Do Different Price Points Exhibit Different Investment Risk and Return in Commercial Real Estate?*

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Abstract

Conventional real estate price indices provide a single measure for the path of asset prices over time (controlling for the quality of the representative or average property). But it could be that properties have different price dynamics based on the price segment they are traded in. On the demand side, investors at different price points are differentiated by the amount of capital the investor has at their disposal and the type and source of financing. Smaller, private investors cluster at lower price points, while large institutions dominate the high price points. On the supply side, properties at different price points may serve different space markets with different types of tenants, and may reflect different supply elasticity and land/structure value ratios. This paper uses an unconventional approach, quantile regression, to estimate price indices for different price segments in commercial real estate. Our results show that there are indeed large differences in price dynamics for different price points. These differences are suggestive of a lack of integration in the property asset market, because we find apparent differences in the risk/return relationship. Lower price point properties exhibit less risk (in the form of volatility and cycle amplitude), but without evidence of lower total returns. Lower price point properties also show greater momentum and hence, predictability.

KEYWORDS: commercial real estate; quantile regression; chained hedonic index; investment property; equilibrium asset pricing; price of risk.
JEL-codes: R32, C01.

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1 Introduction

Asset price dynamics are a fundamental and important consideration for real estate investors. Price dynamics include characteristics such as the long-term trend, volatility, momentum, cyclicality, and co-movements in property prices. These characteristics are important for portfolio analysis including both strategic and tactical investment planning. They largely determine the nature and amount of investment risk. Combined with income metrics such as cap rates or yields, price dynamics determine the total returns that investors care about and relate to the investment risk. Average total returns should reflect investment risk.

What we know about commercial property price dynamics is based on price indices such as NCREIF, Real Capital Analytics, Green Street, NAREIT, etc. These indices reflect aggregate price movements, or central tendencies, in the sense that they generally track with a single index the entire asset class or large sectors of it or entire metro markets. Such aggregate indices represent properties that span a wide range of different price points for investors, from properties that sell for only a few million dollars to those that sell in the hundreds of millions. Yet many institutional investors aim at different price points. Sovereign Wealth Funds (SWF) may only consider very large (high price point) properties, while small institutions and private investors may be confined to smaller (lower) price points. It is also possible in principle to create funds that target different price points, for example, by aggregating portfolios of many small properties, or by dividing up shares in fewer larger properties. Thus, price point analysis has important practical value. Figure 1 shows the proportion of purchases at different price ranges made by different categories of investors, as classified by Real Capital Analytics (RCA). Institutional investors, public REITs, and foreign investors almost solely invest in the higher price points, while private investors and owner/occupiers invest primarily at the lower price points.

The present paper aims to disaggregate the price indices much in the way that a prism breaks white sunlight into its constituent colors. We develop price indices that explicitly, separately track properties at different price points. This allows a more detailed and informed look at whether, and how, price dynamics differ by price point. We also analyze cap rates corresponding to the different price points. This enables an exploration of risk and return by price point, as the risk is largely reflected in the price dynamics, and the total return is reflected in a combination of price trend and income yield. As our analysis is based on the Real Capital Analytics (RCA) sample of transactions over $2,500,000, we do not consider the smallest "general" (or "mom-&-pop") properties. Our analysis is focused on the realm of 'investment' or 'institutional' real estate.

Another way to think about what the analysis in this paper does is that we quantify the *full* distribution of investment property prices at each period of time, and we form price-point

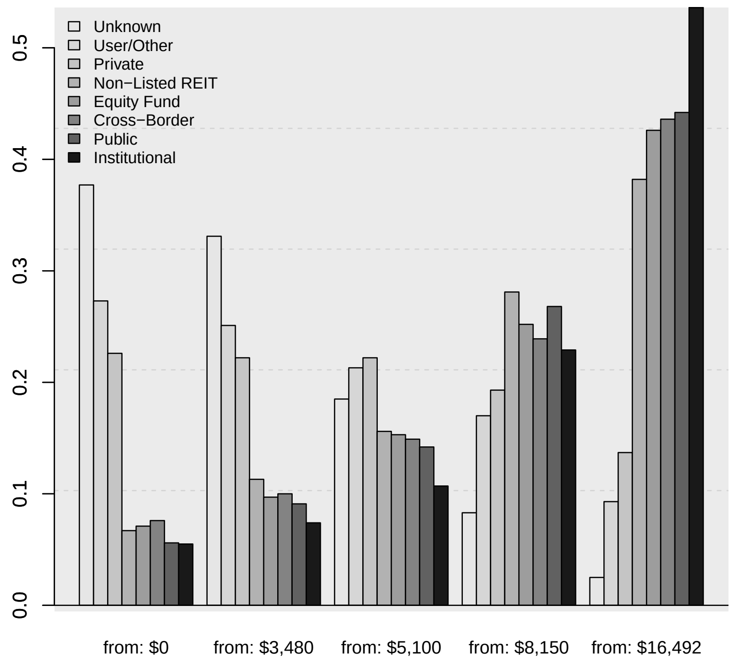


Figure 1: Percentage of properties (not total value) bought by type of investor, per five price-buckets. The starting value of the bucket is given $\times 1,000$. For example, the price of over 50% of all the properties bought by institutional investors was $16.5 million or more between 2005 – 2015. Source of data: Real Capital Analytics.

specific indices from these ditributions. We produce annual indices for the US between 2005 and 2015. This is a particularly interesting span of history because it includes the financial crisis of 2007 – 2010.

Technically, we use the approach developed by Machado and Mata (2005) to estimate the different price indices for commercial real estate based on different price points. In essence, the indices are produced by connecting the same quantile of the price distribution from one period of time to the next.

Our results show a clear pattern. The quantile indices reveal that higher price points in the property distribution have more volatility and greater cyclical amplitude over time, less autocorrelation and move first, compared to the lower price points. While the higher price points have also exhibited greater mean appreciation, this difference is very slight in relation to the difference in apparent risk and predictability. The spread between the high and low price points' average year-to-year price growth is less than 20 basis points. But the cap rate is some 40 basis points lower for the high price point properties in our analyzed period. Thus, the total return is actually higher for the apparently lower risk low price point properties.

The analysis in this paper is closely related to the economic literature on changes in household earnings inequality, as well as studies of price dynamics in the art market, and urban economic studies of housing markets. For example, Buchinsky (1994) and Dickey (2007) find that the earnings of high-income workers have risn more than that of low-income workers. Scorcu and Zanola (2011) find that the same Picasso art-collectors appreciate differently the characteristics of low- and high-price items. Picassos that reach an extraordinary monetary value, the "top lots", appreciated far more than other works of the same artist between 1988 and 2005. However, our work is most closely related to the work done by McMillen (2008). He uses the same Machado and Mata (2005) decomposition that we use, for housing in Chicago for two periods, 1995 and 2005. He concludes that high-priced homes in Chicago appreciated the most between 1995 and 2005. The higher appreciation rates are explained solely by differences in the hedonic price model coefficients between those two dates, the so-called "shadow prices". In other words, it was pure price change, not change in the quantitative or qualitative characteristics of the houses, that drove the differential pricing results. (It was not that houses in the upper-end of the price distribution became relatively larger in size over time, for example.)

The present paper is structured as follows. The background, including a discussion of the possible theoretical reasons for price segmentation in the commercial property market, is given in Section 2. Section 3 provides the empirical analysis methodology. Section 4 describes the data and gives some descriptive statistics. In Section 5 the results are presented, and Section 6 concludes.

2 Background

In real estate, a standard hedonic or repeat-sales price index, or indeed an appraisal-based index for that matter, provides a single measure for the movement in market values of a subject population of properties during any period of time (McMillen, 2008). This should be adequate provided the subject population represents a single, integrated asset market, a single "population" of properties in terms of pricing.

Fundamental micro-economic theory tells us that, within a single market, the "Law of One Price" should prevail. That is, the same good cannot sell at different prices at the same time within the same market. Otherwise, "arbitrage" would be possible, and in a well-functioning, integrated market, the exploitation of any such arbitrage opportunities would drive prices to where no such opportunities exist. Thus, a single index tracking the price of the good in question is all that is needed (Modigliani and Miller, 1958; Rosen, 1974; Freeman III, 1979).

In the context of the asset market for investment properties, the “goods” being traded are, essentially, assets that provide investment risk and return. The Law of One Price applies to the price of risk in these assets, the ex ante investment return premium (over the riskfree rate) per unit of investment risk (as the market perceives and cares about, hence, prices, such risk). That price of risk will be evident in the risk and return quantified by a combination of the income yield and the price dynamics of the population of properties that is the subject of the price index. In particular, the price dynamics will reveal the volatility and cyclicality in the asset returns, the major sources of risk. The average income yield (approximated by the average cap rate) can be combined with the average price growth to provide the average total return, which is how the investor is compensated for taking on the investment risk.

But suppose the subject population of properties does not represent a single, integrated market. This could happen in two different ways that are important for us to distinguish in this paper.

First, we could simply have different “goods”, different products being traded, but no barriers to cross-trading between the market segments. This might typically occur across different space markets, for example, different property usage type sectors or different metro markets. But if the properties are all at similar price points, such that similar investors and sources of capital are involved, then there should be little in the way of barriers impeding trading between the market segments. In that case we might observe different price dynamics in the segments, but we should not observe a different price of risk. The different space markets might drive one market to have a higher growth trend and lower cap rate, for example. Or different space markets could result in different volatility or cyclicality in the asset prices. This could imply different amounts of risk, hence different equilibrium total return expectations. But the relationship between risk and return should be very similar across the market segments. That is, they should all have the same risk premium per unit of risk, the same ”Treynor Ratio,” in principle. Otherwise, investors would trade across the market segments, selling out of the markets with lower returns per unit of risk and buying into the markets with higher returns per unit risk. This would bid down and up the prices in the respective markets until equilibrium (the same price of risk) prevailed.

Such a situation could also exist in the context of price-point indexing. We might observe different price dynamics, separate price-point indices that do not appear as “parallel” lines over time. But the risk/return relationship would be similar across the price-point indices. Indeed, it could be differences in the space markets in which the different price point properties rent which drive the different asset price dynamics. For example, lower price properties might rent to a different class of tenants with different demand drivers.

This type of market segmentation, effectively trading different ”goods”, is very important for investors to understand. It can be very important in portfolio planning. For one thing, it can suggest ways to efficiently diversify the real estate investment portfolio. The study of such segmentation, differential price dynamics, as a function of price points, is the first focus of this paper.

But now consider the second way in which the population of properties tracked by an index might not represent a single, integrated market. Suppose there are barriers to the movement of capital between properties in the different market segments. In the context of the current paper, there could be barriers to capital flow across the different price points. For example, investors in small properties might not be able to invest in large properties, and investors in large properties might not be able to invest in small properties. In this case the property asset market would not only be segmented, but would be what economists refer to as “not integrated.”

When market segments are not integrated, the Law of One Price need not then hold across the price-point segments (or sub-markets). In such a situation we could have multiple equilibria, a different equilibrium price prevailing in the different segments. Not only would the price-point indices probably display different price dynamics (non-parallel lines), but more to the point, the risk/return relationship would be different between the price points, a different price of risk. Different Treynor Ratios could prevail in the different segments.

Clearly, one motivation to study this question is that if different pricing exists in different market segments, then arbitrage opportunities may exist for investors who can figure out how to break through the barriers that prevent the market from integrating.

With the above in mind, let us consider why, or how, different price points of investment property could exhibit different pricing dynamics. There could be several reasons or sources for such a phenomenon, relating to the demand or supply sides of the space or asset markets. Some of these causes do not necessarily or particularly imply differential pricing of risk, but merely different price dynamics. Others possibilities could indeed suggest barriers or asymmetries which could support differential risk pricing.

First, consider the property asset market. There is evidence that different types of investment institutions dominate acquisition markets at different price points. This is true both regarding debt as well as equity capital. In the case of equity, our data shows that institutional investors, public REITs, and foreign investors are responsible for the majority of the purchases at the higher price points. Rather different types of entities, private investors and owner/occupiers, dominate at the lower price points. There are similar distinctions in the debt market. Institutions, life insurance companies, and large national and international banks dominate for large loans, while local and regional commercial banks dominate for small loans. The large institutions buying at the high price points are often tax-exempt and may have less need or desire for debt financing.

These different types of investors may have different investment objectives and constraints, as well as different perceptions and knowledge underlying their expectations. The marginal investors at the various price points may differ sufficiently in objectives, constraints, and outlook, such that a different equilibrium risk/return relationship could hold, provided barriers to capital flow prevent the arbitraging away of such differences.

Now consider the space market. Different price dynamics in the asset market could be attributed in part to different supply elasticity in the space market at different property price points. Smaller, less expensive properties are easier (and quicker) to supply (through the real estate development process). Greater supply elasticity enables faster and larger responses to increases in demand, which prevents rents from increasing as much (Harter-Dreiman, 2004; Saiz, 2010), which in turn mitigates against rents falling as far, making such properties less volatile or cyclical in their fundamentals.

Another consideration regards the characteristic type of tenants in lower versus higher price point properties. The tenant base of cheaper properties is on the whole arguably riskier than the tenants of expensive properties, as larger properties tend to attract credit tenants with more established access to capital. Even if this difference is only true in perception, it may cause the asset market to price less expensive properties at higher cap rates (income yields), other things equal, to compensate for higher perceived rental risk. (Tenant lease defaults are likely to be correlated with financial conditions, hence impossible to fully diversify against.)

In the asset market, if lower price properties tend to have higher cap rates (for example due to lower long-run growth expectations), then this in itself will also tend to make such properties less volatile, by making them less vulnerable to swings in real interest rates. A given shock to real interest rates will produce a lower percentage change in property value in higher cap rate properties. For example, a 100 basis-point shock in cap rates from 5% to 6% produces a 17% drop in asset value other things equal while the same 100 basis-point shock from 8% to 9% produces only an 11% drop in value.

Yet another consideration may be that lower priced properties tend to be located on lower land value sites, which means that more of the built property value is attributable to the structure, less to the land, proportionately (Bourassa et al., 2006). Land is more volatile in value than are structures because land is less elastic in its supply than is construction. This may be the fundamental reason for greater price elasticity of supply in lower-priced properties, hence, lower volatility. (See Davis and Heathcote, 2007; Davis and Palumbo, 2008; Glaeser et al., 2008, for examples in housing.)

The above supply-side and space market considerations could result in different price dynamics across price point segments. But in themselves these considerations do not necessarily imply different risk/return equilibrium pricing. Asset prices could adjust so as to present the same relationship between risk and expected total return (same Treynor Ratio).

For example, higher growth markets could exhibit higher asset prices, lower cap rates, keeping total returns the same (if risk is the same). Indeed, in theory such adjustment would be expected to happen unless there are barriers that prevent capital flowing freely between the price points. However, if capital flow barriers do exist, then differential price dynamics across the price point market segments could be associated with different risk/return pricing. Lower risk price points might not exhibit sufficiently higher prices to bid down the cap rates and total returns, leaving the return risk premium per unit of risk higher than at higher risk price points.

This last possibility is the second major focus in this paper. After identifying the nature of price-point pricing dynamics, we study risk/return pricing by analyzing total returns across the price points, considering both growth and yield. We quantify a simple version of the Treynor Ratio for the various price points.

But first, the threshold empirical question is simply whether different price points exhibit different price dynamics, and the nature of any such differences. This is the indexing question to which we turn in the next section.

3 Model

3.1 Methodology for Price Point Indexing: Traditional and Quantile Approaches

As noted, traditional real estate price indexing methodology produces a single index value or return in each period of time. This single value aims to track a central tendency or average value within the subject population of properties. From the point of view of analyzing price dynamics, the subject population of properties is treated like a single, integrated market. With traditional index methodology, the only way to identify and track different price point dynamics is to segment the sample of properties. We can group the sample into two or more sub-samples based on price level, and construct separate indexes for each sub-sample, using a traditional price index methodology like the repeat-sales model or the hedonic model.

This approach can be interesting, and it is one of the approaches we shall pursue in this paper. However, there are some disadvantages to this approach. First of all, there is considerable dispersion in property transaction prices, and this makes such indices vulnerable to estimation error and noise (Francke, 2010; Bokhari and Geltner, 2012; Francke and Van de Minne, 2016). This puts a premium on the size of the sample, with larger samples being necessary to get sufficiently accurate and reliable indices. When we break up the estimation sample into price point segments, we reduce the effective sample sizes on which the indexes can be based.

Secondly, conventionally constructed price-point indices are either biased, or not truly constant quality (that is, not truly controlling for differences in the properties that trade in one period versus another). If one subdivides the data by price based on the total sample, properties sold just after the crisis might all fall in the lowest 'price bucket', even though they might actually have been relatively highly valued within that down-market period of time. This can give a serious bias to the indices.

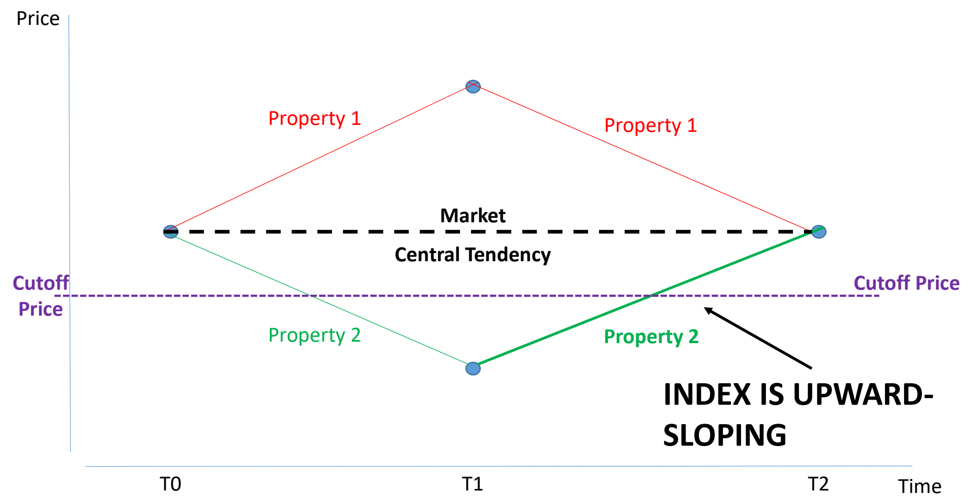
Subdividing the properties into different price buckets on a year by year basis would solve that particular issue, but would create new problems. Investors may 'flee to quality' during an overall market downturn and vice versa. For example, in the previous illustration, one would subdivide properties sold directly after the crisis into three equal buckets. However, in reality all transacted properties might have been relatively high valued (as investors 'fled to quality' during the down market). In this case, the index would not be truly constant quality anymore. We would be comparing "apples" (on average "regular quality" properties before the downturn) with "oranges" (only "high quality" properties during the downturn).

Furthermore, conventional indices are susceptible to bias that can be caused specifically by stratifying the estimation sample based on price, which is what we need to do to produce price point indices with traditional methods. To see this, consider Figure 2. The Figure shows a stylized representation of the effect of random dispersion in property transaction pricing within a single, integrated property market. Such idiosyncratic dispersion means that any given transaction is as likely to be priced "high" as "low", relative to the "true" or mean (central tendency) of the market pricing. (Fundamentally this is because no one ever knows the exact "true" market value of any given property at any given time, and so any given "deal" or negotiation can end up either above or below the central tendency.)

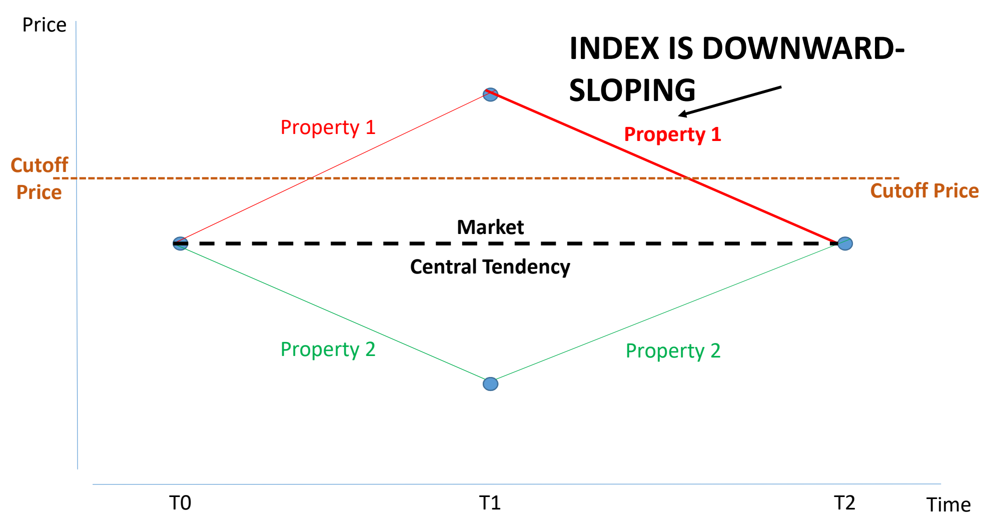
The Figure shows two identical properties transacting at three consecutive points in time labeled T0, T1, and T2. The random price dispersion is represented by the deviation of Property 1 above, and Property 2 below, the market value or central tendency in the market in Period T1. Such idiosyncratic random price dispersion probably tends to be mean reverting or self-correcting over time, as it would be difficult for individual properties to deviate ever father and farther from the market central tendency. We represent this by the two properties both transacting exactly at the market value central tendency in periods T0 and T2, with the dispersion appearing only in T1.

The lower panel depicts the segmentation of the transaction sample to construct a high price point index. A lower bound is placed on the value of the first sales for the same-property transaction pairs that are reflected in the index, so that the index represents a high price segment. This results in a downward bias for the price index for that high price point. The opposite situation is represented in the top panel, which depicts the segmentation for the low price point index, in which an upper bound has been placed on the acceptable first-sale price, in order to represent a low price segment. The result is an upward bias in the low price point index. This type of bias would exist in either a repeat-sales or hedonic index (and even in an appraisal based index), as it is due fundamentally to the truncation, or censoring, of the underlying sample of properties.

Figure 2: Segmentation bias.



(a) Upper-bound cutoff.



(b) Lower-bound cutoff.

There is evidence that this type of censored sample bias is not of serious concern at the aggregate level, for tracking the central tendency of a subject population of properties. But for constructing price point indexes aimed specifically at studying possible differential price dynamics among price point segments, this problem could be more serious, especially at the top and bottom extremes of the price points.

3.2 Quantile Hedonic Index

To circumvent such criticisms regarding price-point indexing with conventional indexing techniques, we study price point dynamics by employing a new and less conventional approach. We employ quantile regression estimation together with representative property simulation. In the context of property price indexing, the quantile regression methodology lends itself particularly well to the price point dynamics question that is our focus. While quantile price indexes lack the direct investor experience basis of repeat-sales indexes, they can address some of the other problems we have described for the traditional price indexing methods. Quantile indexes as developed here effectively separately track the performance of properties at any of a range of price points.

The methodology of price point indexing based on imputed (or “chained”) hedonic quantile regressions that we develop in this paper will be described in detail below. The essence that should be noted here at the outset is that the full transaction sample is used in each period to separately estimate the full distribution of pure prices, defined by the coefficients on the property hedonic characteristics. The notion of ”pure prices” implies value changes holding the ”quantity” of the property constant, where ”quantity” reflects the qualitative (hedonic) attributes of the property, for example, the size or age of the property. These pure prices are estimated based on the coefficients of the hedonic price model, also referred to as ”shadow prices”, or ”implicit prices”.

The estimated distribution of implicit prices (hedonic coefficients) is then applied to a “representative property” that is in principle held constant across all periods. In other words, the ”quantities” of the property attributes (the hedonic variables) will remain constant, so that value changes over time in the indices reflect only changes in the attribute shadow prices. The representative property is defined to reflect the attribute quantities of the average property in the overall transaction sample.

Mechanically, for technical practicality considerations, this operation of pairing shadow prices with representative quantities is done by random sampling with replacement, in effect, simulation, of both the period-specific price distribution and the constant property characteristics distribution, each period. This creates a distribution, in each period, of property values, which can effectively represent price points.

Price point indexes can then be constructed, for any price point, defined as a quantile of the value distribution, by connecting the values over time of the same quantile in the value distributions of each period. (Recall that "values" are pure prices times pure quantities, the product of the time-varying shadow price vector times the longitudinally fixed vector of the representative property's attribute quantities.)

As noted, the type of quantile regression based price point indices that we develop in this paper do not exactly or directly mimic investor experiences by tracing literally the same properties across time. However, they effectively control for the most important types of "apples vs oranges" differences that can cause problems in price point indexing. (And as noted, traditional index methodologies that ostensibly track same properties across time also suffer from biases that undercut the "same property" ideal.) To better understand what is, and is not, controlled for in the quantile regression based price point indexing procedure we use, consider two illustrative examples.

First, suppose that one of the hedonic characteristics in the price model is a New York dummy-variable, and it gets a positive coefficient (NY properties sell at a price premium). Suppose that that NY coefficient remains constant every year when we re-estimate the hedonic model each year on the transactions of that year. Suppose in Year "t" there are almost no NY properties sold, and in Year "$t+1$" a whole lot of NY properties are sold but otherwise the "t" and "$t+1$" samples are very similar. If in constructing our quantile price index we did not hold "quantity" (the characteristics $\hat{x}$ vector) constant between "t" and "$t+1$", this difference in proportion of NY properties in the sample would tend to cause a NY property to "jump buckets" (change quantile) downward from a higher to lower quantile between "t" and "$t+1$", as its NY price premium gives it less higher rank in the distribution of "$t+1$" sales because there are so many other NY properties that year. However, because we DO hold "quantity" constant in constructing the index, the index for the quantile in which our NY property finds itself in "t" would (probably approximately) well represent the actual price-change of our NY property between "t" and "$t+1$".

Now consider a counter-example, of bucket-jumping that we cannot control against in the quantile indices. Suppose the same situation as before, only now assume similar proportion of NY properties traded both years, but now assume that the coefficient on the NY dummy-variable increases substantially from "t" to "$t+1$". In other words, the market values NY properties a lot more (relatively speaking) in $t+1$ than it did in t. Otherwise things are very similar between "t" and "$t+1$". In this case, our NY property would jump UP to a higher quantile in $t+1$ than it was in t (surpassing some otherwise fine non-NY properties in the distribution that had been above it in "t"). In this case, the quantile price index for the quantile in which our property found itself in "t" would not be a good representation of what actually happened to the price of our NY property between "t" and "$t+1$".

Thus, the idea of the quantile index is to track the performance of properties at each price point, at each period of time. It is not generally to track the performance of individual (same) properties across time, such as governs the experience of investors. Properties can change their price quantiles over time, but price-points do not (as we define them in our quantile indices). However, usually price coefficients do not change drastically relative to other price coefficients (of other attributes) from one period to the next. Most properties don't significantly jump across quantiles from one period to the next. So, the quantile index gives an approximation of same-property price-point based performance. Moreover, there is some interest in tracking price-point performance as distinct from same-property performance. Price-point performance reflects the functioning of price-point market segments, consistently reflecting the same price-point segments across time.

One way to "correct" or address the concern raised for investors by the second New York illustrative example above, is that we can try to construct quantile indices at a more granular level, so that all the properties in the samples over the years are more homogeneous (e.g., in the above illustration ALL & ONLY NY properties would be included in the indices). In Section 5.4 in the Results section of this paper, we summarize the key findings from applying the quantile model to a granular set of indices, estimated separately for property type sectors, and for 18 specific metro markets.

A second approach to addressing any concern about the quantile index not being a same-property index is to indeed follow the traditional index methodology described at the outset of this paper. In part as a robustness check, and to compare the two approaches, we also present this traditional approach in this paper. In Section 5.1 of the Results section we estimate classical repeat-sales indices based on price-bucket stratified sub-samples (hypothesized potential price-point market segments). The sub-samples are defined by the price (bucket) of the first-sale in the repeat-sale pairs.

Both of these approaches are trying to represent something closer to the actual price-change experience of investors, holders of individual (same) properties across time. The Repeat-sale approach models this experience directly, but faces problems of limited sample sizes with also some possible sample selection bias (including the asymmetric censoring of the sample by conditioning on a bounded first-sale price: biasing low price-point indices upward and high price-point indices downward). The granular price-point indices based on quantile hedonic regressions are not literally same-property indices, but they should significantly mitigate the major potential sources of any significant differences between price-point indices and same-property indices.

In order to estimate our quantile price indices we use the multi-step procedure proposed by Machado and Mata (2005), in combination with imputed (or "chained") hedonic models (de Haan and Diewert, 2011). The three steps are presented in depth in the following sub-sections.

3.2.1 *Step 1. Quantile hedonic model*

First we estimate a Quantile hedonic model on a period-by-period basis (similar to the "chained" hedonic methodology, see: de Haan and Diewert, 2011). Quantile regression produces estimates of the entire distribution of the predicted (dependent) variable, based on the minimization of weighted absolute errors (rather than the minimization of squared errors in traditional OLS regression). The predicted variable is the transaction prices of the traded properties. The quantile regression analysis models the τ^{th} conditional quantile of p given x as:

$$Q_{p_{it}}(\tau|x_{it}) = x_{it}^T\beta(\tau) \qquad \text{for } t = 1, \ldots, n_t \tag{1}$$

where p is the transaction prices of the traded properties, and x is the vector of hedonic attributes (quantities) in the traded properties. (1) is equivalent to the standard OLS hedonic model with $Q_{\epsilon_{it}}(\tau|x_{it}) = 0$. The τ-specific coefficient vector $\beta_t(\tau)$ for every t is estimated by minimizing the loss function:

$$\min_{\beta_t(\tau)} \sum_{i=1}^{n_t} \rho_\tau(p_{it} - x_{it}^T\beta_t(\tau)) \tag{2}$$

separately for every t, where $\rho_\tau(u) = \mu\tau$ if $\mu \geq 0$, and $\rho_\tau(u) = \mu(\tau - 1)$ if $\mu < \mu$; see Koenker and Bassett Jr (1978). In total we estimate almost 100 quantiles, going through $q = 0.01, 0.02, \ldots, 0.99$ for every year t. We have k explanatory variables in the x vector.

3.2.2 *Step 2. Sampling with Replacement of Coefficients and Data*

Next we draw with replacement a vector of coefficients $(1 \times k)$ from the $(k \times Q)$ coefficient matrix, separately for every t. Every draw is denoted $\hat{\beta}_{tb}$, with $b = 1, \ldots, B$, where every quantile q from 0.01 through 0.99 is equally likely to be drawn. In this paper we set B to $200,000$. Note that $\hat{\beta}_{tb}$ now has dimensions $k \times B$ for every t. We also draw with replacement - again B times - observations $(1 \times k)$ from the $(N \times k)$ matrix of explanatory variables x_b^* across the N property transactions in the entire temporal sample (all periods). Note that x_b^* has no subscript t, as it should be *a representative property.* However, different values of x^* will result in slightly different index numbers during our final step (see de Haan and Diewert, 2011, for more details on this topic). As will be discussed in Section 5 we will use different values for x^* for robustness. As in the previous step, every observation vector is equally likely to be drawn. The new attribute (quantity) matrix, denoted $\hat{x}_b^*$, has dimensions $B \times k$.

Next we calculate $\hat{x}_b^*\hat{\beta}_{t=1,b}$, $\hat{x}_b^*\hat{\beta}_{t=2,b}$, $\ldots$,$\hat{x}_b^*\hat{\beta}_{t=T,b}$ for every b. These values represent the estimated values of p for every b. For every t, the dimensions are therefore $1 \times B$. If x^* represents the full population of buildings, vector $\hat{x}_b^*\hat{\beta}_{t,b}$ gives the full (predicted) price distribution of every single property in period t (including the ones that were not sold in that year). This distribution is denoted $\hat{f}_t(p)$. Note that in a perfect world we would have liked to 'simply' map all quantile coefficients to the entire stock of properties. However, the random sampling process will give us the same target distribution, without storing matrices with potentially millions of rows.[1]

3.2.3 *Step 3. The Quantile Indices*

Price point indexes can then be constructed, for any price point (defined as a quantile of the value distribution, from 0.01 through 0.99), by connecting the values over time of the same quantile in the value distributions of each period. The *full* change in price density is now simply given by the difference between two target price densities ($\Delta(p)$):

$$\hat{f}_{t+1}(p) - \hat{f}_t(p) = \hat{x}_b^*\hat{\beta}_{t+1,b} - \hat{x}_b^*\hat{\beta}_{t,b} \tag{3}$$

Note again, that the price changes are constant quality as we fix x^*. A graphical stylized example of a change in density ($\hat{f}_t(p)$) between two different periods (let's assume the periods 2005 and 2015) is given in Figure 3. The price densities in Figure 3 are simulated and prices are log transformed. The density distributions are for log values holding the distribution of attributes (hedonic quantities) constant, thus representing a "constant quality" price index (pure price changes). The indicated difference in the Figure would be the percentage change in the 50^{th} quantile (median) price-point index.

If we look at the median (or 50^{th} quantile) of the two simulated price densities in Figure 3, we see that that quantile shifted to the right. In 2005 the median (log) price was just under 11 (horizontal axis). In 2015 it was close to 13. Thus, in the median, prices increased: $\exp(13 - 11) - 1 \approx$ sixfold. Note that Figure 3 depicts normal distributions for illustration. But the estimated price densities found in our application can have any shape (i.e. we use simple histograms).

To summarize the price indexing in this paper, we use two approaches: standard repeat-sales indices of price buckets, and quantile price-point indices based on chained hedonic price models. The two approaches can be viewed as "triangulating" onto the price dynamics behavior of different price points in the US commercial properties in the RCA database. But it is also interesting from a methodological perspective to compare the two approaches.

[1] For robustness we re-ran the entire procedure multiple times. The estimated price distributions did not change notably.

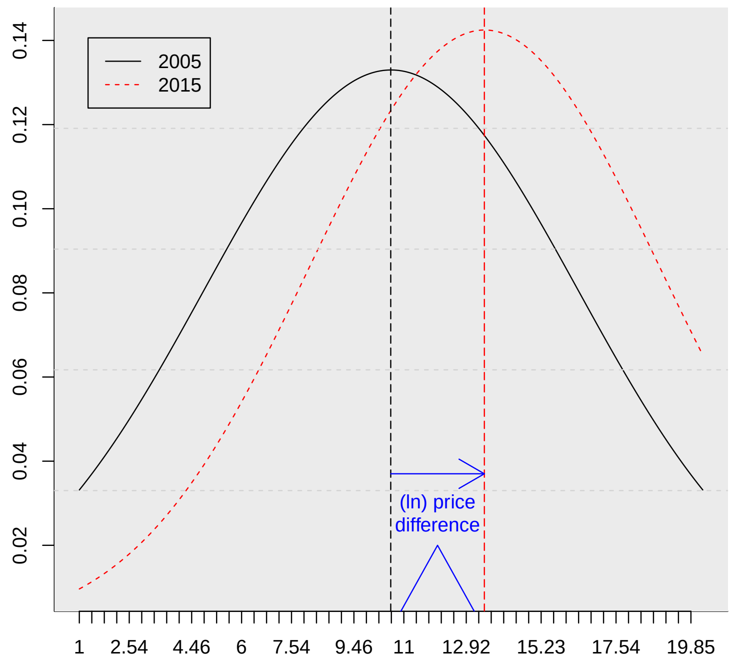


Figure 3: Example of a log price change of the 50^{th} quantile (median) between two periods. The x-axis gives the log values, and the y-axis the densities.

The last part of the analysis in this paper involves modeling total returns, including income. This is to enable us to draw some conclusions about the risk/return performance of the different price buckets or price points. To do this, we develop a cap rate model based on price points that correspond to those in the price dynamics analysis. By combining the estimated cap rates with the corresponding quantile price growth trends, we can relate total returns by price point to the investment risk characteristics by price point. Investment risk is largely determined by asset price evolution, as income returns contain very little volatility or cyclicality compared to price change returns (capital returns). Thus, we obtain the risk measures from the price indices, and income to compute total returns from the cap rate model.

4 Data and Descriptives

Real Capital Analytics. (RCA) provided the data to make this research possible. RCA is a data and analytics company that focuses particularly on the institutional investment market for commercial real estate in the United States and various international markets. RCA has been collecting, analyzing, and interpreting comprehensive commercial real estate transaction information within all investment-grade strata of United States commercial property asset markets since 2000.

Since 2005 RCA tracks all commercial properties sold for $\$2,500,000$ or above (whereas before 2005 the lower limit was $\$5,000,000$). RCA claims to have a capture rate of over 90% after 2005. The transaction database therefore gives a good representation of the entire population of major investment properties in the US, both residential and non-residential.

We are interested in the following characteristics; Size (units for hotels and apartments, square feet for other property types), land site (acres), age (years), property type (we distinguish between warehouses, industrial, full or limited service hotels, garden and mid/highrise apartments, CBD versus non-CBD offices, malls and strips) and metro dummies. For large metros (Los Angeles, New York and San Fransisco) we also define subregions, for a total of 30 (sub)metro dummies. Finally, we also take up a separate CBD dummy (which coexists next to the CBD-office dummy for property types) per region (6 in total) and a dummy which flags distressed sales. The exact definitions used by RCA can be found on the website of RCA.

In total we observe approximately $200,000$ transactions in the data after 2005, of which $178,000$ unique and $60,000$ repeat sales. As we are interested in the hedonics described above, we lose observations due to missing characteristics. After filtering the data for missing characteristics we end up with $110,000$ observations - of which $70,000$ unique sales (important for x_b^*) - or over $10,000$ per year on average. The loss of observations are mostly random and are shared equally by all years (i.e. almost no selection bias is introduced). The means and standard deviation of the sales price (the only variable that is always observed) before and after the filters, together with the results of an ANOVA are printed for a selection of years in Table 1. Table 1 shows that in most years (except 2015) the difference in mean is less than 1%. Only in year 2015 do we find a significant difference, but note that the difference is still small (5%).

Table 1: ANOVA of the data before and after filters: Log price

	means		**std.dev**		**Pr(>F)**
	before	*after*	*before*	*after*	
2005	15.733	15.733	1.044	0.995	0.983
2007	15.821	15.828	1.052	0.998	0.483
2010	15.602	15.609	1.070	1.042	0.656
2015	15.773	15.824	1.044	1.051	0.000

Table 2 gives some descriptive statistics of the data. Table C1 in Appendix C provides

Table 2: Descriptive statistics of the data

Year	*Mean*	*Std. Dev.*	*Min*	*Max*
	2005 – 2015 (111,001 Obs.)			
Log of sales price	15.856	0.910	9.328	20.492
Cap rate (%)	0.068	0.016	0.010	0.135
Log of # of units	4.595	0.944	1.386	7.822
Log of building area	10.884	1.088	6.477	14.809
Log of lot size (ac)	1.072	1.389	-2.303	3.689
Age	31.318	24.604	0	115
Distress dummy	0.078	0.268	0	1
CBD dummy	0.128	0.334	0	1

the same statistics, but for a selection of years (2005, 2007, 2010 and 2015). Interestingly, the years 2005, 2007 and 2015 give almost the same descriptives (except for the distress dummy). In 2010 (just after the crisis), prices are (as expected) lower compared to the other years. The average square feet (second to 2005) and the CBD dummy return the highest value in 2010. This could indicate a 'flight to quality' during the crisis years.

5 Results

5.1 Repeat Sales Model

Before we present the main findings of the quantile regression we first look at the results of a more traditional repeat-sales model, based on the Bailey et al. (1963) methodology. Repeat-sales indexes have the advantage of directly modeling investor (same property) experience. Repeat-sales indexes are based only on the price changes experienced between the "buy" and the "sell" of the same properties over time. This is exactly the type of price change that real estate investors actually experience, as investors must sell the same properties which they have previously bought. However, Repeat-sales indices can only be based on properties that have sold more than once, and this may further exacerbate the sample size problem and the censored sample bias problem noted in Section 3.1.

The standard repeat-sales model is given by:

$$p_{jt} - p_{js} = \mu_t - \mu_s + \epsilon_{jt} - \epsilon_{js} \tag{4}$$

where p are log transaction prices of pair j and μ is a dummy matrix for time of buy (s) and time of sale (t). The error is denoted ϵ which is assumed to be normally distributed with mean 0. The model can be estimated using OLS.

In order to build price segment indices using Eq. (4), we estimate the model on different subsamples of the data, based on price level cohort. First we take properties that were bought in 2005 (approximately 5,000 observations). We group those transactions into three equal sized (observation-wise) price buckets, from lowest to highest. We redo this procedure for every year in our data for a total of approximately 50,000 pairs. This way we circumvent possible correlation between time of sale and the placement in certain buckets (i.e. properties sold during the crisis would all be in the lowest bucket and vice versa).

Next, we trace each property's subsequent sales and create a repeat-sales index for the entire period, one for each price "bucket". In such an index, properties cannot 'jump' to another quantile.

The resulting price indices are shown in Figure 4. Some descriptive statistics on the returns of these indices can be found in Table 3 in Section 5.2. It is evident from both Figure 4 and Table 3 that the index representing low valued properties had less volatility and higher overall price growth than the high price point properties between 2005 and 2015. However, the middle price bucket index does not seem to be consistent with this pattern. Nevertheless, comparing the high and low price points, there is clearly greater risk yet lower return (at least in terms of price growth) in the high price point properties.

Figure 4: Repeat sales model of properties in three price-buckets

As mentioned earlier, this methodology has several drawbacks. Firstly it does not give the full distribution of prices, as subdividing the data into (infinite more) price buckets will decrease the number of observations (per bucket), and therefore make the indices more noisy Francke (2010). Secondly, the buckets are not truly constant quality, as the 'top' bucket in one year might consist of different types of properties compared to another year. Finally, the indices suffer from the censored sample bias stylized in Figure 2.

5.2 Quantile Regression

To calculate the Machado and Mata (2005) decomposition we first estimate the almost 100 quantile regressions for all 10 years, see step (1) in Section 3. For readability we focus on the years 2005, 2007, 2010 and 2015 as they represent the main cycle.

Several variables exhibit significant quantile effects over the analyzed years. By "quantile effects", we mean that the hedonic coefficients, the pure "price" elements (holding quantity and quality of the properties constant), vary across the quantiles (price points), in possibly different ways over time. A few examples are given in Figure A.1 in the Appendix A. For example, the extra price premium purely for being located in the CBD (the shadow price of CBD location) is greater for higher price-point properties than for lower price-point properties, but this quantile effect shifted slightly over time and was greatest in 2015 and least in 2010.

We also estimate the goodness of fit measure developed by Koenker and Machado (1999). The measure itself and the results are given in Appendix B. The R^1 reveals that the goodness of fit increases for high quantiles. The goodness of fit is sufficiently high and stable for quantiles above approximately the 25th quantile. The explanatory power (for the selected years) is lowest for 2010 and equally as good for 2005 and 2007.

Regarding the quantile effects on the prices, Figure A.1 shows that an additional unit or square foot of structure size (both log transformed) adds much more to the price at high quantiles for all the years. The CBD dummy and the dummy for strip retail also shows a strong quantile effect. In some cases we see quantile effects disappear. For example, in 2005 there is still a quantile effect in Washington DC. However, this effect is no longer observed in any of the other years.

Next we draw with replacement from both the quantile coefficients and the data, as described in step (2) in Section 3. In total we use five different samples from which we draw with replacement from the total data, in order to construct $\hat{x}_b^*$. For the main model we draw with replacement from all the properties that were (uniquely) sold in the RCA data, from which we have data for all characteristics. This will essentially give us a representative (i.e. average) property for the RCA database of 2005-15. We denote this as the 'quasi

Fisher' index.[2] In the 'Laspeyeres' ('Paassche') index we only draw with replacement from the properties that were sold in 2005 (2015). The chained returns - step (3) - are indexed in Figure 5 for a selection of quantiles. Some statistics on the returns are given in Table 3.

Table 3: Return statistics for the different quantiles (q).

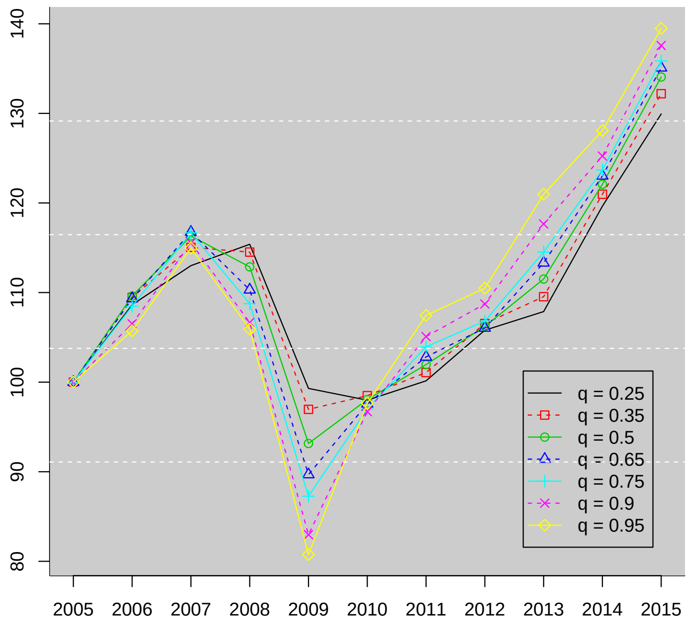
				Panel A: quasi Fisher Index			
quantile	**mean**	**mean***	**sd**	**|mean|**	**ACF1**	**ACF2**	**Crisis**
q = 0.25	0.026	0.024	0.072	0.059	0.271	-0.027	-0.152
q = 0.35	0.028	0.025	0.076	0.062	0.273	-0.105	-0.158
q = 0.50	0.029	0.026	0.086	0.074	0.219	-0.195	-0.200
q = 0.65	0.030	0.026	0.095	0.083	0.174	-0.262	-0.232
q = 0.75	0.031	0.026	0.101	0.089	0.136	-0.317	-0.253
q = 0.90	0.032	0.025	0.115	0.097	0.017	-0.307	-0.279
q = 0.95	0.033	0.025	0.125	0.102	-0.050	-0.301	-0.293
				Panel B: Laspeyeres Index			
quantile	**mean**	**mean***	**sd**	**|mean|**	**ACF1**	**ACF2**	**Crisis**
q = 0.25	0.026	0.023	0.072	0.058	0.297	-0.098	-0.149
q = 0.35	0.027	0.024	0.078	0.067	0.246	-0.171	-0.181
q = 0.50	0.029	0.025	0.088	0.078	0.184	-0.255	-0.219
q = 0.65	0.030	0.025	0.100	0.088	0.097	-0.297	-0.250
q = 0.75	0.031	0.025	0.106	0.092	0.058	-0.302	-0.263
q = 0.90	0.032	0.025	0.120	0.099	-0.046	-0.285	-0.287
q = 0.95	0.034	0.025	0.131	0.105	-0.082	-0.286	-0.301
				Panel C: Paassche Index			
quantile	**mean**	**mean***	**sd**	**|mean|**	**ACF1**	**ACF2**	**Crisis**
q = 0.25	0.026	0.024	0.070	0.059	0.317	-0.040	-0.152
q = 0.35	0.027	0.025	0.075	0.061	0.302	-0.115	-0.154
q = 0.50	0.028	0.025	0.085	0.073	0.224	-0.205	-0.201
q = 0.65	0.029	0.025	0.093	0.082	0.187	-0.280	-0.234
q = 0.75	0.029	0.024	0.101	0.088	0.146	-0.317	-0.255
q = 0.90	0.031	0.024	0.118	0.100	0.049	-0.343	-0.291
q = 0.95	0.033	0.025	0.124	0.104	0.010	-0.340	-0.298
				Panel D: Repeat Sales Index			
bucket	**mean**	**mean***	**sd**	**|mean|**	**ACF1**	**ACF2**	**Crisis**
Lowest	0.002	-0.007	0.133	0.105	0.556	0.043	-0.401
Middle	-0.014	-0.032	0.190	0.135	0.528	0.015	-0.525
Highest	-0.014	-0.031	0.188	0.114	0.272	-0.158	-0.472

*mean** is the Geometric mean, in contrast to the arithmetic *mean*. *sd* stands for the standard deviation of the returns, and *ACF1* and *ACF2* is the serial correlation with either 1 or 2 lags respectively. The crisis range is the % difference between the min of the max (of the index) for the periods 2006 and 2010.

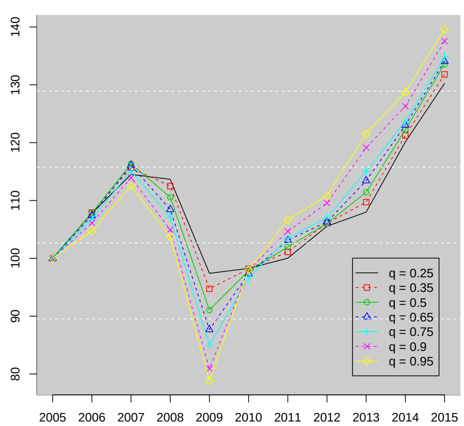
From both the graphical depiction of the indices and from the statistics it is evident that

[2] With a 'real' Fisher index one takes the average between the end (2015) and the beginning (2005) of the characteristics of the good being tracked. Whereas we take the average of the *entire* market.

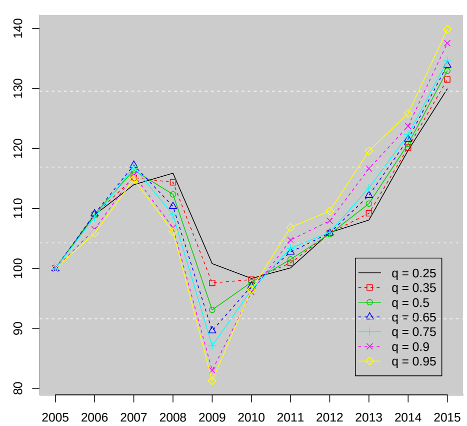
Figure 5: Quantile price indices US, 2005 – 2015



(a) quasi Fisher.



(b) Laspeyeres.



(c) Paassche.

the higher price point indices are more volatile (approximately 70% more) than the lower quantile indices. They also exhibit correspondingly greater down-cycle amplitude. The high price-point indices also display greater overall average return (price growth), but only slightly so. The difference in geometric mean returns (mean*) is less than 20 bps between the high price point and low price point indices.

Finding autocorrelation in real estate prices is not uncommon, as private property markets are not perfectly informationally efficient (Barkham and Geltner, 1995; Case and Shiller, 1989; Quan and Quigley, 1991). However, our findings suggest that there is more autocorrelation (and thus predictability) in lower quantiles. Though not reported here, we also found evidence that higher quantiles Granger-cause lower quantiles (that is, they lead them or "predict" them in time). This can be seen visually in Figure 5. Indeed, the lower quantiles go down (up) one year later than the upper quantiles at the beginning (end) of the crisis. Greater predictability means that the volatility or cyclicality is less of a concern to investors. Predictable price movements are arguably not "risk", but rather, present investors with opportunities (to "buy low and sell high").

5.3 Total Returns and Treynor Ratios

The findings reported above reveal interestingly different price dynamics by price-point. Back in Section 2 we described several reasons and sources for such differential price dynamics. For example, lower price growth trends for the lower price point properties, as well as lower volatility and cyclicality, could be caused by greater supply elasticity in the space market. Different price dynamics suggests some degree of market segmentation by price point. But as noted in Section 2, market segmentation may, or may not, be associated with differential risk pricing, that is, a different risk/return relationship which might possibly suggest the existence of capital flow barriers and some potentiality for arbitrage across the price points. This is the question we turn to next.

In order to study the risk pricing question we must quantify not just price change dynamics, but differences in the average total returns, including income, across the price points. We can do this by combining information about cap rates by price point with our previous information on price dynamics by price point.

The first step in this process is to estimate a (quantile) cap rate model (and the price model), in a seemingly unrelated regression framework. Thus, we regress cap rates (in %, not log transformed) onto the same variables that we previously used in the price model. As before, we draw with replacement from the coefficients of the cap rate model, and multiply them times the *previously* drawn matrix x^*. Thus for every property in x^* we get both the price density for every t, but also now a density of the cap rates. The big difference is that

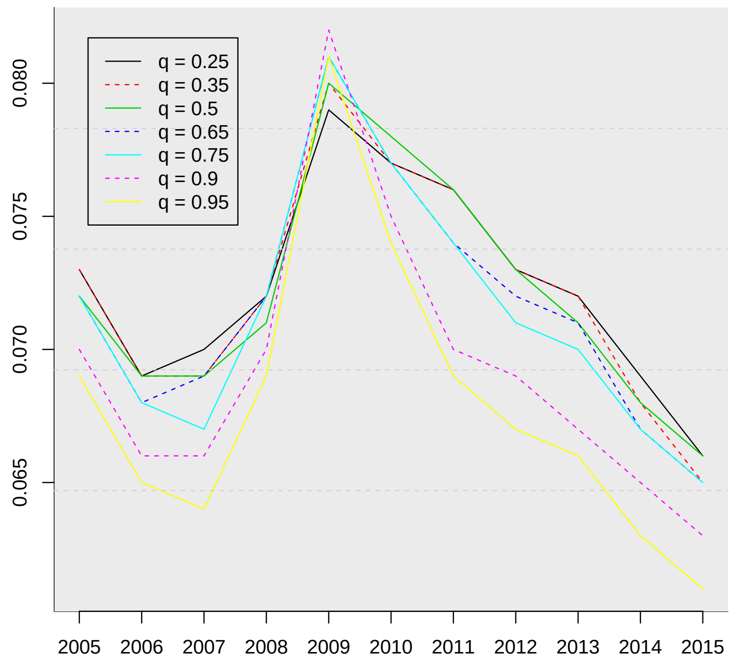


Figure 6: Average cap rates per price quantile, quasi Fisher model

now we are not so much interested in the change in density of the cap rates. We are simply using the cap rate density distributions to identify the typical cap rates for certain *price points*. The purpose is to model total returns by price point, to relate to the investment risk implications by price point that we found in the differential price dynamics of Section 5.2.

As noted previously, and as is obvious in Figure 6, income returns (essentially similar to cap rates) do not exhibit nearly as much volatility as capital returns (price changes). The cap rates in Figure 6 range only over a couple hundred basis-points even across the Financial Crisis. In contrast, the price changes in Figure 5 show over ten times as much percentage variation. Thus, almost all of the risk in real estate returns is reflected in the asset price dynamics, effectively, the capital returns, not in the income returns. (This is characteristic not only of real estate but other asset classes as well, for example, stock dividends move only slightly as a fraction of the stock price, while the stock price itself moves dramatically more.)

Figure 6 shows our estimates of the *average* RCA cap rates over time for the price quantiles used in Section 5.2. Recall that with the quantile model we are estimating a *distribution* of cap rates around different price points, not just a single cap rate. (For simplicity we show the quasi Fisher results, but other definitions of the representative property give similar results.)

We see in Figure 6 that cap rates have been between 6% and 8.5%. And it is clear that higher price point properties have provided lower cap rates. Higher price point cap rates are also more volatile (which is simply a reflection of the greater price volatility we already noted in Section 5.2). It is interesting that during the crisis cap rates more or less coincided over all price points. Other than that, the cap rates show a familiar pattern: a mirror image of the price indices. However, the lead-lag relationship in the quantile price indices seem to absent in the cap rates.

On average the difference between the cap rates associated with the 95$^{\text{th}}$ percentile price point properties was 44 basis points lower compared to its 25$^{\text{th}}$ percentile equivalent. As noted before, the average difference in return was about 15 basis points. Thus in total, the return was not higher for higher price point properties in our analyzed period, even though the high price point displayed greater investment risk.

To quantify this point more explicitly, we calculate the Treynor Ratios for the different quantiles. The Treynor ratio (T) is given by:

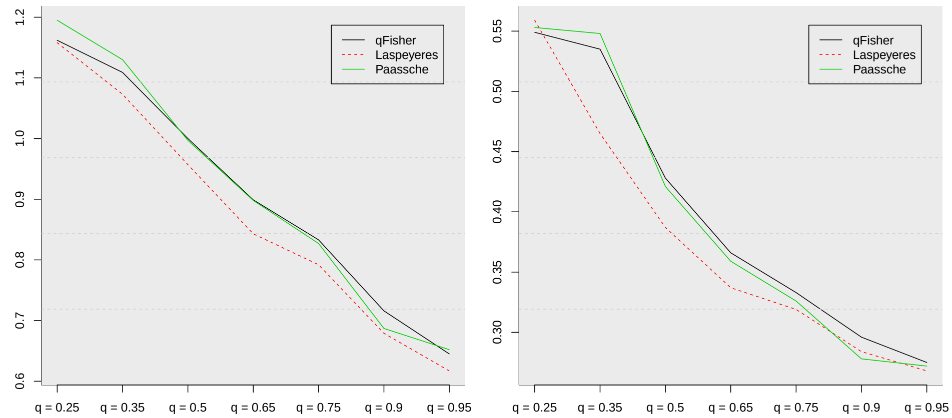
$$T_q = \frac{R_{tot,q} - R_f}{Risk_q} \tag{5}$$

where R_{tot} is the total return (cap rate plus index returns) for quantile q and R_f is the riskfree rate. We used the average 30-day Treasury Bill rate during our analyzed period as the riskfree rate, which was 1.3%. The exact choice of the riskfree rate will not change the ordering of Treynor Ratios as it is constant across the quantiles.

The risk in the denominator of the ratio can be quantified in different ways. Keep in mind that we simply need to quantify risk in a manner that allows a risk adjusted comparison of the returns across real estate price points. We use two alternative metrics: (i) The price index quantile *volatility*, and (ii) The price index quantile *crash magnitude* (% trough/peak). We report the Treynor Ratio based on both risk measures, and for the different definitions of the representative property. Thus, the Treynor Ratio measures risk premium per unit of risk. A higher Treynor Ratio implies a higher price of risk, that the investor gets a "better deal" for taking on risk, at least insofar as the risk that matters is proportional to the risk metric in the denominator of the Treynor Ratio.

A graphical representation of the Treynor ratios as a function of the price points is given in Figure 7. The relationship is strong and nearly linear. The Treynor ratio for the lowest analyzed price point is double that of the Treynor ratio of the highest price point. The results are robust to the two different risk metrics.

Figure 7: Treynor ratios.



(a) Risk measured as volatility. (b) Risk measured by crash magnitude.

5.4 Results from Granular Indices

The above results suggest that real estate markets are not fully integrated across price points. They suggest that there could be capital flow barriers and that arbitrage opportunities could be obtained if any such barriers could be pierced. However, before we draw such a strong conclusion, other possibilities should be considered. We will consider alternative explanations in Section 5.5 below. But a relevant empirical issue can be pursued further here. What if what we think are price point segments are in fact reflecting other types of market segmentation? The analysis in this paper so far has been at a national aggregate level for all property types and locations (albeit with hedonic dummy variables and re-estimation every period, which does provide considerable control).

But suppose the higher price points are dominated by offices (with higher volatility) and major markets, while lower price points are characterized by NNN and owner occupied properties. While that would not change the above finding of differential risk pricing, it would suggest that the cause might not simply be differential price points. And this might cast a different light on the nature of the capital flow barriers or differences in investor clientèles across the price points.

To test if this (or anything similar) is the case, we re-estimate our Treynor Ratio analysis based on more granular indices. As we noted in Section 3.2, increasing the granularity of the quantile indices also helps to address a theoretical issue with these indices, which is the fact that they are not literally "same property" indices.

With this in mind, we replicate the previously described analysis separately for a set of indices that are based on more homogeneous stratified sub-samples of the overall RCA national sample. Thus, we only sample with replacement from the corresponding market/-sector, such as office, or New York. We construct the relevant x^* representative properties in this manner for both the prices and cap rates. Within these more granular and homogeneous indices, the quantile indices will more purely represent the effect of price points. They will also more closely approximate investor experience same-property indices.

To conserve space we only show the difference between the highest and lowest price-point quantiles' Treynor ratios ($T_{q=0.25} - T_{q=0.95}$), based on the 'quasi' Fisher chaining technique. The resulting Treynor Ratios according to both risk measures are given in Figure 8.

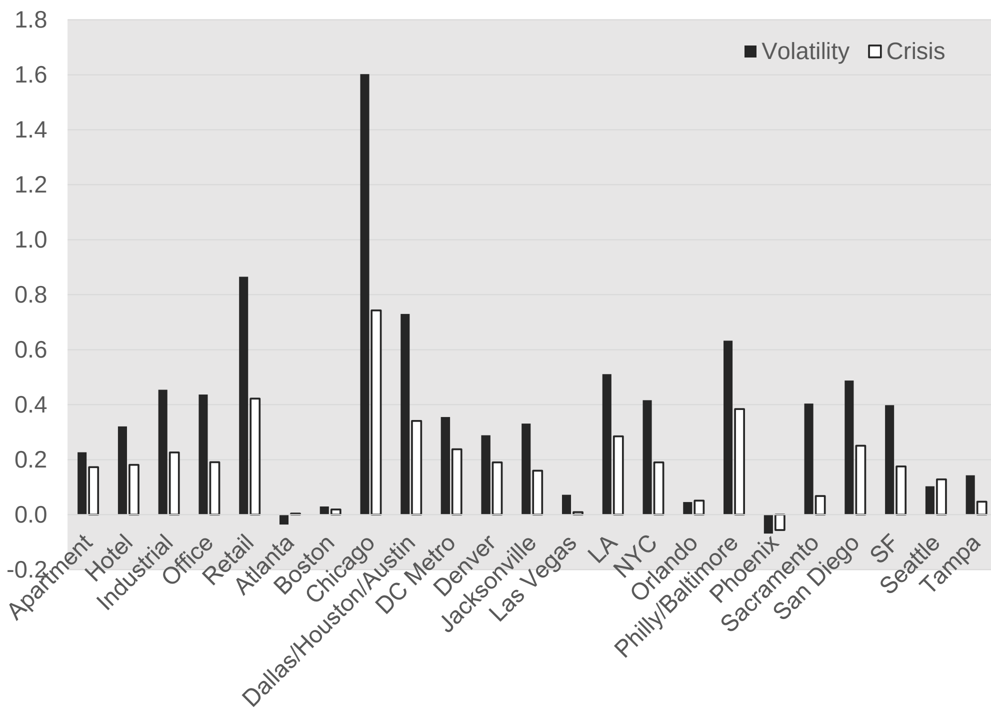


Figure 8: Difference in Treynor ratios ($T_{q=0.25} - T_{q=0.95}$) for different markets and sectors in the US, using both measures for risk. Positive number indicates better risk/return for low price point properties within that sector/market and vice verse.

A positive value in the Figure means that the apparent risk/return pricing differential in favor of the low price points exists in the given market / sector. While it must be born in mind that some of these granular indices are based on small samples, the overall result is again strong and clear. The favorable risk/return pricing for low price points holds for all sectors and for 13 of the 18 metro markets for which we have enough data.

Interestingly, this holds even though the overall level of the price points varies considerably across the sectors and metros. For example, the median ($q = 0.50$) price point in New York is much higher than the median price point in Phoenix, in dollar terms. This applies to any quantile, and also to the difference between the highest and lowest quantiles in dollar terms. We note that where the 'spread' in price points is larger in dollar terms, there is a tendency for that market to show a greater difference in the high-minus-low price point Treynor ratios. Thus for example the relatively small difference in Treynor Ratios for apartments in part reflects relatively little spread in apartment price points in dollar terms.

5.5 Alternative Explanations for our Findings

Back in Section 2 we suggested that different price dynamics across price points do not necessarily imply a violation of the Law of One Price regarding the risk/return relationship. We pointed out that even if different price points exhibit different risk, it is possible for asset pricing to reflect this, such that the same price of risk (expected return risk premium per unit of risk prevails across the price points, i.e. same Treynor Ratio). But the results described in the previous two sections have found apparent violation of the Law of One Price, evidenced by higher Treynor Ratios associated with lower price points. In this case, the suggestion in Section 2 was that barriers likely exist to capital flowing between price points, and that arbitrage type opportunities await the investors fortunate enough to pierce the barriers.

While this could be a reasonable conclusion from our analysis, we should briefly point out a couple of possible alternative explanations, at least in principle, that could explain our findings. In the framework presented in Section 2, these alternative explanations could be viewed as suggesting that the different price points effectively represent different "goods", although the nature of the difference has less to do with space market differences and more to do with particular challenges in the asset market for investment in the lower price point properties.

First, there could be differences in the type and amount and quality of information about the property assets. Perhaps larger properties have better information, less uncertainty about the facts and characteristics and considerations that affect their value at the individual property level. Indeed, our finding noted at the end of Section 5.2 that the lowest price point index is predicted by the higher price point indices suggests that higher price point properties reflect relevant information sooner than the bottom price point properties. (While predictability can provide opportunities for investors in the aggregate, if it reflects greater uncertainty at the individual property level then that poses a challenge for investors who must buy individual properties, not whole indices.) Investors are very averse to uncertainty, more so perhaps than they are averse to risk. (Risk is quantifiable future dispersion in possible outcomes, "known unknowns"; uncertainty is non-quantifiable future dispersion, "unknown unknowns".) If information quality is correlated with asset price point, then this could lead to different equilibrium risk/return pricing, even without implying capital flow barriers or any sort of arbitrage between price points. In effect, due to informational differences, large and small properties would in an important sense be "different goods" (which could therefor sell at different prices).

A second possibility is that search and transaction costs could be greater, per dollar invested, in the low price point properties. To some extent this explanation overlaps with the previous. Perhaps with sufficient time and investment, potential buyers could unearth as much good information about small properties as is available about bigger properties. But the necessary additional search and transaction costs could wipe out the apparent arbitrage.

One might view these alternative explanations as, indeed, a type or cause of the "capital flow barriers" that we have said can allow differential risk pricing. But unless and until such issues can be mitigated, they prevent high price point investors from being able to take advantage of the more favorable risk/return pricing in the lower price points. Another way of looking at it is that information and search/transaction challenges with smaller properties may eliminate any risk/return bargain. If the denominator in our Treynor Ratios could reflect these other dimensions of investment "disutility", then the Treynor Ratios might equalize across the price points.

Finally, there is a third possible alternative explanation that any empirical study of risk and return must not neglect to note. The time period measured could have some bias – starting somewhere around the last peak but not getting to the next peak yet. Maybe there is yet further upside in the future that the market figures will benefit the larger properties.

Unfortunately, we do not presently have the ability to test rigorously for these alternative explanations to the price point risk/return puzzle that this paper has documented. We must leave such a quest for future research.

6 Concluding Remarks

Conventional real estate price indices provide a single measure for the path of asset prices over time (controlling for the quality of the representative or average property). But it could be that properties have different price dynamics based on the price segment they are traded in. On the demand side, investors at different price points are differentiated by the amount of capital the investor has at their disposal and the type and source of financing. Smaller, private investors cluster at lower price points, while large institutions dominate the high price points. On the supply side, properties at different price points may serve different space markets with different types of tenants, and may reflect different supply elasticity and land/structure value ratios. This paper uses an unconventional approach, quantile regression, to estimate price indices for different price segments in commercial real estate. We make our analysis more robust by also analyzing a traditional repeat-sales index, and by developing quantile indices on more homogeous, granular populations of properties.

Our results show that there are indeed large differences in price dynamics for different price points. On the risk side, we find clear evidence that high price-point properties are more risky. The quantile indices show almost *twice* the volatility and twice the downturn magnitude compared to low price-point properties.

At first sight, it might seem that high price point properties are at least partially compensated for their greater risk by a higher providing about 15 bps more growth, as our indices show. However, the cap rate of the high price point properties is 44 bps lower on average. Hence, the average total return of high price point properties is approximately 30 bps *less*, in spite of the greater risk. In addition, lower price point properties show greater momentum and hence, predictability, which should make investment in them more appealing.

This would seem to reveal a violation of Law of One Price, a type of arbitrage opportunity in the investment property market. This should be possible only if there are significant barriers to capital flow across the price points. It is possible that such barriers to exist, and that bargains await investors who can break through the barriers.

On the other hand, there could be mitigating factors, such as poorer information quality (greater uncertainty), and higher search and transaction costs, associated with smaller properties in secondary locations. And our analysis is based on only 10 years of data. The history covers the financial crisis, but it is not clear if it encompasses a complete cycle, much less the multiple cycles that would be necessary to give great confidence to empirical results. It is thus recommended to redo this study in the future.

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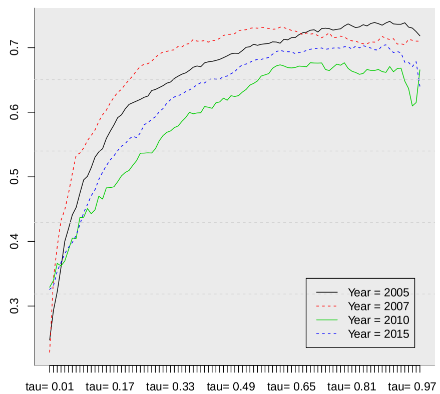
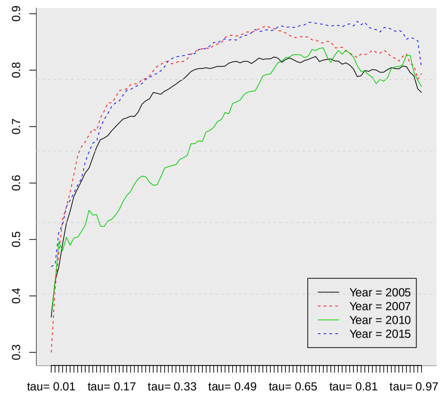
A Quantile Results

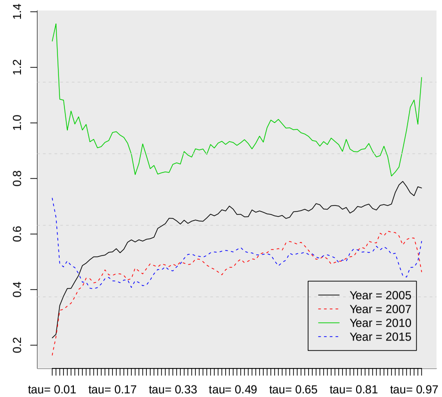
Figure A.1: Coefficient estimates by quantile



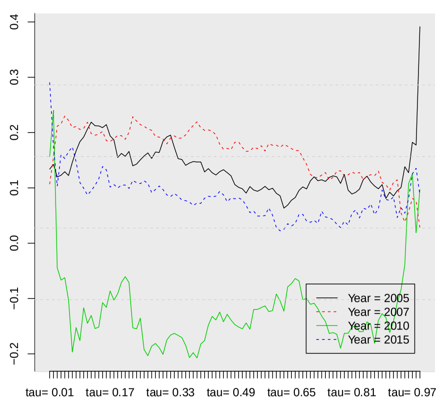
(a) Log of structure size.



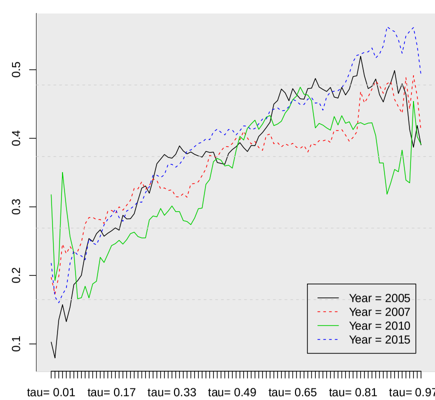
(b) Log of # of units.



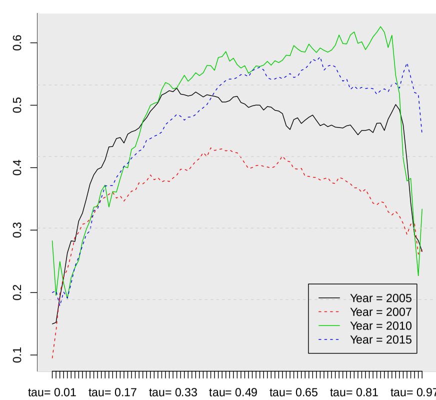
(c) Washington, DC.



(d) Phoenix.



(e) CBD dummy



(f) Strip retail

B Goodness-of-fit for the Quantile Regression

The Koenker and Machado (1999) measure of fit (R^1) is given in Figure B.1. The R^1 is similar to the more conventional R^2 and is calculated for every quantile (q) by estimating a restricted model in parallel to the full model described in Section 3.2. The restricted model is simply the same quantile regression, but with only a constant as explanatory variable (α). The R^1 is calculated as follows (Eq. (6)):

$$R^1(q) = 1 - \frac{\sum \hat{p}_{qit} - x_{it}\hat{\beta}_{qt}}{\sum \bar{p}_{qit} - \bar{\alpha}_{qt}} \tag{6}$$

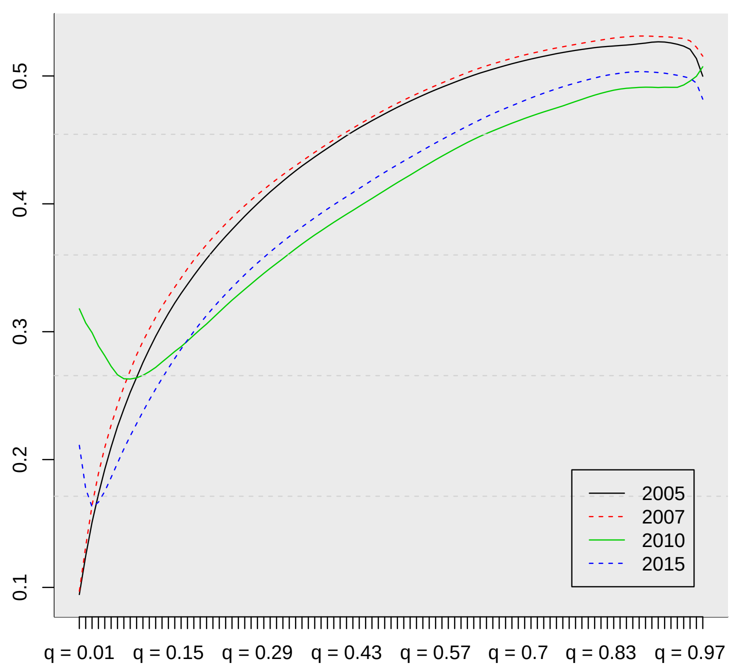


Figure B.1: Goodness-of-fit of the Quantile regressions, R^1

C Descriptive Statistics per Year

Table C1: Descriptive statistics for a selection of years

	Mean	*Std. Dev.*	*Min*	*Max*
Year	**2005 (11,859 Obs.)**			
Log of sales price	15.907	0.890	14.039	19.519
Cap rate (%)	0.069	0.016	0.020	0.135
Log of # of units	4.681	0.945	1.946	7.822
Log of building area	11.019	1.040	7.148	14.557
Log of lot size	1.143	1.364	-2.303	3.689
Age (years)	27.349	22.356	0	105
Distress dummy	0.010	0.085	0	1
CBD dummy	0.117	0.321	0	1
Year	**2007 (11,615 Obs.)**			
Log of sales price	15.937	0.884	12.995	19.467
Cap rate (%)	0.066	0.015	0.020	0.134
Log of # of units	4.676	0.892	1.386	7.256
Log of building area	10.896	1.063	7.116	13.911
Log of lot size	1.112	1.364	-2.303	3.689
Age (years)	28.714	23.128	0	107
Distress dummy	0.119	0.324	0	1
CBD dummy	0.119	0.324	0	1
Year	**2010 (4,835 Obs.)**			
Log of sales price	15.743	0.930	11.918	19.592
Cap rate (%)	0.075	0.016	0.018	0.135
Log of # of units	4.634	0.946	1.792	7.601
Log of building area	10.991	1.087	7.480	14.078
Log of lot size	1.104	1.407	-2.303	3.689
Age (years)	30.263	24.591	0	110
Distress dummy	0.225	0.418	0	1
CBD dummy	0.139	0.346	0	1
Year	**2015 (16,191 Obs.)**			
Log of sales price	15.937	0.932	11.513	19.698
Cap rate (%)	0.064	0.017	0.010	0.134
Log of # of units	4.518	0.972	1.609	7.005
Log of building area	10.798	1.118	6.908	14.241
Log of lot size	1.017	1.395	-2.303	3.689
Age (years)	34.393	25.052	0	115
Distress dummy	0.051	0.220	0	1
CBD dummy	0.125	0.331	0	1