}(F_{t,120} - F_{t,0})$ while the curvature is $(F_{t,120} - 2F_{t,60} + F_{t,0})$. The top (bottom) plot in each figure corresponds to the Class A (B) term structure. Because the key rates are estimates rather than directly observed, we additionally plot 95% confidence intervals for the slope and curvature.[17]

What is apparent from the plots is that, although the term structure is typically upward sloping with negative curvature, this is not always so. Consistent with the naïve OLS estimates in Figure 3, on the heels of the Great Financial Crisis, the term structure briefly flattened and even became slightly downward-sloping. Another main takeaway is that both the positive slope and negative curvature are more pronounced for Class A than Class B properties. Finally, although the slopes and curvatures across the different class of properties are clearly correlated,

[17]The Kalman smoother generates a covariance matrix for the estimation errors of the key rates at each quarter. This, in turn, is used to calculate the confidence intervals.



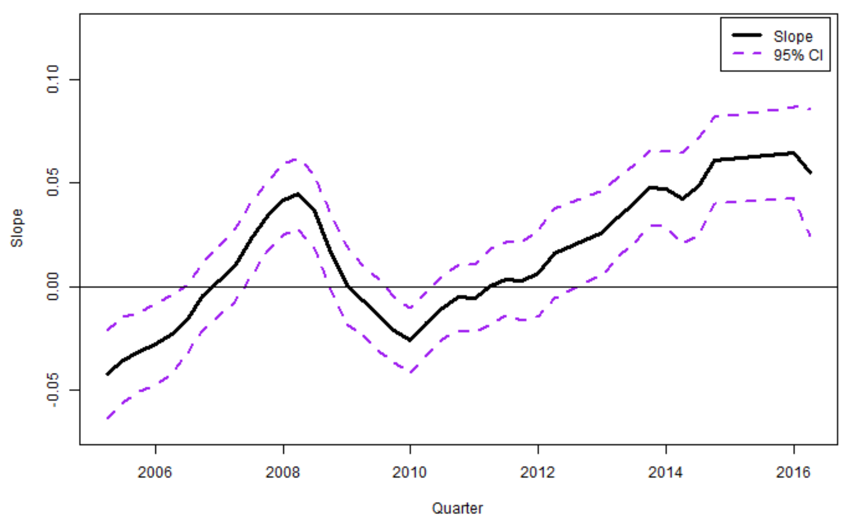


Figure 6. : **Slope of the term structure of lease rates**. The figure shows the slope of the term structure measured as $\frac{1}{10}(F_{10} - F_0)$, one tenth of the difference between the 10-year forward and the spot rates for class A (top) and class B (bottom) properties, as given by the Kalman smoother. The dashed lines represent 95% confidence intervals.

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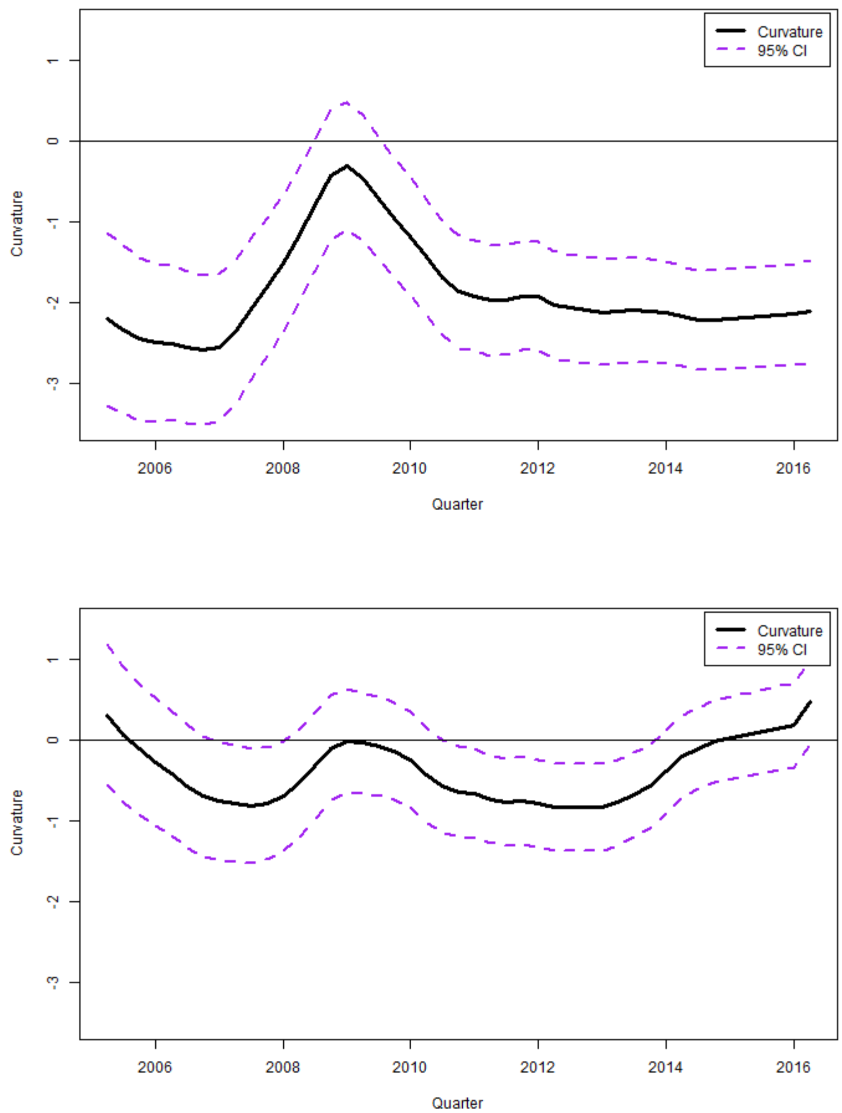

Figure 7. : **Curvature of the term structure of lease rates**. The figure shows the curvature of the term structure measured as $F_0 + F_{10} - 2F_5$, the sum of the spot and the 10-year minus twice the 5-year forward rates for class A (top) and class B (bottom) properties, as given by the Kalman smoother. The dashed lines represent 95% confidence intervals.

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the slope for Class B properties have recently exhibited a steady increase whereas the Class A slope stayed relatively constant.

C. Discussion

It is legitimate to ask whether our estimates of the key rate time series, and therefore the term structure of lease rates, are grounded in substance rather than driven by spurious variables for which we've failed to control. Moreover, it is equally legitimate to ask what kind of economic forces might lead to the imputed shapes of the term structure in our data. We begin by addressing the latter question, and then turn to discussing possible unobserved influences.

Comparison with Other Commodities. — To gain an appreciation for what may drive lease forward rates, it seems natural to compare them with other types of forward contracts. The most important economic force restricting the term structure of forward contracts is the possibility of arbitrage. The price specified by a τ-years forward contract on a dividend paying asset with current value S_t must be given by

$$F_{t,\tau} d_{t,\tau} = S_t d_{t,\tau} - pv_t[Div(t, t+\tau) + f(t, t+\tau)],$$

where $d_{t,\tau}$ is the price of a \$1 zero coupon bond maturing at $t+\tau$, $pv_t[Div(t, t+\tau)]$ is the present value of all benefits ("dividends") derived from ownership of the asset between t and $t+\tau$, and $pv_t[f(t, t+\tau)]$ is the present value of a market friction component, $f(t, t+\tau)$, that must be in the interval $[-bc(t,\tau), sc(t,\tau)]$. Here, $bc(t,\tau)$ is the cost to short sell (or borrow) the asset between t and $t+\tau$, and $sc(t,\tau)$ is the cost to store the asset between t and $t+\tau$. In plain language, absent market frictions, the forward price is the spot, less the capitalized value of cash flow forgone before taking delivery of the asset. As most text books describe it, deviations from this pricing in a frictionless market leads to an arbitrage opportunity from a cash-and-carry or reverse cash-and-carry strategy. In the former, when the forward price is too high, one borrows money risk-free, purchases and stores the asset while simultaneously selling it forward, invests the dividend cash flow until delivery, and upon delivery uses the forward sale price to pay off the debt. In a reverse cash-and-carry, if the forward price is too low, the asset is sold short while simultaneously purchasing the asset forward, the proceeds are invested in a risk-free bond and used to pay off the dividend commitments to the asset holder, and the remainder is used to pay the forward price and settle the asset loan.

The friction working against a (reverse) cash-and-carry strategy is (short-selling) storage costs. The important point we wish to make is that the frictions associated with executing a cash-and-carry strategy (or its reverse) in the space market are prohibitively large, meaning that arguments based on arbitrage are unlikely to bound forward prices in a meaningful way. To see this, contrast the forward market on space with the forward market on, say, copper. Firstly, the underlying commodity on which the forward contract is written in the space market (via a lease) is not homogeneous across time. That's because the 10-year forward component of a lease corresponds to space in a property that is ten years older than the underlying space for a spot component. The quality of Copper, on the other hand, is essentially time-invariant. Second, and relatedly, the copper promised in a ten-year forward commitment can come from anywhere so long as its purity exceeds certain standards; by contrast, it would be generally prohibitively expensive for a tenant to agree to accept the promised space in the ten-year forward component of a lease from anywhere other than the same source (i.e., building) as the spot commitment of space.[18] Finally, a forward component of a lease delivers occupancy to be consumed over a set period of time, whereas a forward commitment to copper delivers a good that can be consumed anytime once it is acquired (subject to storage costs). In other words, the benefits from the delivery of a lease forward cannot be "stored", and this restricts cash-and-carry forms of arbitrage. In this regard, lease forwards in the space market resemble those in the electricity market where storage costs are prohibitive and thus a claim delivered today must be consumed today or foregone. Correspondingly, reverse cash-and-carry is not possible because the cost to borrow spot space is the full spot rental rate whereas the cost to borrow a non-ephemeral commodity (like copper) corresponds more closely to cost of deferring its use.[19]

To summarize, inherent inhomogeneities and the inability to meaningfully store the goods that a lease delivers obviate typical no-arbitrage constraints on lease forwards that might otherwise apply to standard commodities.

Our estimates suggest a term structure that is often upward sloping and potentially concave. Below, we list some economic influences that might explain the imputed shape.

Anticipated price increases. — Inflation and anticipated increases in demand for space relative to supply will place upward pressure on forward lease rates. If the market anticipates rental growth rates that decelerate from an initial high level, then this can lead to an upward sloping and concave term structure.

Quality deterioration and obsolescence. — The Class A leases we "unbundle" are only guaranteed to be deemed Class A when the lease is executed. Prop-

[18]Correspondingly, individual forward lease commitments cannot be costlessly stripped from a lease agreement. I.e., few Class A office tenants can move their operations for only a single month while their leased space is let to another entity for only that month.

[19]The analogy with electricity would have us compare the property itself to the generation plant. One may be able to store or borrow the plant, as one would with a building, but one cannot (at this point) meaningfully store a month's production of electricity from a large plant, and to borrow one month's production one would have to pay its full price because its use could not be deferred.

erties deteriorate and become obsolete, and it's considered received wisdom in the commercial real estate industry that, absent intensive capital investment, most properties will drift down the quality spectrum as newer or more updated space becomes locally available. This means that the 10-year forward lease rates we impute represent space that, on average, is in decline relative to the spot market.

In other words, one would expect deterioration and obsolescence to contribute towards a downward sloping term structure. In addition, because deterioration decelerates, one would expect that these effects would contribute *positively* to the convexity of the term structure. Relatedly, one would expect these considerations to be more impactful for Class A properties than for Class B properties.[20]

Credit losses.. — Typical forward agreements mitigate counterparty risk using margin accounts. Although leases may incorporate some kind of escrowed funds (e.g., damage deposits), these are not typically enough to offset losses due to lease defaults. We are unable to control for tenant credit in the imputation of forward lease rates, meaning that they likely incorporate some credit spread, much as in the case of corporate bonds. With high-quality bonds, one also typically observes default rates that increase with term — this is because a well-underwritten investment-grade issuance is less likely to default in its first year than in subsequent years, and because credit is mean-reverting.

To compensate for possible default, the corresponding forward lease rate would have to increase. In turn, and absent the type of selection bias described below, the default dynamics described above should contribute positively to both the slope and convexity of the term structure.

Lease renewal options.. — Like the option to default on their lease commitment, a renewal option can only be exercised by the tenant. Although renewal can reduce lease commissions paid by landlords, one may generally expect that their value is greater to the tenant, meaning that their value should be expressed through higher rents.

Assortative matching (selection bias).. — Our methodology relies on the assumption that differences in term across tenants and space (for same-class leases) is statistical noise. If there are systematic differences in the quality of tenants or space across lease terms, then this will lead to systematic distortions in our imputed term structure of lease forwards.

Examples of such systematic differences include the possibility that greater default risk is associated with mid-maturity leases. This could happen, for instance, if landlords will only sign long-term leases with the highest-credit tenants. In such a case, long-maturity leases will have lower effective rents as would shorter-term

[20]Class B properties are often properties that, at some point, were Class A.

leases for reasons mentioned earlier. If risk concentrates in mid-maturity leases, then this would contribute to negative convexity in the term structure. Although we cannot rule this out, we do note that the negative convexity of Class B properties is greatly attenuated relative to Class A. It is unclear why risk concentration would be more pronounced in mid-term Class A, relative to Class B, leases.

Likewise, if renewal options are more prevalent in Class A mid-term leases then this too would enhance the negative convexity of the term structure. Here too, it is not clear how such an explanation accounts for the difference between Class A and B properties. Moreover, in examining leases that include some information on renewals, we could not find any significant relationship between the presence of lease renewals and lease term.[21] However, there is limited evidence that lease renewal options tend to cluster around renewal terms of five years. If a five-year renewal option value is "amortized" over the length of the original lease, then this might cause shorter-term leases to exhibit greater effective rent. Overall, to investigate these possibilities further, we would need to explore a complementary dataset that included details on renewal options.

By summarizing the influences identified above, one can attempt to account for the observed term structure of lease rates over our sample period in the absence of selection bias. Specifically, anticipated growth and "normal" default profiles will contribute to an upward sloping term structure. Natural depreciation and obsolescence will pull the term structure the other way. The term structure we observe can be rationalized by dynamics in rental growth expectations for Class A space are themselves concave (i.e., greater growth rates in the short/mid- than in the long-term), and in the long-run are further slowed by expected quality depreciation. For Class B properties throughout much of the sample period, by contrast, growth and depreciation expectations were roughly in balance, leading to a far flatter term structure. A default-free version of the term structures would potentially exhibit similar characteristics but with an even lower slope and more pronounced concavity.

IV. An application

By estimating a model of the term structure of leases, we are in a unique position to address questions concerning the risk and reward associated with spatial market strategies. The recent hype surrounding WeWork focused attention on the viability of short-term leasing of office space. Co-working companies, such as WeWork, provide high-quality office space and amenities/services on a short-term basis where the contracts can range from a day to a year. The business strategy consists of obtaining long-term rights to a space, and then offering it to short-term users. Profitability and risk can arise from what we refer to as the

[21]There are about one thousand leases with some information about lease renewals, although all but a few are missing key information and are not used in our estimation of forward key rates.

spatial and service components of the business strategy. The spatial component consists of benefiting from the difference between short-term and long-term lease rates and/or the intensification of short-term usage of leased space. The latter consists of increasing the number of users of the space per unit of area. The service component comes from profit margins created by the provision of amenities (e.g., internet usage, food/beverage, document printing, storage, furniture, and even atmosphere).

The success of a co-working business relies on both the spatial and the service components. Although we are not in a position to assess the risk and reward from the service component, our model permits us to analyze the spatial component of a co-working business. In particular, we take the position that the more the business model depends on the service component to be profitable, the more it resembles service-intensive real estate investments (e.g., hotels) rather than traditional office properties. Indeed, our analysis in this section suggests that, absent non-trivial intensification or skill in obtaining long-term claims to high-quality space at below market prices, the spatial component is a drag on profits and a source of substantial financial risk.

Our analysis is based on estimating, in each quarter of our sample period, the profits and risk from paying to lease a space for 10 years, and financing this by turning around and leasing the space to users on a short term (quarterly) basis. Because our model allows us to both forecast the dynamics of lease rates *and* calculate the standard error around that forecast, we can calculate both the expected profit and the profitability standard deviation from this strategy. Moreover, we can vary assumptions about average vacancy, intensification, and skill required to secure a below-average rate for the 10-year lease.

The appendix details how we calculate the expected profit and associated standard deviation for the strategy. Figure 8 depicts the results. To crystallize ideas, consider executing the strategy in 2010Q1. We first calculate the *filtered* key rates in 2010Q1 from the Kalman filter (using only information dating from 2010Q1 or earlier). These roughly correspond to the smoothed 2010Q1 key rates seen in Figure 5. The lease forward key rates can then be used to price an average 10-year lease (i.e., derive the average rent that would have been paid on a 10-year lease in 2010Q1). Next, we use the 2010Q1 filtered key rates to forecast quarterly lease rates for the next 40 quarters, together with their Gaussian forecast error. The difference between the forecasted quarterly lease rates and the fixed 10-year rental payment is then discounted to the present using 2010Q1 zero coupon bond prices. This is a measure of the (discounted) expected profitability of the strategy, as might have been assessed in 2010Q1. The forecast errors are then used to derive a 95% confidence interval for this quantity. The procedure is then repeated for 2010Q2, and so on. Figure 8 indicates that during most of the sample window, the strategy has not been profitable in Class A properties. This is because the lease term structure is mostly upward sloping. The only time the strategy would have been profitable was in the two years following the Great Financial Crisis.

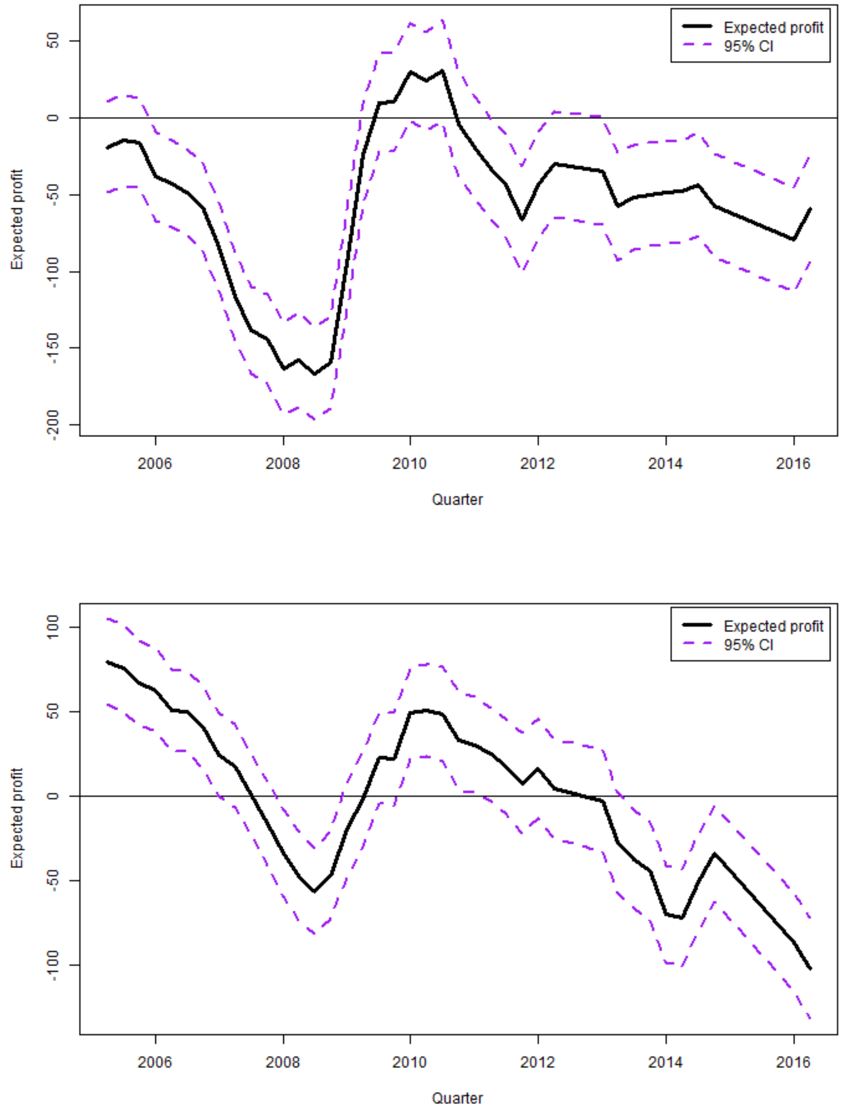


Figure 8. : **Expected profit from long-short space strategy**. The figure depicts the time series of expected profits for a strategy in which a 10-year lease is financed by leasing the space on a rolling quarterly basis. Profits from class A (B) properties are depicted in the top (bottom) panel. The strategy assumes 100% occupancy (there is a short-term occupant in every quarter over the 10 years). The profits are in dollars per square foot and discounted to the present using the contemporaneous zero-coupon yield curve. Dashed lines denote 95% confidence intervals.

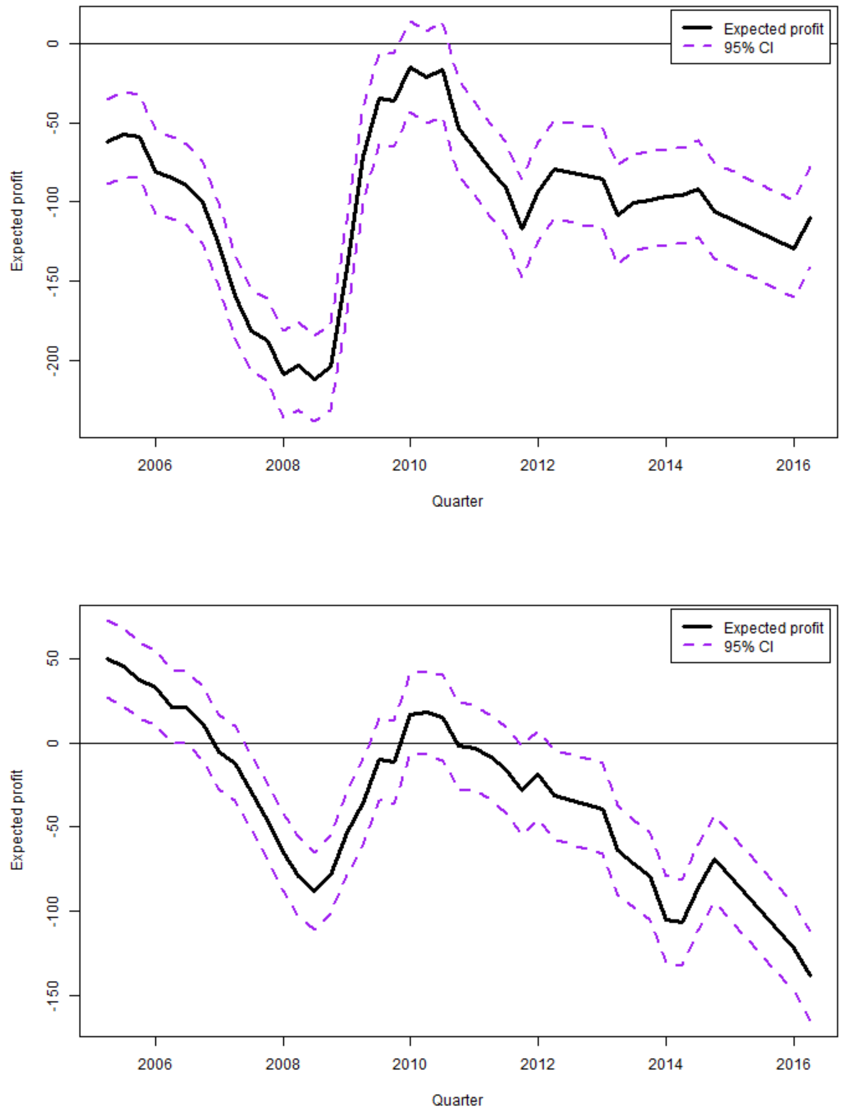


Figure 9. : **Expected profit from long-short space strategy**. The figure depicts the time series of expected profits for a strategy in which a 10-year lease is financed by leasing the space on a rolling quarterly basis. Profits from class A (B) properties are depicted in the top (bottom) panel. The strategy assumes 90% average short-term occupancy. The profits are in dollars per square foot and discounted to the present using the contemporaneous zero-coupon yield curve. Dashed lines denote 95% confidence intervals.

On the other hand, save for the Great Financial Crisis years, the strategy has been profitable in Class B space prior to 2013. After 2013, it was not longer so.

In calculating the profitability of the long-short space strategy outlined above, we assumed that the space was successfully leased out on a short-term basis in each of the 40 quarters that the 10-year lease was in place. Under a more reasonable 90% occupancy assumption, there are no quarters in which the strategy is profitable in Class A space, and it becomes unprofitable throughout most of the sample period for Class B space. This is depicted in Figure 9.

To provide a sense of the riskiness involved, we calculate a "Sharpe Ratio" for the various strategies. This is done by dividing the annualized expected strategy profits in a given quarter by the corresponding annualized standard deviation of the profits. For comparison, consider that typical equity market risk corresponds to annualized Sharpe Ratios of roughly 50%. What the figures show is that even when the long-short space strategy is profitable, it rarely provides high compensation for the risk entailed. By and large, our takeaway is that without additional advantages, the long-short space strategy yields negative profits at substantial risk.

A. Turning a profit in a long-short space strategy

The preceding analysis of risk and reward to the spatial component of a co-working business model assumes that space is acquired and re-leased at average market rates. Table 5, which documents estimated dispersion in observed effective rents, indicates that observed lease rates can vary widely from average market rates. This suggests that a talented (or lucky) negotiator may be able to obtain a below-market rate on the 10-year lease. Likewise, our analysis assumes that the use intensivity of the space obtained with the 10-year lease is equal to the use intensivity of the short-term renter. Because co-working businesses may be able to configure their space to accommodate a higher density of users per square foot, they may be able to generate more revenues through increased intensivity.

For each quarter in our sample period, we shift the 10-year lease rate until the long-short space strategy breaks even (zero profits) at 90% occupancy. We then continue to shift the 10-year rent until the strategy achieves a 50% Sharpe ratio (also at 90% occupancy). Next, we calculate the percentile in the distribution of observed leases to which these thresholds correspond. The results are depicted in Figure 11. On average, for the strategy to be profitable, the 10-year lease would have to be struck around the bottom tercile or quartile of average market rental rates. A firm that created new co-working spaces every quarter, or renewed "expiring" space, would have to sustainably negotiate leases with rents in the bottom tercile or quarter, and this would likely require skill. The message is similar from Class A and B, despite the lower average profitability of Class A, because Class A leases exhibit substantially greater dispersion in effective rents, which might make it relatively easier to find "good deals" in Class A.

It is worth mentioning that the estimates in Figure 11 are likely to be optimistic in the assumption that a below-market 10-year lease can achieve average short-term rents in the same property. In practice, we expect that at least some of the

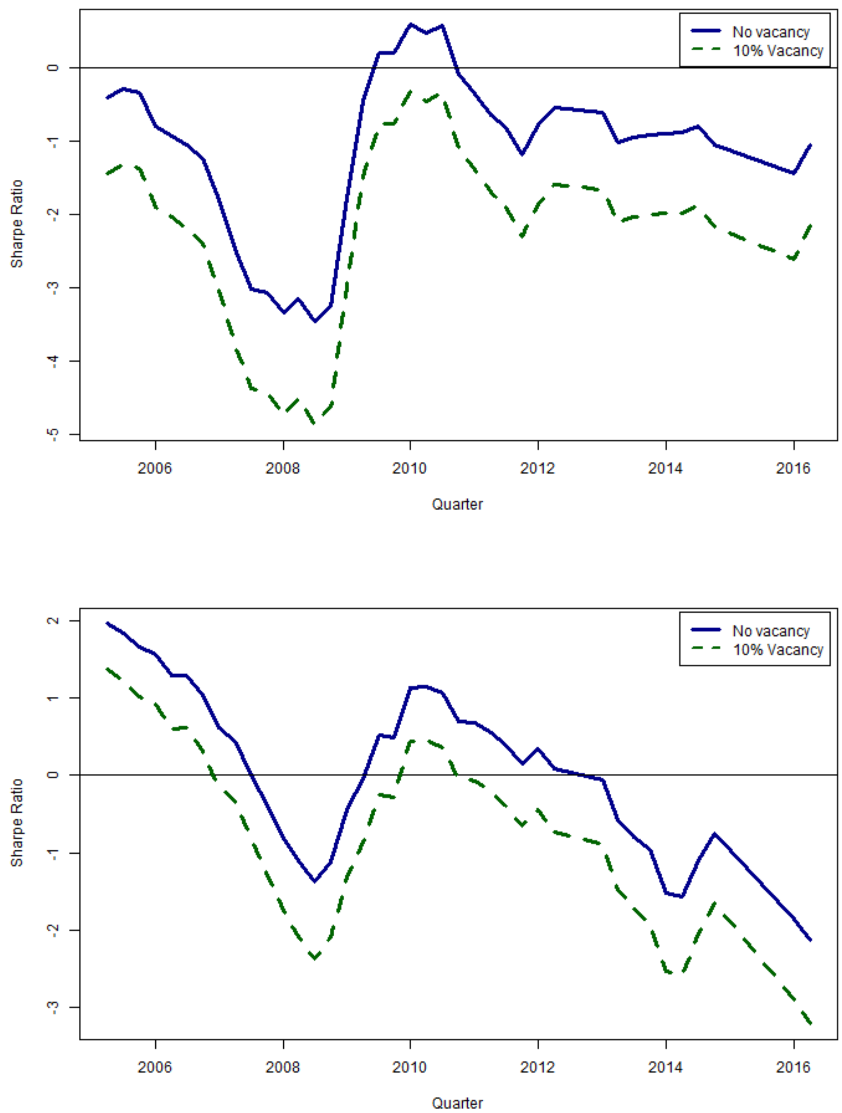
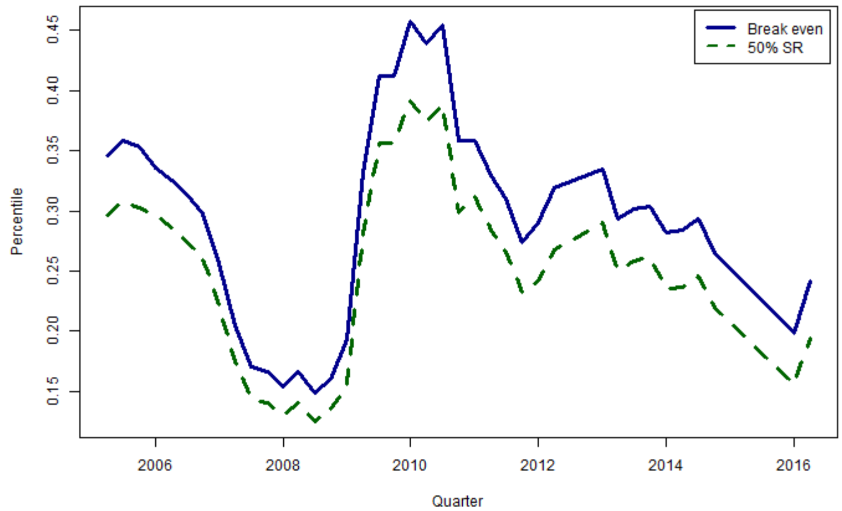


Figure 10. : **Sharpe ratios of long-short space strategy**. The figure depicts the time series of Sharpe ratios for a strategy in which a 10-year lease is financed by leasing the space on a rolling quarterly basis. Ratios for class A (B) properties are depicted in the top (bottom) panel. The Sharpe ratio is calculated by dividing the annualized expected strategy profits in a given quarter by the corresponding annualized standard deviation of the profits

observation error arises from unobserved quality characteristics within each Class. Thus, it seems likely that a building featuring a below-market 10-year lease will also be a building exhibiting below-market short-term rates.



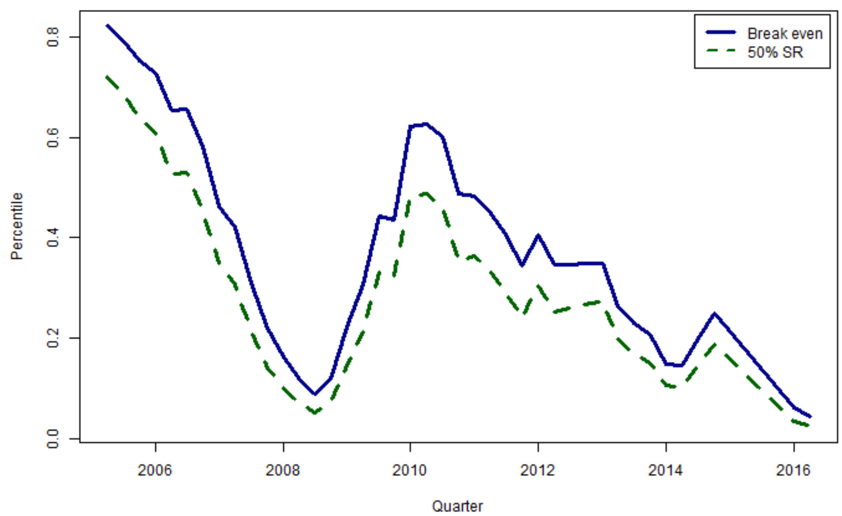


Figure 11. : **Percentage of profitable 10-year leases in a long-short strategy**. The figure displays an estimate of the proportion of Class A (top) and Class B (bottom) leases that would allow an investor to achieve different levels of profitability. For each quarter in our analysis, we calculate the the 10-year lease rate that allows the strategy to break-even (blue, solid line) or attain a 50% Sharpe ratio (green, dashed line), and show its rank in the conditional distribution of 10-year leases given that year's observation error variance (Table 5).

In our dataset, we were only able to identify 12 full-information leases corresponding to co-working tenants (six, each, from Class A and B). Lease terms ranged from 10 to 21 years. The earliest lease was signed in 2007Q3 and the latest in 2014Q1. When comparing the effective rents paid by these tenants to the break-even thresholds we calculated for their their hypothesized spatial strategies, we found that seven paid effective rents that led to better than break-even profits, and six of these paid effective rents that led to a 50% Sharpe ratio or more. All but one, however, signed leases with lower than average rates.

Figure 12 similarly depicts the threshold of intensification factor needed to achieve zero profitability or a 50% annualized Sharpe ratio at 90% occupancy. The intensification factor is used to multiply the short-term rents from the strategy. For instance, an intensification factor of 1.20 corresponds to a 20% increase in the short-term rents achieved throughout the 10-years the strategy is in play. Here, the takeaway is that Class A long-short strategies call for an intensification factor of roughly 1.25 or more if they are to be profitably implemented throughout our sample period. Because, until 2013, the strategy was generally profitable in Class B spaces, the required intensification in such spaces is smaller. Note, however, that since 2013 this has markedly changed.

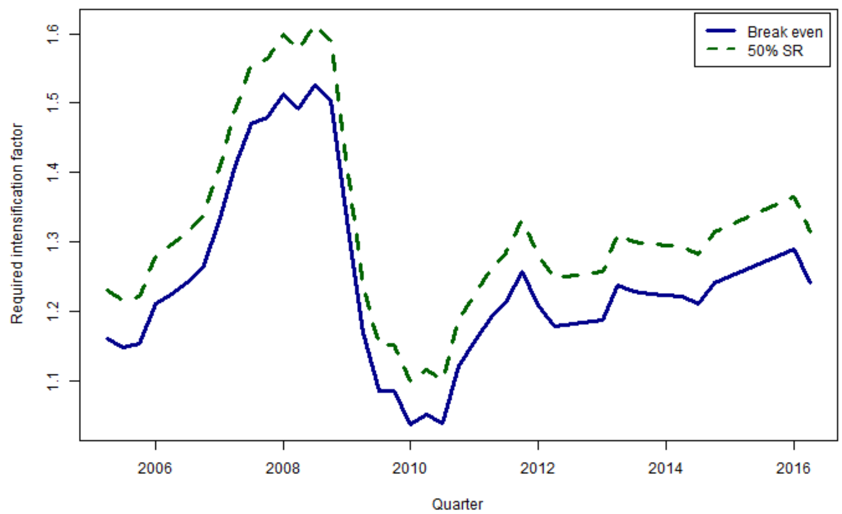
In practice, skill at obtaining space at cheaper prices and intensification of use may be combined. Overall, our analysis allows us to quantify metrics that would translate into a profitable strategy as well one that would mitigate risk. We re-emphasize that our focus here is on the spatial component of the co-working strategy. To the extent that services are key to co-working profitability, its risk profile might better resemble that of service intensive real estate (e.g., hotels).

V. Conclusion

Rental income is fundamental to the valuation and return dynamics of real estate investments, and rental lease contracts dictate effective rent terms and cash flow duration. However, relatively little work has been done to explore how the prices of newly originated leases with different maturities evolve over time. We use data on Manhattan rental transactions from Compstak during 2005-2016 to estimate a model of the evolution of the term structure of lease rates over the business cycle.

Estimated model dynamics imply a term structure of forward lease rates that is often upward sloping, and more likely (than not) to be concave. These attributes are more pronounced for Class A leases. This is consistent with high, but decelerating, rental growth expectations that are tempered by eventual same-building quality depreciation or obsolescence.

Despite allowing for three types of shocks to impact the term structure, the best model fit corresponds to one where a single factor drives lease rate dynamics. The best fit dynamics describe a market where, when the economic environment changes unexpectedly, the "shock" is felt first in long-dated components of newly issued leases and then transmitted to shorter-maturity components in subsequent quarters. Shocks, after their arrival, take a couple of quarters to be fully incorporated into leases, suggesting that the space market suffers from informational



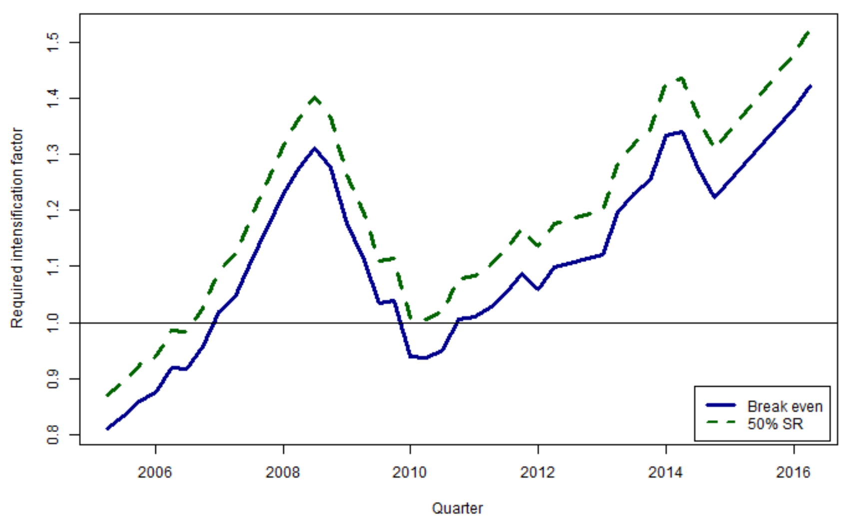


Figure 12. : **Required intensification factor for profitable long-short strategy**. The figure displays the factor by which short term lease rates must be multiplied for the strategy to achieve different levels of profitability (zero-profit or 50% Sharpe ratio). The top graph corresponds to class A properties, while the bottom one depicts the required intensification factor for class B leases. Occupancy of 90% is assumed.

inefficiencies.

Our model can be used to assess the profits and returns of leasing strategies. We apply it to conclude that the *spatial component* of co-working (i.e., acquiring space on a long-term basis and then selling it to short-term tenants) is, on its own, generally not profitable in NYC. To make co-working profitable, a long-short leasing strategy must be accompanied by advantageous acquisition of the space, and/or its usage intensification, and/or a profitable service component.

Some helpful results on model dynamics. The key rate model itself takes the form:

$$F_{t+1} = \rho F_t + \bar{F} + \Sigma \tilde{\varepsilon}_{t+1}.$$

where $F_t = (F_{t,0}, F_{t,5}, F_{t,10})'$ is the vector of key rates and ε_s is a three-vector of independent and normal Gaussian random variables. Setting $F_U \equiv (I - \rho)^{-1}\bar{F}$, one can rewrite the dynamics of the key rates as follows

$$F_{t+1} - F_U = \rho(F_t - F_U) + \Sigma \tilde{\varepsilon}_{t+1}, \tag{A1}$$

which describes the demeaned dynamics of the key rates.

The distribution of future key rates is obtained by iterating equation (A1):

$$F_{t+\tau} - F_U = \rho^{\tau}(F_t - F_U) + \sum_{j=1}^{\tau} \rho^{j-1} \Sigma \tilde{\varepsilon}_{t+\tau-j+1}. \tag{A2}$$

From this, one can immediately deduce the Gaussian distribution of $F_{t+\tau}$:

$$F_{t+\tau} \sim F_U + N\left(\rho^{\tau}(F_t - F_U),\ \sum_{j=1}^{\tau} \rho^{j-1} \Sigma \Sigma' (\rho')^{j-1}\right). \tag{A3}$$

A1. Distribution of long-short portfolio profits

To simplify matters, we assume that each rental period corresponds to a quarter (i.e., rents are paid quarterly), but rents are quoted in monthly terms. We do this to match the estimated model dynamics to the lease contract. Consider a strategy where, beginning at date t, a short position is undertaken in a long-term lease with maturity of T years, together with a rolling long position in a one-quarter lease every quarter from date t until one quarter prior to T. The question at hand: What is the expected distribution of cash flow?

To address this, first consider the fixed (short) leg of the transaction (the long-term lease) and denote the model-implied monthly lease rate at date t for a T-year lease as $\ell_t(T)$. The forward equivalence relation implies that

$$3 \sum_{m=1}^{4\times T} d_{j,t} v_j' \cdot F_t = 3\ell_t(T) \sum_{m=1}^{4\times T} d_{j,t},$$

where $d_{j,t}$ is the price of a j-quarter strip bond at date t and v_j is the vector of weights applied to the key-rate vector, F_t to arrive at a j-month forward price (e.g., $v_1' = (0.05, 0.95, 0)$ and $v_{30}' = (0.5, 0.5, 0)$). The factor of 3 corresponds to the fact that the lease rate is quoted as monthly but each lease period is a quarter. Thus,

$$\text{(A4)} \qquad \ell_t(T) = \frac{\sum_{m=1}^{4\times T} d_{j,t} v_j' \cdot F_t}{\sum_{m=1}^{4\times T} d_{j,t}}.$$

Each quarter starting at date t the strategy rolls over a new one-quarter lease. The profit at date $t+\tau$ is therefore $3(v_1' \cdot F_{t+\tau} - \ell_t(T))$. To assess the overall strategy profitability and its riskiness, one has to settle on a convention for accumulating them over time. This could be done by discounting the strategy cash flow to date t, capitalizing it forward to date $t+T$, or even using a simple sum. Each approach has its benefits and shortcomings. Consider for now the sum of strategy profits, discounted to date t:

$$\text{(A5)} \qquad P(t,T) = 3\sum_{\tau=1}^{4T} d_{\tau,t}\Big(v_1' \cdot F_{t+\tau-1} - \ell_t(T)\Big).$$

Correspondingly, from equation (A2),

$$\text{(A6)} \qquad E_t[P(t,T)] = 3\sum_{\tau=1}^{4T} d_{\tau,t}\Big(v_1' \cdot \big(F_U + \rho^\tau(F_t - F_U)\big) - \ell_t(T)\Big).$$

To calculate the variance, first observe that

$$\begin{aligned}
F_{t+1} - E_t[F_{t+1}] &= \Sigma\tilde{\varepsilon}_{t+1} \\
F_{t+2} - E_t[F_{t+2}] &= \rho\Sigma\tilde{\varepsilon}_{t+1} + \Sigma\tilde{\varepsilon}_{t+2} \\
F_{t+3} - E_t[F_{t+3}] &= \rho^2\Sigma\tilde{\varepsilon}_{t+1} + \Sigma\rho\tilde{\varepsilon}_{t+2} + \Sigma\tilde{\varepsilon}_{t+3} \\
&\vdots \\
F_{t+S} - E_t[F_{t+S}] &= \rho^{S-1}\Sigma\tilde{\varepsilon}_{t+1} + \Sigma\rho^{S-2}\tilde{\varepsilon}_{t+2} + \rho^{S-3}\tilde{\varepsilon}_{t+3} + \ldots + \Sigma\tilde{\varepsilon}_{t+S} \\
F_{t+\tau} - E_t[F_{t+\tau}] &= \sum_{\tau'=1}^{\tau} \rho^{\tau-\tau'}\Sigma\varepsilon_{t+\tau'}
\end{aligned}$$

From this telescoping pattern, one can rewrite any sum over demeaned forwards as a sum over distinct shocks:

$$\sum_{\tau=1}^{S} w_\tau' \cdot \big(F_{t+\tau} - E_t[F_{t+\tau}]\big) = \sum_{\tau=1}^{S}\Big(\sum_{\tau'=\tau}^{S} w_{\tau'}' \cdot \rho^{\tau'-\tau}\Big)\Sigma\tilde{\varepsilon}_{t+\tau}.$$

Because the shocks are orthogonal, one can calculate the variance as:

$$\mathrm{VAR}_t\Big[\sum_{\tau=1}^{S} w'_\tau \cdot \big(F_{t+\tau} - E_t[F_{t+\tau}]\big)\Big] = \sum_{\tau=1}^{S}\Big(\sum_{\tau'=\tau}^{S} w'_{\tau'} \cdot \rho^{\tau'-\tau}\Big)\Sigma\Sigma'\Big(\sum_{\tau'=\tau}^{S} w'_{\tau'} \cdot \rho^{\tau'-\tau}\Big)'$$

Returning to the calculation of the profit variance using equation (A2), identify w_τ with $3d_{\tau,t}v_1$ and set $S = 4T$ to yield

$$\text{(A7)} \qquad \mathrm{VAR}_t[P(t,T)] = 9\sum_{\tau=1}^{4T}\Big(\sum_{\tau'=\tau}^{4T} d_{\tau',t}v'_1 \cdot \rho^{\tau'-\tau}\Big)\Sigma\Sigma'\Big(\sum_{\tau'=\tau}^{4T} d_{\tau',t}v'_1 \cdot \rho^{\tau'-\tau}\Big)'.$$

NOTE: To calculate these statistics for the forward capitalized profits we would replace $d_{\tau,t}$ with $\frac{d_{\tau,t}}{d_{4T,t}}$. If the aggregated strategy profits are measured as a simple sum, we'd replace $d_{\tau,t}$ with 1.

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REFERENCES

Agarwal, Sumit, Brent W. Ambrose, Hongming Huang, and Yildiray Yildirim. 2011. "The term structure of lease rates with endogenous default triggers and tenant capital structure: Theory and evidence." *Journal of Financial and Quantitative Analysis*, 46(2): 553–584.

Ambrose, Brent W., and Yildiray Yildirim. 2008. "Credit risk and the term structure of lease rates: a reduced form approach." *The Journal of Real Estate Finance and Economics*, 37(3): 281–298.

Ambrose, Brent W., Patric H. Hendershott, and Małgorzata M. Kłosek. 2002. "Pricing upward-only adjusting leases." *The Journal of Real Estate Finance and Economics*, 25(1): 33–49.

Benjamin, John D., Glenn W. Boyle, and C. F. Sirmans. 1990. "Retail Leasing: The Determinants of Shopping Center Rents." *Real Estate Economics*, 18(3): 302–312.

Bond, Shaun A., Pavlos Loizou, and Patrick McAllister. 2008. "Lease Maturity and Initial Rent: Is There a Term Structure for UK Commercial Property Leases?" *The Journal of Real Estate Finance and Economics*, 36(4): 451–469.

Brennan, Thomas P., Roger E. Cannaday, and Peter F. Colwell. 1984. "Office rent in the Chicago CBD." *Real Estate Economics*, 12(3): 243–260.

Chegut, Andrea, and Mike Langen. 2019. "The Financial Impacts of Coworking: Rental Prices and Market Dynamics in the Commercial Office Market." *Available at SSRN 3481142*.

Chiang, I-Hsuan Ethan, W Keener Hughen, and Jacob S Sagi. 2015. "Estimating oil risk factors using information from equity and derivatives markets." *The Journal of Finance*, 70(2): 769–804.

Clapham, Eric, and Åke Gunnelin. 2003. "Rental expectations and the term structure of lease rates." *Real Estate Economics*, 31(4): 647–670.

Englund, Peter, Åke Gunnelin, Martin Hoesli, and Bo Söderberg. 2004. "Implicit forward rents as predictors of future rents." *Real Estate Economics*, 32(2): 183–215.

Fang, Fang, and Lu Ruichang. 2009. "Is there a term structure? Empirical evidence from shanghai office rental market." *International Real Estate Review*, 12(1): 23–38.

Grenadier, Steven R. 1995. "Valuing lease contracts A real-options approach." *Journal of Financial Economics*, 38(3): 297 – 331.

Grenadier, Steven R. 2005. "An equilibrium analysis of real estate leases." *The Journal of Business*, 78(4): 1173–1214.

Gunnelin, Åke, and Bo Söderberg. 2003. "Term structures in the office rental market in Stockholm." *The Journal of Real Estate Finance and Economics*, 26(2-3): 241–265.

Hendershott, Patric H, Colin M Lizieri, and Bryan D MacGregor. 2010. "Asymmetric adjustment in the city of London office market." *The Journal of Real Estate Finance and Economics*, 41(1): 80–101.

Hendershott, Patric H, Colin M Lizieri, and George A Matysiak. 1999. "The workings of the London office market." *Real Estate Economics*, 27(2): 365–387.

Hüttel, Silke, Matthias Ritter, Viacheslav Esaulov, and Martin Odening. 2016. "Is there a term structure in land lease rates?" *European Review of Agricultural Economics*, 43(1): 165–187.

McConnell, John J., and James S. Schallheim. 1983. "Valuation of asset leasing contracts." *Journal of Financial Economics*, 12(2): 237–261.

Miller, Merton H., and Charles W. Upton. 1976. "Leasing, buying, and the cost of capital services." *The Journal of Finance*, 31(3): 761–786.

Schallheim, James S., and John J. McConnell. 1985. "A model for the determination of "fair" premiums on lease cancellation insurance policies." *The Journal of Finance*, 40(5): 1439–1457.

Stanton, Richard, and Nancy Wallace. 2009. "An empirical test of a contingent claims lease valuation model." *Journal of Real Estate Research*, 31(1): 1–26.

Webb, R. Brian, and Jeffrey D. Fisher. 1996. "Development of an effective rent (lease) index for the Chicago CBD." *Journal of urban Economics*, 39(1): 1–19.

Wheaton, William C., and Raymond G. Torto. 1994. "Office rent indices and their behavior over time." *Journal of urban Economics*, 35(2): 121–139.

Yoshida, Jiro, Miki Seko, and Kazuto Sumita. 2016. "The rent term premium for cancellable leases." *The Journal of Real Estate Finance and Economics*, 52(4): 480–511.